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KILLING VECTOR FIELDS ON NON-COMPACT RIEMANNIAN MANIFOLDS WITH BOUNDARY*

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1. Introduction.

The study of Killing vector fields on compact Riemannian manifolds with boundary had been started by K. Yano [3]. In a previous paper [5], we discussed non-existence of Killing vector fields with finite global norms on complete Riemannian manifolds (without boundary).

The purpose of the present paper is to discuss non-existence of Killing vector fields with finite global norms on non-compact Riemannian manifolds with boundary.

We shall be in C^∞-category. Latin indices run from 1 to $n+1$ and Greek ones from 1 to n, and the Einstein summation convention will be used.

2. Riemannian manifold with boundary.

Let $\mathcal{M}$ be a complete, non-compact, connected and orientable Riemannian manifold of dimension $n+1$ and g (resp. ∇) the Riemannian metric (resp. the Riemannian connection) on $\mathcal{M}$. We take a non-compact manifold $\bar{M}=\partial M \cup M$ such that M is a noncompact, connected, open submanifold of $\mathcal{M}$ and $\partial M=\bar{M}-M$ is an n dimensional, compact, connected submanifold of $\mathcal{M}$, where $\bar{M}$ denotes the closure of M in $\mathcal{M}$. Then $\bar{M}$ is a Riemannian manifold with boundary ∂M, and the Riemannian metric on $\bar{M}$ is induced from the Riemannian metric g on $\mathcal{M}$. $\bar{M}$ is complete as a metric space with the distance determined by the induced Riemannian metric on $\bar{M}$. For simplicity, hereafter, we denote by g the induced Riemannian metric on $\bar{M}$ and by ∇ the Riemannian connection on $\bar{M}$.

At each point p of ∂M, there exists a coordinate neighborhood system $\{U; (x^i)\}$ of p in $\mathcal{M}$ such that $U \cap \bar{M}$ is represented by $x^{n+1} \geqq 0$ and $U \cap \partial M$ is represented by $x^{n+1}=0$. Such a coordinate neighborhood system is called a boundary coordinate system. And $\{U \cap \partial M; (x^\alpha)\}$ is the induced coordinate system on ∂M. If $\{U; (x^i)\}$ and $\{V; (y^i)\}$ are boundary coordinate systems satisfying $U \cap V \neq \emptyset$, then we have that

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(2.1) $$\frac{\partial y^{n+1}}{\partial x^{n+1}}>0 \quad\text{and}\quad \frac{\partial y^{n+1}}{\partial x^{\alpha}}=0 \quad\text{on}\quad \partial M\cap U\cap V \quad (\text{for any } \alpha).$$

Since the Jacobian of the coordinate transformation of $\{U; (x^i)\}$ and $\{V; (y^i)\}$ is positive, the Jacobian of the coordinate transformation of $\{U\cap\partial M; (x^{\alpha})\}$ and $\{V\cap\partial M; (y^{\alpha})\}$ is positive. Thus ∂M is orientable.

Let $\iota: \partial M\to M$ be the inclusion. If $\{U; (x^i)\}$ is a boundary coordinate system of a point p of ∂M in $\mathcal{M}$ and $\{U'; (u^{\alpha})\}$ is a coordinate system of p such that $U'\subset U\cap\partial M$, then the inclusion ι may be represented locally by

(2.2) $$x^i=x^i(u^{\alpha}).$$

We denote by B the differential of the inclusion ι, that is,

(2.3) $$B=(B_{\alpha}^{i})=(\partial x^i/\partial u^{\alpha}).$$

Then the induced metric $'g=('g_{\alpha\beta})$ on ∂M is given by

(2.4) $$'g_{\alpha\beta}=B_{\alpha}^{i}B_{\beta}^{j}g_{ij},$$

where $g=(g_{ij})$. We may choose the unit outer normal vector field N to ∂M (cf. [3, 4]). We denote by $'\nabla$ the Riemannian connection on ∂M with respect to the Riemannian metric $'g$. Then the equations of Gauss and Weingarten is stated as follows:

(2.5) $$\nabla_{\iota_*X}\iota_*Y=\iota_*('\nabla_X Y)+h(X, Y)\cdot N,$$

(2.6) $$\nabla_{\iota_*X}N=\iota_*(-AX)$$

for any vector fields X and Y on ∂M, where h denotes the second fundamental form of ∂M with respect to N and A is defined by $h(X, Y)={}'g(AX, Y)$.

3. Vector field with finite global norm.

Let $\wedge^s(\overline{M})$ (resp. $\wedge^s(\partial M)$) be the space of all smooth s-forms on $\overline{M}$ (resp. ∂M). Let d denote the exterior derivative on $\wedge^s(\overline{M})$ (or $\wedge^s(\partial M)$), and δ is defined by

(3.1) $$\delta=(-1)^m*d*$$

on $\wedge^s(\overline{M})$ (resp. $\wedge^s(\partial M)$) where $m=sn+s+n$ (resp. $sn+n+1$) and $*$ denotes the star operator (cf. [4]).

We denote by $\langle\ ,\ \rangle$ the local scalar product on $\wedge^s(\overline{M})$ (or $\wedge^s(\partial M)$). The global scalar product $\ll\ ,\ \gg$ is defined by

(3.2) $$\ll\xi, \eta\gg=\int_{\overline{M}}\langle\xi, \eta\rangle*1=\int_{\overline{M}}\xi\wedge*\eta$$

for any $\xi, \eta\in\wedge_0^s(\overline{M})$ where $\wedge_0^s(\overline{M})$ denotes the subspace of $\wedge^s(\overline{M})$ composed of forms with compact supports. We have $\ll d\xi, \eta\gg=\ll\xi, \delta\eta\gg$ for any $\xi\in\wedge_0^s(\overline{M})$ and any $\eta\in\wedge_0^{s+1}(\overline{M})$. Let $L_2^s(\overline{M})$ be the completion of $\wedge_0^s(\overline{M})$ with respect to the scalar product $\ll\ ,\ \gg$. We set $\|\cdot\|=\ll\cdot, \cdot\gg^{1/2}$.

For any $\xi\in\wedge^1(\overline{M})$, we define $t\xi\in\wedge^1(\partial M)$ and $n\xi\in\wedge^0(\partial M)$ by

$$(t\xi)(X)=\xi(\iota_*X), \tag{3.3}$$

$$n\xi=\xi(N) \tag{3.4}$$

for any vector field X on ∂M, where $\wedge^0(\partial M)$ denotes the space of all functions on ∂M. For any $\zeta\in\wedge^1(\partial M)$, we define $C\zeta$ by

$$C\zeta(X)=\zeta(AX) \tag{3.5}$$

for any vector field X on ∂M.

A form $\xi\in\wedge^s(\overline{M})$ is represented by locally

$$\xi=\frac{1}{s!}\xi_{i_1\cdots i_s}dx^{i_1}\wedge\cdots\wedge dx^{i_s}.$$

We set $\nabla_i=\nabla_{\partial/\partial x^i}$ and $\nabla^i=g^{ij}\nabla_j$ where (g^{ij}) denotes the inverse matrix of (g_{ij}). For any $\xi, \eta\in\wedge^s(\overline{M})$, we have

$$\langle\xi, \eta\rangle=\frac{1}{s!}g^{i_1j_1}\cdots g^{i_sj_s}\xi_{i_1\cdots i_s}\eta_{j_1\cdots j_s}. \tag{3.6}$$

For any $\xi\in\wedge^1(\overline{M})$, we have

$$(d\xi)_{ij}=\nabla_i\xi_j-\nabla_j\xi_i, \tag{3.7}$$

$$\delta\xi=-\nabla^i\xi_i, \tag{3.8}$$

$$\nabla_i\nabla_j\xi^i-\nabla_j\nabla_i\xi^i=R_{ji}\xi^i, \tag{3.9}$$

$$(t\xi)_\alpha=B_\alpha^i\xi_i, \tag{3.10}$$

$$n\xi=\xi_iN^i, \tag{3.11}$$

$$C(t\xi)_\alpha=A_\alpha^\beta B_\beta^i\xi_i, \tag{3.12}$$

where $\xi^i=g^{ij}\xi_j$ and R_{ij} denotes the components of the Ricci tensor field of ∇ (cf. [4]).

For a vector field $X=\xi^i\frac{\partial}{\partial x^i}$ on $\overline{M}$, a 1-form ξ associated with X is defined by $\xi=\xi_idx^i=g_{ij}\xi^jdx^i$.

DEFINITION 3.1. A vector field X on $\overline{M}$ is called *tangential* (resp. *normal*) *to* ∂M if $n\xi=0$ (resp. $t\xi=0$) for the 1-form ξ associated with X.

DEFINITION 3.2. A vector field X on $\overline{M}$ is called a *Killing vector field if* $\mathcal{L}_Xg=0$ where $\mathcal{L}$ denotes the Lie derivative operator.

A Killing vector field X (or 1-form ξ associated with X) on $\overline{M}$ satisfies the following:

(3.13) $$\nabla_i\xi_j+\nabla_j\xi_i=0$$

and, from this, we have

(3.14) $$\nabla_i\xi^i=0\,.$$

DEFINITION 3.3. A vector field X on $\overline{M}$ is called "*with finite global norm*" if $\xi\in L_2^1(\overline{M})\cap\wedge^1(\overline{M})$ for the 1-form ξ associated with X.

4. Non-existence of Killing vector field with finite global norm.

For each point p of $\overline{M}$, we denote by $\rho(p)$ the distance from p to ∂M. Since $\overline{M}$ has a compact, conneted boundary ∂M, ρ is well-defined. ρ is a locally Lipschitz function on $\overline{M}$. We set

(4.1) $$B(2k)=\{p\in\overline{M}\,;\ \rho(p)\leqq 2k\}$$

for any $k>0$.

We consider a function μ on $\boldsymbol{R}$ satisfying

(i) $$0\leqq\mu\leqq 1 \quad \text{on} \quad \boldsymbol{R}\,,$$

(ii) $$\mu(t)=1 \quad \text{for} \quad t\leqq 1\,,$$

(iii) $$\mu(t)=0 \quad \text{for} \quad t\geqq 2\,.$$

Then we define functions w_k on $\overline{M}$ by

(4.2) $$w_k(p)=\mu(\rho(p)/k) \qquad k=1,\,2,\,3,\,\cdots$$

for any point p of $\overline{M}$.

LEMMA 4.1. (cf. [1], [5]). *There exists a positive number D, depending only on μ, such that*

(i) $$\|dw_k\wedge\xi\|^2_{B(2k)}\leqq\frac{(n+1)D}{k^2}\|\xi\|^2_{B(2k)}\,,$$

(ii) $$\|dw_k\wedge{}^*\xi\|^2_{B(2k)}\leqq\frac{(n+1)D}{k^2}\|\xi\|^2_{B(2k)}$$

for any $\xi\in\wedge^s(\overline{M})$, where $\|\xi\|^2_{B(2k)}=\langle\!\langle\xi,\,\xi\rangle\!\rangle_{B(2k)}=\int_{B(2k)}\langle\xi,\,\xi\rangle{}^*1$.

We remark that $w_k\xi\in\wedge_0^s(\overline{M})$ for any $\xi\in L_2^s(\overline{M})\cap\wedge^s(\overline{M})$ and $w_k\xi\to\xi\,(k\to\infty)$ in the strong sense. For any $\xi\in L_2^1(\overline{M})\cap\wedge^1(\overline{M})$, we have

(4.3) $$d(w_k^2\xi)=w_k^2d\xi+2w_kdw_k\wedge\xi\,,$$

(4.4) $$\delta(w_k^2\xi)=w_k^2\delta\xi-{}^*(2w_kdw_k\wedge{}^*\xi)\,.$$

PROPOSITION 4.1. *For any Killing vector field X on $\bar{M}$,*

$$\ll w_k \mathcal{R}\xi, w_k\xi \gg_{B(2k)}$$
$$=2\|w_k\nabla\xi\|^2_{B(2k)}+\ll 2w_k dw_k\wedge\xi, \nabla\xi\gg_{B(2k)}$$
$$+\int_{\partial M}\{\langle d(n\xi), t\xi\rangle+\langle C(t\xi), t\xi\rangle\}*1\,,$$

where $\mathcal{R}$ denotes the Ricci transformation on $\wedge^1(\bar{M})$ defined by $(\mathcal{R}\xi)_i=R_i{}^h\xi_h$, ξ is the 1-form associated with X and $(\nabla\xi)_{ij}=\nabla_i\xi_j$.

Proof. We define a 1-from η on $\bar{M}$ by

$$\eta=(\nabla_j\xi_i)\xi^j dx^i\,.$$

By (4.4), we have

$$d(*(w_k^2\eta))=-*\delta(w_k^2\eta)$$
$$=*(-w_k^2\delta\eta+*(2w_k dw_k\wedge*\eta))\,.$$

Then, by Stokes' theorem, we have

$$\int_{B(2k)}d(*(w_k^2\eta))=\int_{\partial B(2k)}\langle N, w_k^2\eta\rangle*1\,.$$

We have, by (3.8), (3.9) and (3.13),

$$-\delta\eta=\langle\mathcal{R}\xi, \xi\rangle-2\langle\nabla\xi, \nabla\xi\rangle$$

and

$$*(2w_k dw_k\wedge*\eta)=-\langle 2w_k dw_k\wedge\xi, \nabla\xi\rangle\,.$$

Next, on ∂M, we have

$$\langle N, \eta\rangle=\langle d(n\xi), t\xi\rangle+\langle C(t\xi), t\xi\rangle\,,$$

since $(\nabla_j\xi_i)N^iN^j=0$. Thus we have

$$\ll w_k\mathcal{R}\xi, w_k\xi\gg_{B(2k)}-2\ll w_k\nabla\xi, w_k\nabla\xi\gg_{B(2k)}$$
$$-\ll 2dw_k\wedge\xi, w_k\nabla\xi\gg_{B(2k)}$$
$$=\int_{\partial B(2k)}w_k^2\{\langle d(n\xi), t\xi\rangle+\langle C(t\xi), t\xi\rangle\}*1$$
$$=\int_{\partial M}\{\langle d(n\xi), t\xi\rangle+\langle C(t\xi), t\xi\rangle\}*1\,.$$

Because $\partial B(2k)=\partial M\cup\{p\in\bar{M}\,;\,\rho(p)=2k\}$, $w_k=0$ on $\{p\in\bar{M}\,;\,\rho(p)=2k\}$ and $w_k=1$ on ∂M. Q. E. D.

PROPOSITION 4.2. *For any Killing vector field X (1-form ξ associated with*

X) on $\overline{M}$ with finite global norm, if $\limsup_{k\to\infty}\ll w_k\mathcal{R}\xi,\ w_k\xi\gg_{B(2k)}<\infty$, *then*

$$\limsup_{k\to\infty}\ll w_k\mathcal{R}\xi,\ w_k\xi\gg_{B(2k)}$$

$$\geqq\|\nabla\xi\|^2+\int_{\partial M}\{\langle d(n\xi),\ t\xi\rangle+\langle C(t\xi),\ t\xi\rangle\}*1\,.$$

Proof. By Lemma 4.1, we have

$$|\ll 2dw_k\wedge\xi,\ w_k\nabla\xi\gg_{B(2k)}|\leqq\|w_k\nabla\xi\|^2_{B(2k)}+\frac{(n+1)D}{k^2}\|\xi\|^2_{B(2k)}\,.$$

Thus we have

$$\ll w_k\mathcal{R}\xi,\ w_k\xi\gg_{B(2k)}$$

$$\geqq 2\|w_k\nabla\xi\|^2_{B(2k)}-\left(\|w_k\nabla\xi\|^2_{B(2k)}+\frac{(n+1)D}{k^2}\|\xi\|^2_{B(2k)}\right)$$

$$+\int_{\partial M}\{\langle d(n\xi),\ t\xi\rangle+\langle C(t\xi),\ t\xi\rangle\}*1$$

$$=\|w_k\nabla\xi\|^2_{B(2k)}-\frac{(n+1)D}{k^2}\|\xi\|^2_{B(2k)}$$

$$+\int_{\partial M}\{\langle d(n\xi),\ t\xi\rangle+\langle C(t\xi),\ t\xi\rangle\}*1\,.$$

Therefore we have

$$\limsup_{k\to\infty}\ll w_k\mathcal{R}\xi,\ w_k\xi\gg_{B(2k)}$$

$$\geqq\|\nabla\xi\|^2+\int_{\partial M}\{\langle d(n\xi),\ t\xi\rangle+\langle C(t\xi),\ t\xi\rangle\}*1\,.$$

Q. E. D.

THEOREM 4.1. *Let $\mathcal{M}$ be a complete, non-compact, connected and orientable Riemannian manifold of dimension $n+1$. Let $\overline{M}=\partial M\cup M$ be a non-compact Riemannian manifold such that M is a non-compact, connected, open submanifold of $\mathcal{M}$ and $\partial M=\overline{M}-M$ (boundary of $\overline{M}$) is an n dimensional, compact, connected submanifold of $\mathcal{M}$. The Riemannian metric on $\overline{M}$ is induced from $\mathcal{M}$.*

(i) *Suppose that $\limsup_{k\to\infty}\ll w_k\mathcal{R}\xi,\ w_k\xi\gg_{B(2k)}\leqq 0$ for any $\xi\in L^1_2(\overline{M})\cap\wedge^1(\overline{M})$ and the second fundamental form of ∂M with respect to the unit outer normal vector field is non-negative. If every Killing vector field on $\overline{M}$ with finite global norm is tangential to ∂M, then it is a parallel vector field.*

(ii) *Suppose that $\limsup_{k\to\infty}\ll w_k\mathcal{R}\xi,\ w_k\xi\gg_{B(2k)}\leqq 0$ for any $\xi\in L^1_2(\overline{M})\cap\wedge^1(\overline{M})$. If every Killing vector field on $\overline{M}$ with finite global norm is normal to ∂M, then it is a parallel vector field.*

Proof. Since the second fundamental form of ∂M is nonnegative, we have

$$\int_{\partial M}\langle C(t\xi),\ t\xi\rangle{*}1\geqq 0\,.$$

Thus, by Proposition 4.2, we have the assersions. Q. E. D.

We have easily the following theorem from above theorem.

THEOREM 4.2. *Let $\bar{M}=\partial M\cup M$ be as Theorem 4.1.*

(i) *If $\bar{M}$ is of negative Ricci curvature and the second fundamental form of ∂M with respect to the unit outer normal vector field is non-negative, then there is no non-zero Killing vector field on $\bar{M}$ with finite global norm and tangential to ∂M.*

(ii) *If $\bar{M}$ is of negative Ricci curvature, then there is no non-zero Killing vector field on $\bar{M}$ with finite global norm and normal to ∂M.*

This theorem is a generalization of the results of K. Yano [3] (cf. [4]).

EXAMPLE 4.1. We set $r=(x^2+y^2+z^2)^{1/2}$ for any point (x, y, z) of $\boldsymbol{R}^3$ and $x=r\cos\theta_1$, $y=r\sin\theta_1\cos\theta_2$, $z=r\sin\theta_1\sin\theta_2$ (that is, (θ_1, θ_2, r) is the spherical coordinates in $\boldsymbol{R}^3$). For two positive constant numbers a_1 and $a_2 (a_1<a_2)$, we consider a metric ds^2 on $\boldsymbol{R}^3$ such that $ds^2=r^2((d\theta_1)^2+\sin^2\theta_1(d\theta_2)^2)+(dr)^2$ for $r\leqq a_1$, $=r^{-2/3}((d\theta_1)^2+\sin^2\theta_1(d\theta_2)^2)+(dr)^2$ for $r\geqq(a_1+a_2)/2$. Then $\mathcal{M}=(\boldsymbol{R}^3, ds^2)$ is a complete, non-compact, connected and orientable Riemannian manifold. We set $\bar{M}=\{(\theta_1, \theta_2, r)\in\mathcal{M};\ r\geqq a_2\}$, then $\bar{M}$ is a non-compact, connected and orientable Riemannian manifold with a compact, connected boundary $\partial M=\{(\theta_1, \theta_2, r)\in\mathcal{M};\ r=a_2\}$. Then we have

(i) The volume of $\bar{M}$ is infinite.

(ii) A vector field $X=\partial/\partial\theta_2$ on $\bar{M}$ is a Killing vector field with finite global norm and tangential to ∂M.

(iii) $\langle\!\langle\mathcal{R}\xi, \xi\rangle\!\rangle=\infty$ for the 1-form ξ associated with X.

(iv) The second fundamental form of ∂M with respect to the unit outer normal vector field is negative.

EXAMPLE 4.2. Let $\mathcal{M}$ be a surface of revolution in $\boldsymbol{R}^3$ defined by $x=e^{-u^2/2}\cos v$, $y=e^{-u^2/2}\sin v$, $z=u$ $(-\infty<u<\infty,\ 0\leqq v\leqq 2\pi)$. Then $\mathcal{M}$ is a complete, non-compact, connected and orientable Riemannian manifold with the Riemannian metric $ds^2=e^{-u^2}(dv)^2+(1+u^2e^{-u^2})(du)^2$. For a constant number $a_0>1$, we set $\bar{M}=\{(v, u)\in\mathcal{M};\ u\geqq a_0\}$. Then $\bar{M}$ is a noncompact, connected and orientable Riemannian manifold with a compact, connected boundary $\partial M=\{(v, u)\in\mathcal{M};\ u=a_0\}$. Then we have

(i) The volume of $\bar{M}$ is finite.

(ii) The Ricci curvature of $\bar{M}$ is negative.

(iii) The second fundamental form of ∂M with respect to the unit outer normal vector field is negative.

(iv) A vector field $X=\partial/\partial v$ on $\bar{M}$ is a Killing vector field with finite global norm and tangential to ∂M.

EXAMPLE 4.3. Let (θ, r) be the polar coordinates in $\boldsymbol{R}^2$ and $\mathcal{M}=\boldsymbol{R}^2-\{(0, 0)\}$. We take four constant numbers a_1, a_2, a_3, a_4 such that $0<a_1<a_2<a_3<a_4<1$, and we consider two functions $h_1, h_2: (0, \infty)\to\boldsymbol{R}$ satisfying $0\leqq h_i(r)\leqq 1$ $(i=1, 2)$ for $0<r$ and

$$h_1(r)=1,\ h_2(r)=0 \quad \text{for} \quad 0<r\leqq a_2\,,$$

$$h_1(r)=0,\ h_2(r)=1 \quad \text{for} \quad a_3\leqq r\,.$$

Then we define a Riemannian metric ds^2 on $\mathcal{M}$ by

$$ds^2=(h_1(r)(\log r)^{-2}+h_2(r)r^{-4/3})(d\theta)^2$$
$$+(h_1(r)r^{-2}(\log r)^{-2}+h_2(r))(dr)^2\,.$$

$(\mathcal{M}, ds^2)$ is a complete, non-compact, connected and orientable Riemannian manifold. We set $\bar{M}=\{(\theta, r)\in\mathcal{M};\ r\geqq a_1\}$. Then $\bar{M}$ is a non-compact, connected and orientable Riemannian manifold with a compact, connected boundary $\partial M=\{(\theta, r)\in\mathcal{M},\ r=a_1\}$. Then we have

(i) The volume of $\bar{M}$ is infinite.

(ii) A vector field $X=\partial/\partial\theta$ on $\bar{M}$ is a Killing vector field with finite global norm and tangential to ∂M.

(iii) $0<\langle\langle\mathcal{R}\xi, \xi\rangle\rangle<\infty$ for the 1-form ξ associated with X.

(iv) The second fundamental form of ∂M with respect to the unit outer normal vector field is positive.

We also have examples of Killing vector fields on $\bar{M}$ with finite global norms and normal to ∂M.

The non-existence of harmonic forms on $\bar{M}$ is discussed by H. Kitahara and H. Matsuda [2].

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