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# The graded Witt ring of a quasi-pythagorean field

Dedicated to the memory of Professor Akira Hattori

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Let $F$ be a field of characteristic different from 2, $\dot{F}$ be the multiplicative group $F\setminus\{0\}$, and $WF$ be the Witt ring of quadratic forms over $F$. We denote by $IF$ the ideal of even dimensional forms in $WF$, by $I^nF$ its $n$-th power and by $\langle a_1,\dots, a_n\rangle$ the diagonalized form $a_1x_1^2+\cdots+a_nx_n^2$ for $a_1,\dots, a_n\in\dot{F}$. We also denote by $D_F\langle a_1,\dots, a_n\rangle$ the set of elements of $\dot{F}$ represented by $\langle a_1,\dots, a_n\rangle$ and we put $D_F(n)=D_F\langle 1,\dots, 1\rangle$ ($n$ terms), $D_F(\infty)=\bigcup_{n=1}^{\infty} D_F(n)$.

A field $F$ is called formally real if $-1\notin D_F(\infty)$, pythagorean if $D_F(2)=\dot{F}^2$, and quasi-pythagorean if $D_F(2)=R(F)$, where $R(F)$ denotes Kaplansky's radical $\{a\in\dot{F}\mid D_F\langle 1, -a\rangle=\dot{F}\}$.

For a pythagorean field $F$, the structure of $WF$ and especially the relations of the graded Witt ring $GWF=\oplus_{n=0}^{\infty} I^nF/I^{n+1}F$ to the rings $k_*F$ and $H^*(F, 2)$ have been studied in [4] and [6].

We shall study in this paper the same subject for a quasi-pythagorean field to obtain Theorem 1.5 and Proposition 2.2 below, with some additional results.

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## §1. $GWF$ and $k_*F$

First we recall the definition of Milnor's $K$-ring $K_*F$ for any field $F$. Let $K_1F$ be an additive group with a fixed isomorphism $l$: $\dot{F}\to K_1F$, and $T(K_1F)$ be the tensor algebra on $K_1F$ over the ring $\boldsymbol{Z}$ of integers. $K_*F=\boldsymbol{Z}\oplus K_1F\oplus K_2F\oplus\cdots$ is defined to be $T(K_1F)/I$, where $I$ is the two-sided ideal of $T(K_1F)$ generated by $\{l(a)\otimes l(1-a)\mid a\in\dot{F}, a\neq 1\}$.

We denote by $l(a_1)\cdots l(a_n)$ the image of $l(a_1)\otimes\cdots\otimes l(a_n)$ in $K_nF$. Then the basic properties of $K^*F$ are as follows.

PROPOSITION 1.1 ([13]).
(1) $\eta\xi=(-1)^{mn}\xi\eta$ *for every* $\xi\in K_m$, $\eta\in K_n$.
(2) $l(a)l(-a)=0$ *in* $K_2F$ *for every* $a\in\dot{F}$.
(3) $l(a)^2=l(a)l(-1)$ *in* $K_2F$ *for every* $a\in\dot{F}$.

$k_*F$ is defined to be $K_*F/2K_*F$ and is a commutative graded algebra over the field $Z/2Z$ by Proposition 1.1 (1). The group $k_1F$ is isomorphic to $\dot{F}/\dot{F}^2$. We shall write, by abuse of notation, $l(a_1)\cdots l(a_n)$ for $l(a_1)\cdots l(a_n) \bmod 2K^*F$ and $a$ for $a\dot{F}^2$, so that the isomorphism is expressed by $l(a)\leftrightarrow a$.

LEMMA 1.2. *$k_*F$ is isomorphic to the factor ring $T(k_1F)/J$ of the tensor algebra $T(k_1F)$ on $k_1F$ over $Z/2Z$, by the ideal $J$ generated by $\{l(a)\otimes l(b) \mid a, b\in \dot{F}, D_F\langle a, b\rangle \ni 1\}$.*

PROOF. If we write $T$ for $T(K_1F)$, then

$$k_*F = (T/I)/2(T/I) \cong T/(2T+I).$$

So $k_*F\cong(T/2T)/(2T+I/2T)$, and $T/2T$ is the tensor algebra $T(k_1F)$ on $k_1F$ over $Z/2Z$. Then it suffices to show that the image $J'$ of $2T+I$ in $T(k_1F)$ is equal to $J$. The inclusion $J'\subseteq J$ is obvious. On the other hand, if $1\in D_F\langle a, b\rangle$, then $ax^2+by^2=1$ for some $x, y\in F$. In case $x, y\in \dot{F}$, we have $l(ax^2)\otimes l(by^2)\in I$, that is, $\{l(a)+2l(x)\}\otimes\{l(b)+2l(y)\}\in I$, which implies $l(a)\otimes l(b)\in J'$. In case one of $x$ and $y$ (say $y$) is zero, we have $l(ax^2)=0$ in $K_1F$ and $l(a)\in 2T$. So $l(a)\otimes l(b)\in J'$. Q. E. D.

Lemma 1.2 is stated in another way, as follows.

LEMMA 1.3. *For $a_{ij}\in \dot{F}$ $(i=1,\dots, r; j=1,\dots, n)$, $\xi=\sum_{i=1}^{r} l(a_{i1})\cdots l(a_{in})$ is equal to zero in $k_nF$ if and only if there exist $b_{pq}\in \dot{F}$ $(p=1,\dots, s;\ q=1,\dots, n)$ such that $\sum_{i=1}^{r} l(a_{i1})\otimes\cdots\otimes l(a_{in})=\sum_{p=1}^{s} l(b_{p1})\otimes\cdots\otimes l(b_{pn})$ in $T(k_1F)$ and such that, for each $p$, $D_F\langle b_{p\nu(p)}, b_{p,\nu(p)+1}\rangle \ni 1$ for some $\nu(p)$ $(1\le \nu(p)\le n-1)$.*

A homomorphism $s_1\colon k_1F\to IF/I^2F$ is defined by $s_1(l(a))=\langle 1, -a\rangle$ in $WF$ for $a\in \dot{F}$. If $D_F\langle a, b\rangle\ni 1$, then $\langle 1, -a\rangle\langle 1, -b\rangle=0$ in $WF$. So $s_1$ induces a homomorphism $s_*\colon K_*F\to GWF$ of graded algebras. Since the $n$-fold Pfister forms $\langle\langle a_1,\dots, a_n\rangle\rangle=\langle 1, a_1\rangle\cdots\langle 1, a_n\rangle$ $(a_i\in \dot{F})$ generate $I^nF$ as an additive group, $s_n$ is surjective for any $n$. It was proved in [13] that $s_1$, $s_2$ are isomorphisms for any field $F$.

For a formally real field $F$, we denote by $X_F$ the set of all orderings on $F$. Then $(X_F, \dot{F}/D_F(\infty))$ is a space of orderings in the sense of M. Marshall [9], [11]. In a space of orderings $(X, G)$, $H(a)=\{\sigma\in X \mid \sigma(a)=1\}$ for $a\in G$ are open and closed, and constitute a subbasis for the topology on $X$. We define the chain length of $X$, denoted by $cl(X)$, to be the supremum of the set of integers $k$ for which there exists a chain

$$H(a_0)\subset H(a_1)\subset\cdots\subset H(a_k)$$

of length $k$ in $X$.

Two spaces of orderings $(X, G)$ and $(X', G')$ are said to be equivalent, and denoted by $(X, G)\sim(X', G')$, if there exists a group isomorphism $\alpha: G\to G'$ such that the dual isomorphism $\alpha^*: \mathrm{Hom}(G', \{\pm 1\})\to\mathrm{Hom}(G, \{\pm 1\})$ carries $X'$ onto $X$.

PROPOSITION 1.4 ([3], [11]). *Suppose $(X, G)$ is a space of orderings with $cl(X)<\infty$. Then there exists a pythagorean field $K$ such that $(X, G)\sim(X_K, \dot K/\dot K^2)$.*

For a pythagorean field $K$ with $cl(X_K)<\infty$, B. Jacob [6] proved that $s_*$: $k_*F\to GWF$ is an isomorphism. We shall generalize this as follows.

THEOREM 1.5. *Let $F$ be a formally real, quasi-pythagorean field with $cl(X_F)<\infty$. Then $s_*: k_*F\to GWF$ is an isomorphism.*

PROOF. By Proposition 1.4, there exists a pythagorean field $K$ such that $(X_F, \dot F/R(F))\sim(X_K, \dot K/\dot K^2)$, since $D_F(\infty)=R(F)$ for any quasi-pythagorean field $F$ ([7], Lemma 2.2). Then we have $\dot F/R(F)\cong\dot K/\dot K^2$. We denote the isomorphism by $aR(F)\mapsto a'\dot K^2$. Composing it with the natural homomorphism $\dot F/\dot F^2\to\dot F/R(F)$, we obtain a homomorphism $\psi: \dot F/\dot F^2\to\dot K/\dot K^2$ which we identify with a homomorphism $\varphi_1: k_1F\to k_1K$. Now $X_F\sim X_K$ also implies $WF/W_tF\cong WK$ by [10], Theorem (2.6), where $W_tF=\{\langle 1, -a\rangle \mid a\in R(F)\}$ is the nilradical of $WF$ ([7], Proposition 2.3). So $\psi$ induces an isomorphism $\psi_n: I^nF/I^{n+1}F\to I^nK/I^{n+1}K$ for any $n\geq 2$. It is clear that, for any field $L$ and $x, y\in\dot L$, $D_L\langle x, y\rangle\ni 1$ if and only if $\langle\langle -x, -y\rangle\rangle=0$ in $WL$. Hence it follows from $WF/W_tF\cong WK$ that $D_F\langle a, b\rangle\ni 1$ if and only if $D_K\langle a', b'\rangle\ni 1$. Thus $\varphi_1$ induces a surjective homomorphism $\varphi_*: k_*F\to k_*K$, and for $n\geq 2$, $\varphi_n$ is injective by the above fact and Lemma 1.3, since $D_F\langle a, b\rangle\ni 1$ for $a\in R(F)$, $b\in\dot F$ (note that $D_F\langle a, b\rangle\ni 1$ if and only if $D_F\langle 1, -a\rangle\ni b$). We have a commutative diagram

$$\begin{array}{ccc} k_nF & \xrightarrow{\varphi_n} & k_nK \\ {\scriptstyle s_n(F)}\downarrow & & \downarrow{\scriptstyle s_n(K)} \\ I^nF/I^{n+1}F & \xrightarrow[\psi_n]{} & I^nK/I^{n+1}K \end{array}$$

where $\varphi_n$ and $\psi_n$ are isomorphisms for $n\geq 2$, and $s_n(K)$ is an isomorphism for any $n$ by [6], Theorem 5. So we see that $s_n(F)$ is an isomorphism for $n\geq 2$. Since $s_0(F)$ and $s_1(F)$ are isomorphisms for any field $F$, $s_*(F)$ is an isomorphism.

Q.E.D.

For any field $F$, $W_{red}F=WF/W_tF$ may be considered as a subring of the ring $C(X_F, \boldsymbol{Z})$ of continuous functions from $X_F$ to $\boldsymbol{Z}$ with the discrete topology, i.e., we identify $\phi$ mod $W_tF$ with $\hat\phi: X_F\to\boldsymbol{Z}$ defined by $\hat\phi(\sigma)=\sum_{i=1}^n\sigma(a_i)$ for $\sigma\in X_F$, $\phi=\langle a_1,\dots, a_n\rangle\in WF$. Then the stability index of $X_F$, denoted by $st(X_F)$, is defined to be the infimum of the set of integers $k$ such that $2^k C(X_F, \boldsymbol{Z}) \subseteq W_{red}F$. It is known that $X_F$ is finite if and only if both $cl(X_F)$ and $st(X_F)$ are finite ([8], Theorem 13.9).

PROPOSITION 1.6. *Let $F$ be a formally real, quasi-pythagorean field with $st(X_F)=i<\infty$. Then $s_n$ is an isomorphism for $n \geq 2^{i-1}$.*

PROOF. The proof of [4], Theorem 5.9 for a pythagorean field is valid for a quasi-pythagorean field with trivial modifications. Q. E. D.

PROPOSITION 1.7. *Let $F$ be a formally real, quasi-pythagorean field such that $st(X_F) \leq 1$ and $|X_F|=n<\infty$. Then the following statements hold:*

(1) *$WF$ is a trivial extension of the ring $W_{red}F$ by the ideal $W_tF$. In other words, we have $(W_tF)^2=0$ and there exists a subring $A$ of $WF$, which is mapped isomorphically onto $W_{red}F$ by the canonical homomorphism, such that $WF=W_tF \oplus A$ as an additive group.*

(2) *For any $r \geq 2$, $I^rF/I^{r+1}F$ is an $n$-dimensional vector space over $\boldsymbol{Z}/2\boldsymbol{Z}$.*

PROOF. (1) Let $X_F=\{\sigma_1,\dots, \sigma_n\}$. Since $st(X_F) \leq 1$, there exist $a_i \in \dot{F}$ $(i=1,\dots, n)$ such that $\sigma_i(a_i)=-1$, $\sigma_j(a_i)=1$ $(j \neq i)$. Then $\{-1, a_2,\dots, a_n\}$ form a basis of $\dot{F}/R(F)$ over $\boldsymbol{Z}/2\boldsymbol{Z}$ ([4], Proposition 5.8). Let $A$ be the subgroup of $WF$ generated by $\{\langle 1\rangle, \langle a_2\rangle,\dots, \langle a_n\rangle\}$. We see that $A$ is isomorphic to $W_{red}F$ as an additive group. For $\phi=\langle 1, -a_i\rangle\langle 1, -a_j\rangle$ $(2 \leq i<j \leq n)$, $\hat{\phi}=0$ in $C(X_F, \boldsymbol{Z})$. Hence we have $\phi \in W_tF \cap I^2F=0$. So $\langle a_i\rangle\langle a_j\rangle=-\langle 1\rangle+\langle a_i\rangle+\langle a_j\rangle \in A$ for $i$, $j$ $(2 \leq i<j \leq n)$. Thus $A$ is indeed a subring. Since $W_tF=\{\langle 1, -b\rangle | b \in R(F)\}$, it is easy to see that $(W_tF)^2=0$ and $WF=W_tF \oplus A$.

(2) This follows from the fact that the image of $\{2^{r-1}\langle\langle 1\rangle\rangle, 2^{r-1}\langle\langle -a_2\rangle\rangle,\dots, 2^{r-1}\langle\langle -a_n\rangle\rangle\}$ form a $\boldsymbol{Z}$-free basis for the image of $I^rF$ in $W_{red}F$ ([4], Proposition 5.8). Q. E. D.

Finally, we remark that if $F$ is a quasi-pythagorean field which is not formally real, then $WF$ is isomorphic to $\boldsymbol{Z}/2\boldsymbol{Z} \times \dot{F}/\dot{F}^2$ in which the ring structure is defined by

$$(\varepsilon, a)+(\delta, b)=(\varepsilon+\delta, (-1)^{\varepsilon\delta}ab)$$
$$(\varepsilon, a)\cdot(\delta, b)=(\varepsilon \cdot \delta, a^{\delta}b^{\varepsilon})$$

for $\varepsilon$, $\delta \in \boldsymbol{Z}/2\boldsymbol{Z}$, $a$, $b \in \dot{F}/\dot{F}^2$ ([12], p. 49).

In this case, we have $R(F)=\dot{F}$, $I^2F=0$ and $k_2F=0$. So it is clear that $s_*$ is an isomorphism.

## §2. $H^*(F, 2)$

Let $F(2)$ be the maximal 2-extension of a field $F$ of characteristic different from 2, i.e., $F(2)$ is the composite, in a fixed algebraic closure of $F$, of all the finite galois extensions whose degrees are 2-powers. Then $G(2)=Gal(F(2)/F)$ is a pro 2-group. We put $H^n(F, 2)=H^n(G(2), \boldsymbol{Z}/2\boldsymbol{Z})$, where we identify $\boldsymbol{Z}/2\boldsymbol{Z}$ with the subgroup $\{\pm 1\}\subseteq F(2)^{\cdot}$.

From the exact sequence

$$1 \longrightarrow \{\pm 1\} \longrightarrow F(2)^{\cdot} \xrightarrow{\varphi} F(2)^{\cdot} \longrightarrow 1 \quad (\varphi(x)=x^2)$$

of $G(2)$-modules, we deduce the cohomology exact sequence. Since $H^1(G(2), F(2)^{\cdot})=0$ by Hilbert Theorem 90, we have the following exact sequences:

(1) $\dot{F}\to\dot{F}\to H^1(F, 2)\to 0$,

(2) $0\to H^2(F, 2)\to H^2(G(2), F(2)^{\cdot})\to H^2(G(2), F(2)^{\cdot})$.

The sequence (1) shows that

$$\delta\colon \dot{F}/\dot{F}^2 \longrightarrow H^1(F, 2), \quad \delta(a) = \frac{\sigma(\sqrt{a})}{\sqrt{a}} \quad (a\in\dot{F},\ \sigma\in G(2))$$

is an isomorphism, and (2) shows that $H^2(F, 2)$ is isomorphic to $Br_2(F)$, the subgroup generated by the elements of order 2 in the Brauer group of $F$, since these elements are split by $F(2)$.

$H^*(F, 2)=\bigoplus_{n=0}^{\infty} H^n(F, 2)$, in which the multiplication is defined by the cup product, is a commutative graded algebra over $\boldsymbol{Z}/2\boldsymbol{Z}$. We have seen that $k_1F\cong\dot{F}/\dot{F}^2\cong H^1(F, 2)$, and as shown in [13], the isomorphism $l(a)\mapsto\delta(a)$ induces a ring homomorphism

$$h_*\colon k_*F \longrightarrow H^*(F, 2), \quad h_n(l(a_1)\cdots l(a_n)) = \delta(a_1)\cup\cdots\cup\delta(a_n),$$

since $\delta(a)\cup\delta(b)$ corresponds to the Brauer class of the quaternion algebra $\left(\frac{a, b}{F}\right)$ which splits if $D_F\langle a, b\rangle\ni 1$.

To state the following proposition, we have to recall one more definition. Let $F$ be a formally real field and $\sigma\in X_F$. Then the euclidean closure $F_\sigma$ of $F$ with respect to $\sigma$ is an extension of $F$ contained in $F(2)$ and is pythagorean with unique ordering which induces $\sigma$ on $F$.

The existence and the uniqueness, up to conjugacy, of the euclidean closure was shown in [2].

PROPOSITION 2.1. *Let $F$ be a formally real, quasi-pythagorean field with $st(X_F)=i<\infty$, and $\{F_\sigma\}_{\sigma\in X_F}$ be the family of all the euclidean closures of $F$. If $n\geq 2^{i-1}$, then $f\colon k_nF\to\Pi_\sigma k_nF_\sigma$ and $h_n\colon k_nF\to H^n(F, 2)$ are injective.*

PROOF. If $F$ is a pythagorean field, this proposition is a part of [4], Theorem 5.9. The same proof applies to a quasi-pythagorean field without essential change. Q. E. D.

PROPOSITION 2.2. *Let $F$ be a formally real, quasi-pythagorean field with $st(X_F)\leq 1$, and suppose $X_F$ is finite. Then $h_*$ is an isomorphism.*

PROOF. Let $K=F(\sqrt{-1})$. Then $Gal(F(2)/K)$ is a free pro 2-group by [15], Proposition 3.2. We put $G=G(2)$, $N=Gal(F(2)/K)$ and consider the group extension

(1) $1\to N\to G\to G/N\to 1$.

In the Lyndon-Hochschild-Serre spectral sequence

$$E_2^{p,q}\Longrightarrow H^{p+q}(F, 2), \quad E_2^{p,q}\cong H^p(G/N, H^q(N, \mathbf{Z}/2\mathbf{Z})),$$

we have $E_2^{p,q}=0$ for $q\geq 2$, since $N$ is free ([14], Theorem 6.5). Moreover $d^2=0$, by [5], Theorem 4, since (1) is a split extension. So we have

$$H^n(F, 2)\cong E_2^{n,0}\oplus E_2^{n-1,1}.$$

Now $G/N\cong Gal(K/F)$ is a cyclic group of order 2, and $H^1(N, \mathbf{Z}/2\mathbf{Z})\cong \dot{K}/\dot{K}^2$. Since there exists an exact sequence

$$1\longrightarrow \{\pm\dot{F}^2\}/\dot{F}^2\longrightarrow \dot{F}/\dot{F}^2\longrightarrow \dot{K}/\dot{K}^2\xrightarrow{\ \nu\ }\dot{F}/\dot{F}^2,$$

where $\nu$ is the norm from $K$ to $F$ ([8], 5.20), we have

$$E_2^{n-1,1}\cong \dot{F}/\{\pm R(F)\}\quad (n\geq 2).$$

It is clear that $E_2^{n,0}\cong \mathbf{Z}/2\mathbf{Z}$ and $E_2^{0,1}\cong \dot{F}/\{\pm\dot{F}^2\}$. Thus we have

(2) $H^1(F, 2)\cong \mathbf{Z}/2\mathbf{Z}\oplus\dot{F}/\{\pm\dot{F}^2\}\cong\dot{F}/\dot{F}^2$

(3) $H^n(F, 2)\cong \mathbf{Z}/2\mathbf{Z}\oplus\dot{F}/\{\pm R(F)\}=\dot{F}/R(F)\quad (n\geq 2)$.

We know, from Proposition 2.1, that $h_n$ is injective. So Theorem 1.5, Proposition 1.7 (2) and (2), (3) above show that $h_*$ is an isomorphism. Q. E. D.

REMARK 2.3. The above proposition is contained, as a special case, in [1], Theorem 4.3.

PROPOSITION 2.4. *Let $F$ be a formally real, quasi-pythagorean field with $cl(X_F)\leq 1$. Let $\{-1, x_i(i\in I)\}$ be a basis of $\dot{F}/R(F)$ over $\mathbf{Z}/2\mathbf{Z}$. Then the following statements hold:*

(1) *The canonical image of*

$$\{\langle 1\rangle, \langle\langle -x_i\rangle\rangle, \langle\langle -x_i, -x_j\rangle\rangle, \langle\langle -x_i, -x_j, -x_k\rangle\rangle, \ldots \mid i, j, k, \ldots \text{are distinct}\}$$

*forms a free $\mathbf{Z}$-basis for $W_{red}F$, and $W_{red}F$ is isomorphic, as a ring, to the group ring $\mathbf{Z}[H]$, where $H$ is the subgroup of $\dot{F}/R(F)$ generated by $\{-x_i\ (i\in I)\}$.*

(2) *$WF$ is a trivial extension of $W_{red}F$ by the ideal $W_tF$.*

(3) *$h_*$: $k_*F \to H^*(F, 2)$ is injective.*

PROOF. (1) and (3) are shown by modifying the proof of [4], Theorem 5.13. (2) is proved in ths same way as Proposition 1.7 (1), using (1) of this proposition. Q.E.D.

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