Foreword to the special issue on symbolic computation with Sage in mathematical education and research

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Welcome to this first special issue of the *Tbilisi Mathematical Journal* (TMJ). Special in that it is devoted to a single topic; and also because it deals with one of the most important developments of todays mathematics: Symbolic Computation.

There is already an impressive list of Computer Algebra Systems (CAS), software programs for manipulating mathematical expressions in symbolic form. Among all of them, this issue has chosen to pay attention to a new CAS, born as recently as in 2005: Sage (see http://sagemath.org), led by William Stein. In the words of Sage creators, *Sage is a free open-source mathematics software system licensed under the GPL*. It combines the power of many existing open-source packages into a common Python-based interface. With its auto-imposed mission, *creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab*, Sage is rapidly evolving to be a strong competitor of commercial packages in the field.

The aim of this issue is providing a snapshot on some current trends in mathematics education and research on symbolic computation, developed through Sage. We are aware that a single issue of a journal can only attempt to provide a partial picture on the accomplishments related to these broad topics. Yet, we hope that the quality and variety of selected papers will give the reader at least an interesting glimpse on the state of the art concerning the issue goals.

The call for contributions to this special issue was launched at March 2012, contemporary to the organization of the third Spanish meeting on Sage/Python, http://webs.uvigo.es/sage2012. Roughly speaking, the received submissions fell (almost proportionally) into one of the following three categories: using Sage to help improving mathematics education (at different levels), using Sage to perform a particular mathematics research, and developing Sage tools for general mathematics research purposes.

After a rigorous reviewing process, following the TMJ refereeing guidelines, eight papers, coming from Australia, Finland, Macedonia, Portugal, Spain and USA, were finally selected. Let us include here a short description for each of them.

The paper 'Sage computations of $\mathfrak{sl}_2(k)$-Levi extensions' shows the impossibility to find Levi extensions for characteristically nilpotent (non-filiform) Lie algebras. The authors use Sage to find examples of nilpotent Lie algebras admitting Levi extensions with $\mathfrak{sl}_2(k)$ as factor and not being abelian or Heissenberg.

In the paper 'On the Walsh-Hadamard transform of monotone Boolean functions' the authors study and characterize monotone Boolean functions through their associated Cayley graphs. The spectra of these graphs are computed in terms of the Walsh-Hadamard transform of the function. Sage is used to compute some of these functions, and their Cayley graphs and spectra.

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A package enabling creation and modifications in parameterized exercise templates is reported in 'Exercise templates with Sage'. Assuming user knowledge of LaTeX and of elementary Python skills, basic undergraduate mathematical exercises can be easily personalized and distributed in a wide variety of formats.

A fresh view on teaching mathematics to engineering students is the subject of 'Experiences in using Sage to integrate good study habits and problem solving techniques in Engineering Mathematics'. An informal discussion about using technology (and Sage specifically) is carried on and illustrated through a detailed example.

The paper 'Experimental evidence for Maeda's conjecture on modular forms' significantly extends the computational verification of this conjecture. Combining a known technique with a randomized search algorithm, the authors make a valuable contribution to the experimental evidence supporting the conjecture.

Algorithms and the corresponding Sage code for plotting attraction basins are the main subject of 'Plotting basins of end points of rational maps with Sage'. As the authors state, enriching the public domain with a tool as the one here reported will enhance free access to mathematics.

In 'Numerical methods with Sage' the role of Sage as an integrated Python–based environment is emphasized. By selecting some case studies, teaching examples using Sage in the field of numerical analysis are discussed. Special attention is paid to teaching possibilities of the integration of specific numerical libraries under the framework of Sage.

Finally, the paper 'Multilingual Sage' describes an approach to multilingual mathematics. The authors study the problem of producing natural language interfaces to CAS, and introduce such an interface to Sage, `gfsage`. Details and examples of `gfsage` are also provided.

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