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CORRECTION TO THE PAPER

"APPLICATION OF THE THEORY OF MARKOV PROCESSES TO COMMINUTION I. THE CASE OF DISCRETE TIME PARAMETER"

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The statement and proof of Lemma 2 on page 183 is incorrect. It should be replaced by the following

LEMMA 2′. *Let* $\{\zeta_n\}$ *be a sequence of nonnegative random variables and* $\{w_n\}$ *a sequence of positive numbers, and suppose that*

$$\frac{\zeta_n}{w_n}\to 1 \qquad \textit{in probability} \tag{5. 19′}$$

as $n\to\infty$. *It is further assumed that for an integer* $k\geqq 1$ *the* k*-th moment* $E\{\zeta_n{}^k\}$ *exists and*

$$\lim_{n\to\infty} E\left\{\left(\frac{\zeta_n}{w_n}-1\right)^k\right\}=0. \tag{5. 20′}$$

Then for all l *with* $0<l\leqq k$

$$\lim_{n\to\infty}\frac{E\{\zeta_n{}^l\}}{w_n{}^l}=1. \tag{5. 21′}$$

Proof. We begin by showing that under the conditions of this lemma

$$\lim_{n\to\infty} E\left\{\left|\frac{\zeta_n}{w_n}-1\right|^k\right\}=0 \tag{5. 22′}$$

holds. Now put $u_n=(\zeta_n/w_n-1)^k\geqq -1$ and $|u_n|=u_n^+ + u_n^- = u_n+2u_n^-$, where u_n^+ and u_n^- are the positive and negative parts of u_n, respectively. Then we have $0\leqq u_n^-\leqq 1$, so that (5. 19′) implies $E\{u_n^-\}\to 0$. Hence $E\{|u_n|\}=E\{u_n\}+2E\{u_n^-\}\to 0$, as was to be proved.

It is easy to derive (5. 21′) from (5. 19′) and (5. 22′). By Minkowski's inequality,

$$\left[E\left\{\left(\frac{\zeta_n}{w_n}\right)^l\right\}\right]^{1/l}\leqq\left[E\left\{\left(\frac{\zeta_n}{w_n}\right)^k\right\}\right]^{1/k}\leqq 1+\left[E\left\{\left|\frac{\zeta_n}{w_n}-1\right|^k\right\}\right]^{1/k},$$

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whence (5. 22′) reduces to

$$\limsup_{n\to\infty} \frac{E\{\zeta_n{}^l\}}{w_n{}^l} \leqq 1. \tag{5. 23′ a}$$

If n is sufficiently large, on the other hand, it follows from (5. 19′) that

$$E\left\{\left(\frac{\zeta_n}{w_n}\right)^l\right\} \geqq (1-\varepsilon)^l P\left\{\left|\frac{\zeta_n}{w_n}-1\right|<\varepsilon\right\} \geqq (1-\varepsilon)^{l+1}$$

for every fixed $\varepsilon>0$. Since ε is arbitrary, one can conclude

$$\liminf_{n\to\infty} \frac{E\{\zeta_n{}^l\}}{w_n{}^l} \geqq 1, \tag{5. 23′ b}$$

which, together with (5. 23′ a), accomplishes the proof.

In the case of our comminution process, let $\zeta_n = X_n{}^*$, $l=rb$, $k=[rb]$, $w_n = E\{\zeta_n\} = E\{X_n{}^*\} = \nu n$. Considering that $X_n{}^*$ is the sum of mutually independent random variables with a common distribution, the law of large numbers states that the condition (5. 19′) should be satisfied. The validity of (5. 20′) can also be verified by direct calculation. Thus we observe that formula (5. 18) and the subsequent theorems in the paper hold without change.

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