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Exact solutions of a competition-diffusion system

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ABSTRACT. In this paper, we consider a two-component competition-diffusion system of Lotka-Volterra type which arises in mathematical ecology. By introducing an appropriate ansatz, we look for exact travelling and standing wave solutions of this system.

1. Introduction

In mathematical ecology, it has been proposed that systems of reaction-diffusion equations can describe the interaction of biological species which move by diffusion. A frequently used model is the following Lotka-Volterra competition-diffusion system [7]:

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= d_u \frac{\partial^2 u}{\partial x^2} + u(a_u - b_u u - c_u v), \\
\frac{\partial v}{\partial t} &= d_v \frac{\partial^2 v}{\partial x^2} + v(a_v - b_v u - c_v v),
\end{aligned} \tag{1.1}
$$

where $u = u(x,t)$ and $v = v(x,t)$ represent population densities of two competing species which move by diffusion. The constants a_u and a_v are the intrinsic growth rates, b_u and c_v are the coefficients of intraspecific competition, b_v and c_u are the coefficients of interspecific competition, and d_u and d_v are the diffusion rates. We assume that all of these quantities are positive.

By a suitable transformation, we can rewrite (1.1) as

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + u(1 - u - cv), \\
\frac{\partial v}{\partial t} &= d \frac{\partial^2 v}{\partial x^2} + v(a - bu - v),
\end{aligned} \tag{1.2}
$$

where the constants a, b, c, and d are positive. For the initial value problem of (1.2) with initial data $(u,v)(x,0) \geq 0$, the asymptotic behavior of (u,v) can be classified into the following four cases [2]:

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(I) If $a < \min(b, 1/c)$, then $\lim_{t\to\infty}(u,v)(x,t) = (1,0)$;

(II) If $b < a < 1/c$, then

$$\lim_{t\to\infty}(u,v)(x,t) = \left(\frac{1-ac}{1-bc}, \frac{a-b}{1-bc}\right);$$

(III) If $1/c < a < b$, then $(1,0)$ and $(0,a)$ are locally stable rest points;

(IV) If $a > \max(b, 1/c)$, then $\lim_{t\to\infty}(u,v)(x,t) = (0,a)$.

For Cases (I), (II), and (IV), the convergence is compact uniformly in **R**. From an ecological viewpoint, Case (III) is the most interesting one since it implies that, depending on the initial data, only one species survives while the other becomes extinct.

Travelling wave solutions of (1.2) play an important role in determining which species can survive in competition when diffusion is taken into consideration. These solutions are of the form $(u,v)(x,t) = (U,V)(z)$, where $z = x - \theta t$ and θ is the propagation speed. Rewriting (1.2) in terms of z, we obtain

$$(1.3)\qquad \begin{aligned} 0 &= \frac{d^2U}{dz^2} + \theta\frac{dU}{dz} + U(1 - U - cV), \\ 0 &= d\frac{d^2V}{dz^2} + \theta\frac{dV}{dz} + V(a - bU - V). \end{aligned}$$

In this paper, we will consider (1.3) with the boundary conditions

$$(1.4)\qquad (U,V)(-\infty) = (0,a), \qquad (U,V)(+\infty) = (1,0).$$

When a, b, and c satisfy the bistability condition

$$(1.5)\qquad \frac{1}{c} < a < b,$$

Kan-on [4] showed that for a fixed positive d, there exists a unique $\theta = \theta(a,b,c)$ such that (1.3), (1.4) has a solution $(U,V)(z)$ satisfying $dU/dz > 0$ and $dV/dz < 0$ for all $z \in \mathbf{R}$. Furthermore, the speed $\theta = \theta(a,b,c)$ depends monotonically with respect to its arguments, that is,

$$\frac{\partial\theta}{\partial a} > 0, \qquad \frac{\partial\theta}{\partial b} < 0, \qquad \frac{\partial\theta}{\partial c} > 0.$$

The sign of the propagation speed determines which species will survive. If $\theta > 0$ (resp. $\theta < 0$), then the species v (resp. u) becomes dominant and eventually occupies the whole domain. When $\theta = 0$, the two competing species coexist due to the balance between diffusion and competitive interaction. Thus, one would like to know the sign of θ. We shall give an explicit representation of θ in this paper.

The aim of this paper is to show that the above mentioned monotone solution and propagation speed can be represented explicitly under some parameter restrictions. We will look for three classes of parameter restrictions where in each class, the speed θ is determined as a function of the parameters.

Besides travelling wave solutions, we will also look for exact standing ($\theta = 0$) wave solutions of (1.3) which are of two types: periodic-type and pulse-type. The former are equivalent to solutions of (1.3) defined on a finite interval with periodic boundary conditions, while the latter are solutions which approach the limiting value $(0, a)$ as $x \to \pm\infty$.

The essential idea of the method we use in this paper is to introduce an ansatz which will reformulate (1.3) to a new system where the independent variable is U (or V). In the reformulated system, the solutions are just polynomials in U (or V). The solutions to (1.3) are then obtained by integrating the first-order differential equation arising from the ansatz introduced. The result of this integration yields a solution with a hyperbolic tangent profile for the monotone travelling wave case.

Our method can also be used for other reaction-diffusion systems with *polynomial* nonlinearities, e.g., the Gray-Scott model for a cubic autocatalytic reaction and a simplification of the Noyes-Field model for the Belousov-Zhabotinskii reaction [8]. Using our approach, the resulting calculations are not as complicated as the one using Painlevé analysis, for example [3]. In addition, an ansatz-based approach allows us to find standing wave solutions of periodic-type and pulse-type which are not obtained from the usual Painlevé analysis.

This paper is organized as follows: in Section 2, we look for monotone travelling wave solutions. In Section 3, by modifying the ansatz in the previous section, we look for standing wave solutions of periodic-type and pulse-type. Finally, in Section 4, we give a short discussion of our results.

2. Travelling wave solutions

In order to solve (1.3), (1.4), we introduce some ansatz. The idea is to look for a monotone solution with a hyperbolic tangent profile. Since the derivative of the hyperbolic tangent function is expressible in terms of itself, we assume that

$$\frac{dU}{dz} = F(U). \tag{2.1}$$

Moreover, we assume that U and V are related by

$$V = G(U). \tag{2.2}$$

Applying this ansatz to (1.3), we obtain

$$
(2.3)\qquad
\begin{aligned}
0 &= F\frac{dF}{dU} + \theta F + U(1 - U - cG),\\
0 &= d\left(F\frac{dF}{dU}\frac{dG}{dU} + F^2\frac{d^2G}{dU^2}\right) + \theta F\frac{dG}{dU} + G(a - bU - G).
\end{aligned}
$$

We look for F and G in the forms

$$
F(U) = \sum_{i=0}^{m} a_i U^i, \qquad G(U) = \sum_{i=0}^{n} b_i U^i,
$$

where $m, n > 0$. The polynomial dependence for G is also suggested by the profile of the numerical solutions of (1.3) in the (U, V)-plane.

Substituting these choices of F and G in (2.3) and balancing the highest-ordered derivative terms and the highest nonlinear terms, we get the relation

$$
(2.4)\qquad 2m = n + 2.
$$

The boundary conditions (1.4) are transformed to $F(0) = F(1) = 0$, $G(0) = a$, and $G(1) = 0$. Therefore, the constant a_0 is necessarily zero.

Here, we shall consider only three cases. For other values of m and n, the system obtained by substituting F and G in (2.3) yields an overdetermined system where the number of equations is greater than the total number of unknown constants and parameters. Thus, only trivial solutions of this system are obtained.

2.1. $m = 2, n = 2$

We have $F(U) = a_1U + a_2U^2$ and $G(U) = b_0 + b_1U + b_2U^2$. From (1.4), we must have $a_2 = -a_1$, $b_0 = a$, and $b_1 = -a - b_2$.

Substituting in (2.3) and equating coefficients of powers of U to zero, we obtain the following system of nonlinear equations:

$$
(2.5)\qquad 2a_1^2 - b_2c = 0,
$$

$$
(2.6)\qquad -1 - 3a_1^2 + ac + b_2c - a_1\theta = 0,
$$

$$
(2.7)\qquad 1 + a_1^2 - ac + a_1\theta = 0,
$$

$$
(2.8)\qquad b_2 - 6a_1^2d = 0,
$$

$$
(2.9)\qquad 2ab_2 - bb_2 + 2b_2^2 - 2aa_1^2d - 12a_1^2b_2d - 2a_1b_2\theta = 0,
$$

$$
(2.10)\qquad -a^2 + ab - 3ab_2 + bb_2 - b_2^2 + 3aa_1^2d + 7a_1^2b_2d + aa_1\theta + 3a_1b_2\theta = 0,
$$

$$
(2.11)\qquad a^2 - ab + ab_2 - aa_1^2d - a_1^2b_2d - aa_1\theta - a_1b_2\theta = 0.
$$

We now solve for the unknown constants a_1, b_2, and θ. Since the above system is overdetermined, we have to impose some restrictions on the parameters. From (2.5), (2.6), and (2.8), we obtain

$$a_1 = \frac{\sqrt{2b_2c}}{2}, \qquad d = \frac{1}{3c}, \qquad \theta = -\frac{2 - 2ac + b_2c}{\sqrt{2b_2c}}.$$

We note that (2.7) is satisfied trivially. From (2.9) and (2.10), we get

$$b = 2 + \frac{5a}{3} - ac, \qquad b_2 = a.$$

Again, equation (2.11) is satisfied trivially.

Simplifying the obtained expressions for the unknown constants yields the following:

$$d = \frac{1}{3c}, \qquad b = 2 + \frac{5a}{3} - ac, \qquad \theta = \frac{-2 + ac}{\sqrt{2ac}}, \tag{2.12}$$

$$a_1 = -a_2 = \frac{\sqrt{2ac}}{2}, \qquad b_0 = b_2 = -\frac{b_1}{2} = a. \tag{2.13}$$

Integrating (2.1), which is a Bernoulli's equation, and substituting the result in (2.2), we obtain an exact monotone travelling wave solution of (1.2):

$$U(z) = \frac{1}{2}\left[1 + \tanh\left(\frac{\sqrt{2ac}}{4}z\right)\right], \tag{2.14}$$

$$V(z) = \frac{a}{4}\left[1 - \tanh\left(\frac{\sqrt{2ac}}{4}z\right)\right]^2. \tag{2.15}$$

This solution is valid under the parameter restrictions (2.12). To satisfy the bistability condition (1.5), the parameters a and c have to lie in the region defined by $\mathscr{R}_1 \equiv \{(a,c) \mid 6 + 2a - 3ac > 0, -1 + ac > 0\}$. Here, and in the following cases, we of course assume that both a and c are positive. Profiles of (2.14), (2.15) are shown in Figure 1.

The manner of solving the following cases is similar to the preceding one so we will just give the final results.

2.2. $m = 3/2, n = 1$

Here, we have $F(U) = a_1U + a_2U^{3/2}$ and $G(U) = b_0 + b_1U^{1/2} + b_2U$. Then, for

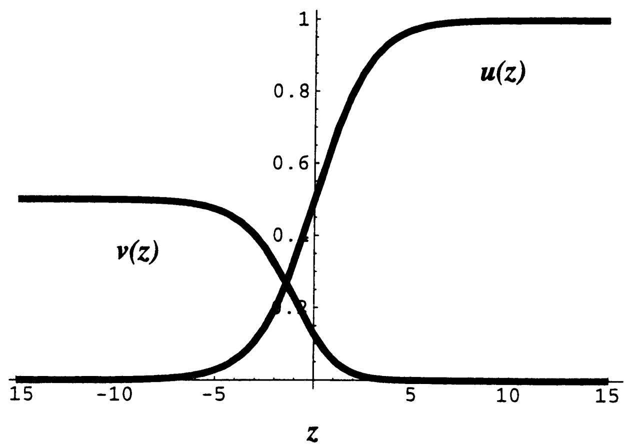

Fig. 1.

$$d = \frac{5 + 6a - ac}{1 + ac}, \qquad b = 5 + 5a - ac, \qquad \theta = \frac{-5 + ac}{\sqrt{6 + 6ac}}, \tag{2.16}$$

$$a_1 = -a_2 = \frac{2\sqrt{1 + ac}}{\sqrt{6}}, \qquad b_0 = b_2 = -\frac{b_1}{2} = a, \tag{2.17}$$

we obtain the following travelling wave solution:

$$U(z) = \frac{1}{4}\left[1 + \tanh\left(\frac{\sqrt{1 + ac}}{2\sqrt{6}} z\right)\right]^2, \tag{2.18}$$

$$V(z) = \frac{a}{4}\left[1 - \tanh\left(\frac{\sqrt{1 + ac}}{2\sqrt{6}} z\right)\right]^2. \tag{2.19}$$

This solution will satisfy (1.5) if a and c lie in the region which is defined by $\mathscr{R}_2 \equiv \{(a, c) \mid 5 + 4a - ac > 0, -1 + ac > 0\}$. Profiles of (2.18), (2.19) are similar to those in Figure 1.

2.3. $m = 3/2, n = 1/2$

Here, we have $F(U) = a_1 U + a_2 U^{3/2}$ and $G(U) = b_0 + b_1 U^{1/2}$. Then, it follows that

$$d = 5 + 6a - 3ac, \qquad b = \frac{5}{3} + 2a - ac, \qquad \theta = \frac{-5 + 3ac}{\sqrt{6}}, \tag{2.20}$$

$$a_1 = -a_2 = -\frac{\sqrt{6}}{3}, \qquad b_0 = -b_1 = a, \tag{2.21}$$

(Travelling Wave Solution No. 3)

$$U(z) = \frac{1}{4}\left[1 + \tanh\left(\frac{1}{2\sqrt{6}}z\right)\right]^2, \tag{2.22}$$

$$V(z) = \frac{a}{2}\left[1 - \tanh\left(\frac{1}{2\sqrt{6}}z\right)\right]. \tag{2.23}$$

This solution will satisfy (1.5) if a and c lie in the region which is defined by $\mathscr{R}_3 \equiv \{(a,c) \mid 5 + 3a - 3ac > 0, -1 + ac > 0\}$. Profiles of (2.22), (2.23) are similar to those in Figure 1.

By choosing other values for m and n in (2.4), we may be able to find other monotone travelling wave solutions of (1.2). In addition, by modifying (2.1), (2.2) to

$$\frac{dV}{dz} = F(V), \qquad U = G(V)$$

and proceeding as before, further solutions may also be obtained.

3. Standing wave solutions

In this section, we will look for standing wave solutions of (1.3) of periodic-type and pulse-type, i.e., $\theta = 0$ and $z = x$. For the periodic solutions, we shall consider periodic boundary conditions while for the pulse solutions, we shall consider the boundary conditions $(U, V)(\pm\infty) = (0, a)$.

We modify (2.1), (2.2) by introducing the ansatz

$$\left(\frac{dU}{dx}\right)^2 = H(U), \qquad V = G(U) \tag{3.1}$$

and substitute in (1.3), giving

$$\begin{aligned} 0 &= \frac{1}{2}\frac{dH}{dU} + U(1 - U - cG), \\ 0 &= d\left(\frac{1}{2}\frac{dH}{dU}\frac{dG}{dU} + H\frac{d^2G}{dU^2}\right) + G(a - bU - G). \end{aligned} \tag{3.2}$$

We look for H and G in the forms

$$H(U) = \sum_{i=0}^{m} a_i U^i, \qquad G(U) = \sum_{i=0}^{n} b_i U^i,$$

where $m, n > 0$. Substituting these choices of H and G in (3.2) and balancing the highest-ordered derivative terms and the highest nonlinear terms, we get the relation

$$m = n + 2. \tag{3.3}$$

For simplicity, we shall only consider two cases:

3.1. $m = 3, n = 1$

We have $H(U) = a_0 + a_1 U + a_2 U^2 + a_3 U^3$ and $G(U) = b_0 + b_1 U$. Substituting in (3.2) and equating coefficients of powers of U to zero, we obtain the following system of nonlinear equations:

$$-2 + 3a_3 - 2b_1 c = 0, \tag{3.4}$$

$$1 + a_2 - b_0 c = 0, \tag{3.5}$$

$$a_1 = 0, \tag{3.6}$$

$$2b + 2b_1 - 3a_3 d = 0, \tag{3.7}$$

$$-bb_0 + ab_1 - 2b_0 b_1 + a_2 b_1 d = 0, \tag{3.8}$$

$$2ab_0 - 2b_0^2 + a_1 b_1 d = 0. \tag{3.9}$$

Solving the above system is straightforward, giving us

$$a_0 : \text{arbitrary}, \qquad a_1 = 0, \qquad a_2 = -1 + ac, \qquad a_3 = -\frac{2}{3}(-1 + ac), \tag{3.10}$$

$$b_0 = -b_1 = a, \qquad d = \frac{a - b}{-1 + ac}. \tag{3.11}$$

Integrating the first equation in (3.1) and substituting the result in the second equation, we get the following periodic standing wave solution of (1.2):

$$U(x) = \frac{1}{2} + \frac{6}{1 - ac} \wp(x; g_2, g_3), \tag{3.12}$$

$$V(x) = a\left[\frac{1}{2} - \frac{6}{1 - ac} \wp(x; g_2, g_3)\right], \tag{3.13}$$

where $\wp$ is the Weierstrass elliptic function and g_2, g_3 are the invariants given by

$$g_2 = \frac{1}{12}(-1 + ac)^2, \qquad g_3 = -\frac{1}{216}(-1 + ac)^2(-1 + 6a_0 + ac). \tag{3.14}$$

For real-valued solutions, the constant a_0 must satisfy

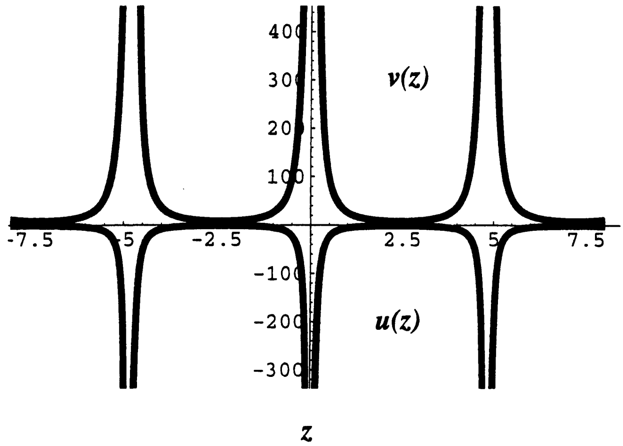

Fig. 2.

$$27g_3^2 - g_2^3 = \frac{a_0(-1+ac)^4}{144}(-1+3a_0+ac) \geq 0. \tag{3.15}$$

Profiles of (3.12), (3.13) are shown in Figure 2.

Setting $a_0 = 0$, integrating the first equation in (3.1), and substituting the result in the second equation, we get the following pulse standing wave solution of (1.2):

$$U(x) = \frac{36e^{\sqrt{\eta}x}}{(1+6e^{\sqrt{\eta}x})^2}, \tag{3.16}$$

$$V(x) = a\left[1 - \frac{36e^{\sqrt{\eta}x}}{(1+6e^{\sqrt{\eta}x})^2}\right], \tag{3.17}$$

where $\eta = -1 + ac$. Profiles of (3.16), (3.17) are shown in Figure 3.

The manner of solving the following case is similar to the preceding one so we will just give the final results.

3.2. $m = 4, n = 2$

Here, we have $H(U) = a_0 + a_1U + a_2U^2 + a_3U^3 + a_4U^4$ and $G(U) = b_0 + b_1U + b_2U^2$. Then, after some calculcations, we obtain the following standing wave solutions of periodic-type:

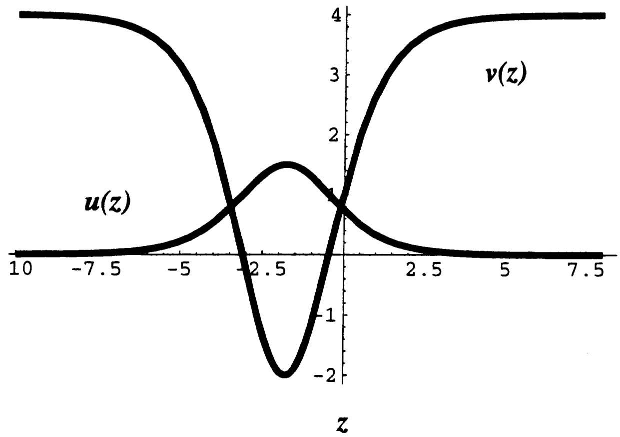

Fig. 3.

$$a_0 = -\frac{125(-2500 + 6000bc - 1575b^2c^2 - 2430b^3c^3 + 729b^4c^4)}{\eta^3}, \tag{3.18}$$

$$a_1 = 0, \qquad a_2 = \frac{10(5 - 3bc)^2}{\eta}, \qquad a_3 = \frac{10 - 6bc}{5}, \qquad a_4 = \frac{\eta}{250}, \tag{3.19}$$

$$b_0 = \frac{50 - 135bc + 81b^2c^2}{c\eta}, \qquad b_1 = \frac{10 - 9bc}{5c}, \qquad b_2 = \frac{\eta}{125c}, \tag{3.20}$$

$$d = \frac{1}{3c}, \qquad a = \frac{50 - 435bc + 261b^2c^2}{3\eta}, \tag{3.21}$$

$$\eta = -200 + 165bc - 9b^2c^2, \qquad \xi = \frac{(5 - 3bc)^2}{1 + k^2}, \tag{3.22}$$

$$U(x) = \frac{25(-5 + 3bc)}{\eta} + \frac{100k\sqrt{\xi}}{\sqrt{8\eta^2}}\operatorname{sn}\left(\frac{\sqrt{5\xi}}{\sqrt{\eta}}x, k\right), \tag{3.23}$$

$$V(x) = -\frac{3(25 - 30bc + 3b^2c^2)}{c\eta} - \frac{156k\sqrt{2\xi}}{\sqrt{\eta^2}}\operatorname{sn}\left(\frac{\sqrt{5\xi}}{\sqrt{\eta}}x, k\right) + \frac{10k^2\xi}{c\eta}\operatorname{sn}\left(\frac{\sqrt{5\xi}}{\sqrt{\eta}}x, k\right)^2, \tag{3.24}$$

where k is an arbitrary positive constant. Profiles of (3.23), (3.24) are shown in Figure 4.

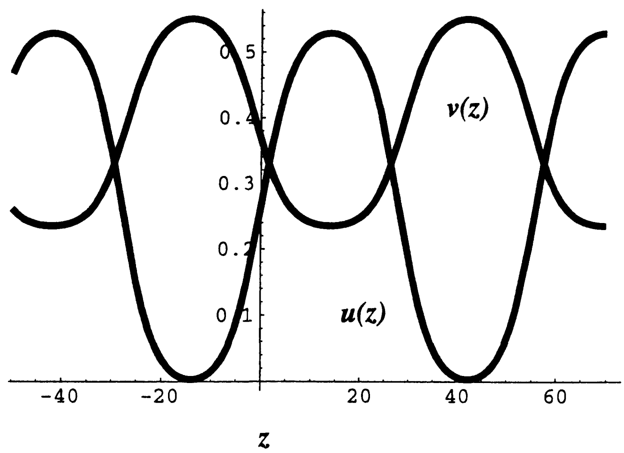

Fig. 4.

Moreover, we obtain the following standing wave solution of pulse type:

$$a_0 = a_1 = 0, \quad a_2 = -1+ac, \quad a_3 = \frac{2-6ac}{3+ac}, \quad a_4 = \frac{5ac(-7+ac)}{(-4+ac)(3+ac)^2}, \tag{3.25}$$

$$b_0 = a, \qquad b_1 = -\frac{10a}{3+ac}, \qquad b_2 = \frac{10a(-7+ac)}{(-4+ac)(3+ac)^2}, \tag{3.26}$$

$$d = \frac{1}{3c}, \qquad b = \frac{10(1+2ac)}{3c(3+ac)}, \tag{3.27}$$

$$\eta = 2 + 5ac + a^2c^2 - 2a^3c^3, \quad \xi = -3 + 2ac + a^2c^2, \quad \omega = -1 + ac, \tag{3.28}$$

$$U(x) = \frac{e^{\sqrt{\omega}x}}{-\frac{\eta}{2(-4+ac)\xi^2} + \frac{-1+3ac}{\xi}e^{\sqrt{\omega}x} + e^{2\sqrt{\omega}x}}, \tag{3.29}$$

$$V(x) = a - \frac{10a}{3+ac}\left[\frac{e^{\sqrt{\omega}x}}{-\frac{\eta}{2(-4+ac)\xi^2} + \frac{-1+3ac}{\xi}e^{\sqrt{\omega}x} + e^{2\sqrt{\omega}x}}\right] + \frac{10a(-7+ac)}{(-4+ac)(3+ac)^2}\left[\frac{e^{\sqrt{\omega}x}}{-\frac{\eta}{2(-4+ac)\xi^2} + \frac{-1+3ac}{\xi}e^{\sqrt{\omega}x} + e^{2\sqrt{\omega}x}}\right]^2. \tag{3.30}$$

Profiles of (3.29), (3.30) are shown in Figure 5.

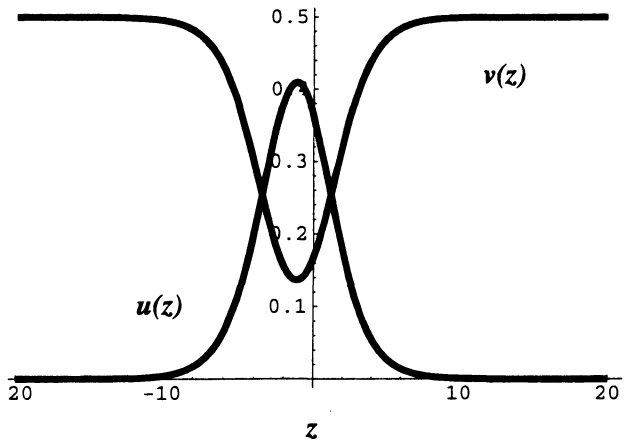

Fig. 5.

By choosing other values for m and n in (3.3), we may be able to find other standing wave solutions of (1.2). In addition, by modifying (3.1) to

$$\left(\frac{dV}{dx}\right)^2 = H(V), \qquad U = G(V)$$

and proceeding as before, further solutions may also be obtained.

4. Discussions

The system (1.2) has four parameters a, b, c, and d. As mentioned in the Introduction, for any fixed positive d and a, b, and c satisfying the bistability condition (1.5), the speed $\theta = \theta(a, b, c)$ is uniquely determined. Let $\mathscr{R}$ be the set of parameters (a, b, c) satisfying (1.5). Then $\mathscr{R}$ describes the region in parameter space where the existence and uniqueness of a monotone travelling wave solution is guaranteed.

In Section 2, we found three classes of parameter restrictions where in each class, the parameters b and d are expressed in terms of a and c. The equation relating b in terms of a and c can be thought of as some surface $\mathscr{S}$ in (a, b, c)-space which intersects $\mathscr{R}$. The sets $\mathscr{R}_i$ $(i = 1, 2, 3)$ are just the projections of $\mathscr{R} \cap \mathscr{S}$ on the (a, c)-plane. If $(a, b, c) \in \mathscr{R} \cap \mathscr{S}$, then it is possible to find an exact representation of the monotone solution. We have found three surfaces for which an exact representation can be given. These are certainly not the only ones for, as mentioned in the last part of Section 2, if we modify the ansatz (2.1), (2.2) by expressing (1.3) in terms of V, we may be able to find other classes. However, the functional forms for U and V are essentially the same as those found in Section 2. In other words, the possible functional forms for U and V are either both quadratic in tanh or one linear in tanh and the other quadratic in tanh.

The solutions found in Section 2 are also valid if a, b, and c satisfy the monostability assumptions of Cases (I) and (IV), since they were not considered at all in the calculations. However, it is known that under this situation, the uniqueness of travelling wave solutions can never hold. This property is sometimes called the *Fisher-type property* because in the monostable Fisher's equation, there is a continuum of wave speeds for which a monotone travelling wave solution exists. The only difference between the exact solutions found for the monostable and bistable cases is in the definition of the $\mathscr{R}_i$'s.

If we set $c = 0$ in (1.2), then the first equation reduces to Fisher's equation for u. From (2.18) and (2.22), we get

$$U(z) = \frac{1}{4}\left[1 + \tanh\left(\frac{1}{2\sqrt{6}}z\right)\right]^2,$$

which is just the solution to Fisher's equation for the special wave speed $\theta = -5/\sqrt{6}$ obtained by Ablowitz and Zeppetella [1]. The same reduction can be said of (2.14) if we set $d = a = 1$ and $b = 0$.

The existence, uniqueness, and instability of a standing pulse solution of (1.2) satisfying (1.5) such that $(U, V)(\pm\infty) = (0, a)$ has been shown in [5], [6]. We note that this solution plays the role of a separator between the constant equilibria $(1, 0)$ and $(0, a)$. An explicit representation of this solution is given by (3.29), (3.30).

Acknowledgments

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