J. Math. Soc. Japan
Vol. 68, No. 1 (2016) pp. 51–69
doi: 10.2969/jmsj/06810051

Self-injective artin algebras without short cycles in the component quiver

By Maciej Karpicz

(Received May 20, 2013)
(Revised Jan. 7, 2014)

Abstract. We give a complete description of all self-injective artin algebras of infinite representation type whose component quiver has no short cycles.

1. Introduction and the main result.

Throughout the paper, by an algebra we mean a basic, connected, artin algebra over a commutative artinian ring k. For an algebra A, we denote by $\operatorname{mod} A$ the category of finitely generated right A-modules and by $\operatorname{ind} A$ the full subcategory of $\operatorname{mod} A$ given by the indecomposable modules. If a category $\operatorname{mod} A$ admits only finitely many pairwise, non-isomorphic indecomposable modules, then A is said to be of finite representation type. Moreover, an algebra A is called self-injective if A_A is an injective module, or equivalently, the projective and injective modules in $\operatorname{mod} A$ coincide. A prominent class of self-injective algebras is formed by the orbit algebras $\widehat{B}/G$, where $\widehat{B}$ is the repetitive category of an algebra B and G is an admissible group of automorphisms of $\widehat{B}$.

An important combinatorial and homological invariant of the module category $\operatorname{mod} A$ of an algebra A is its Auslander–Reiten quiver $\Gamma_A = \Gamma(\operatorname{mod} A)$. It describes the structure of the quotient category $\operatorname{mod} A/\operatorname{rad}_A^{\infty}$, where $\operatorname{rad}_A^{\infty}$ is the infinite Jacobson radical of $\operatorname{mod} A$. By a result of Auslander, A is of finite representation type if and only if $\operatorname{rad}_A^{\infty} = 0$.

In general, it is important to study the behavior of the components of Γ_A in the category $\operatorname{mod} A$. Following [**24**], a component $\mathcal{C}$ of Γ_A is called *generalized standard* if $\operatorname{rad}_A^{\infty}(X, Y) = 0$ for all modules X and Y in $\mathcal{C}$. It has been proved in [**24**] that every generalized standard component $\mathcal{C}$ of Γ_A is *almost periodic*, that is, all but finitely many τ_A-orbits in $\mathcal{C}$ are periodic. Moreover, the additive closure $\operatorname{add}(\mathcal{C})$ of a generalized standard component $\mathcal{C}$ of Γ_A is closed under extensions in $\operatorname{mod} A$. A component of Γ_A of the form $\mathbb{Z}\mathbb{A}_{\infty}/(\tau_A^r)$, where r is a positive integer, is called a *stable tube* of rank r. We note that (see [**24**, Theorem 2.3]), for A self-injective, every infinite, generalized standard component $\mathcal{C}$ of Γ_A is either acyclic with finitely many τ_A-orbits or is a *quasi-tube* (the stable part $\mathcal{C}^s$ of $\mathcal{C}$ is a stable tube). Following [**23**], the *component quiver* Σ_A of an algebra A has the components of Γ_A as vertices and two components $\mathcal{C}$ and $\mathcal{D}$ of Γ_A are linked in Σ_A by an arrow $\mathcal{C} \to \mathcal{D}$ if $\operatorname{rad}_A^{\infty}(X, Y) \neq 0$ for some modules X in $\mathcal{C}$ and Y in

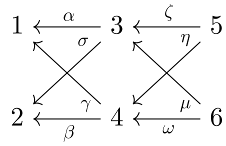
---

2010 *Mathematics Subject Classification.* Primary 16D50; Secondary 16G10, 16G70.
Key Words and Phrases. self-injective artin algebra, orbit algebra, quasitilted algebra, quasi-tube, short cycle, component quiver.

$\mathcal{D}$. In particular, a component $\mathcal{C}$ of Γ_A is generalized standard if and only if Σ_A has no loop at $\mathcal{C}$. By a *short cycle* in Σ_A we mean a cycle $\mathcal{C} \to \mathcal{D} \to \mathcal{C}$, where possibly $\mathcal{C} = \mathcal{D}$. We also mention that the component quiver Σ_A of a self-injective algebra A of infinite representation type is fully cyclic, that is, any finite number of components of Γ_A lies on a common cycle in Σ_A.

Often, in the representation theory of artin algebras, it is possible to recover the ring structure of an algebra from the structure of components in its Auslander–Reiten quiver and their properties in the module category. Moreover, in [**27**], Skowroński proved that every finite dimensional algebra is a factor algebra of a symmetric algebra whose Auslander–Reiten quiver contains a generalized standard stable tube. Therefore, an interesting open problem is to provide a description of all self-injective artin algebras whose Auslander–Reiten quiver contains a generalized standard component. In [**29**] Skowroński and Yamagata gave a complete description of those self-injective artin algebras, which contain a non-periodic, generalized standard component. Hence, it remains to describe the self-injective artin algebras for which the Auslander–Reiten quiver contains a generalized standard quasi-tube. However, as it was shown in [**27**], indecomposable, finite-dimensional modules with periodic syzygies over symmetric algebras may be arbitrarily complicated, hence one should impose on a self-injective algebra additional conditions, like relationship between components, which is reflected in the component quiver (see [**11**] for another result it this direction).

In order to formulate the main result we need a special class of algebras. For a field k of characteristic different from 2, the *exceptional tubular algebra* B_{ex} is the tubular algebra of the tubular type $(2, 2, 2, 2)$ in the sense of [**18**], which is given by the following ordinary quiver



and the relations $\zeta\alpha = \eta\gamma$, $\mu\alpha = \omega\gamma$, $\zeta\sigma = \eta\beta$ and $\mu\sigma = -\omega\beta$. Moreover, an automorphism φ of the exceptional tubular algebra B_{ex} is said to be *distinguished* if $\varphi(\gamma) = a\sigma$, $\varphi(\sigma) = b\gamma$, $\varphi(\beta) = c\alpha$, $\varphi(\alpha) = d\beta$, $\varphi(\mu) = e\eta$, $\varphi(\eta) = r\mu$, $\varphi(\omega) = u\zeta$ and $\varphi(\zeta) = v\omega$ for $a, b, c, d, e, r, u, v \in k \setminus \{0\}$ satisfying the following relations $dv = -ar$, $de = au$, $bv = cr$ and $be = -cu$. In fact, $a = -dv/r$, $b = cr/v$, $e = -uv/r$, for $c, d, r, u, v \in k \setminus \{0\}$.

The aim of the paper is to prove the following theorem characterizing the class of representation-infinite self-injective artin algebras whose component quiver Σ_A contains no short cycles. For definitions of the concepts, used in the formulation of the theorem below, see the Preliminaries. We denote by $\nu_{\widehat{B}}$ the Nakayama automorphism of $\widehat{B}$.

Theorem 1.1. *Let A be a basic, connected, self-injective artin algebra of infinite representation type over an artinian ring k. The following statements are equivalent.*

(i) *The component quiver Σ_A has no short cycles.*
(ii) *k is a field, there exists a tilted algebra of Euclidean type or a tubular algebra B*

and an infinite cyclic group G of automorphisms of $\widehat{B}$ such that A is isomorphic to the orbit algebra $\widehat{B}/G$ and moreover:

(a) *either there exists a strictly positive automorphism φ of $\widehat{B}$ such that $G = (\varphi\nu_{\widehat{B}}^2)$, or*

(b) *B is an exceptional tubular algebra and there exists a rigid automorphism φ of $\widehat{B}$ such that $G = (\varphi\nu_{\widehat{B}}^2)$, whose restriction to B is a distinguished automorphism.*

By a *short cycle* in $\operatorname{mod} A$ we mean a sequence $M \xrightarrow{f} N \xrightarrow{g} M$ of non-zero non-isomorphisms between indecomposable modules in $\operatorname{mod} A$ [**17**], and such a cycle is said to be *infinite* if at least one of the homomorphisms f or g belongs to $\operatorname{rad}_A^\infty$. Moreover, following [**16**], by an *external short path* of a component $\mathcal{C}$ of Γ_A we mean a sequence $X \to Y \to Z$ of non-zero homomorphisms between indecomposable modules in $\operatorname{mod} A$ with X and Z in $\mathcal{C}$ but Y not in $\mathcal{C}$. The assumption (i) of Theorem 1.1 implies that the module category $\operatorname{mod} A$ of A contains no infinite short cycles and every component $\mathcal{C}$ in Γ_A has no external short paths. Therefore, in the proof of Theorem 1.1, we use [**11**, Theorem 1.1], and hence results obtained by Skowroński and Yamagata in [**28**], [**29**] and [**30**].

We note that in our paper [**10**] we presented a similar classification of self-injective algebras of infinite representation type over an algebraically closed field whose component quiver has no short cycles. However, due to a mistake in one result in [**22**] (see Remark 3.2), the characterization given in [**10**] omits one important case of tubular algebras and therefore is incorrect.

The paper is organized as follows. In Section 2 we recall the essential background. In Section 3 we show that every automorphism of non-exceptional tubular algebra fixes a point. Finally, Section 4 is devoted to the proof of Theorem 1.1.

For a basic background on the representation theory of algebras applied in the paper we refer to the books [**18**], [**20**], [**21**] and [**31**].

2. Preliminaries.

Let A be an artin algebra over a commutative artin ring k, D the standard duality $\operatorname{Hom}_k(-, E)$ on $\operatorname{mod} A$, where E is a minimal injective cogenerator of $\operatorname{mod} k$. We denote by Γ_A the Auslander–Reiten quiver of A. Recall that Γ_A is a valued translation quiver whose vertices are the isomorphism classes of modules X in $\operatorname{ind} A$, the valued arrows of Γ_A describe minimal left almost split morphisms with indecomposable domain and minimal right almost split morphisms with indecomposable codomain, and the translation is given by the Auslander–Reiten translations $\tau_A = \mathrm{DTr}$ and $\tau_A^{-1} = \mathrm{TrD}$ (see [**31**, Chapter III] for details). We will identify vertices of Γ_A with the corresponding indecomposable modules and by a component in Γ_A we mean a connected component. Let $\mathcal{C}$ be a family of components of Γ_A. Then $\mathcal{C}$ is said to be *sincere* if any simple A-module occurs as a composition factor of a module in $\mathcal{C}$, and *faithful* if its annihilator $\operatorname{ann}_A(\mathcal{C}) = \bigcap_{X\in\mathcal{C}} \operatorname{ann}_A(X)$ in A is zero. Observe that if $\mathcal{C}$ is faithful, then $\mathcal{C}$ is sincere. Moreover, the family $\mathcal{C}$ is said to be *separating* in $\operatorname{mod} A$ if the indecomposable modules in $\operatorname{mod} A$ split into three disjoint classes $\mathcal{P}^A$, $\mathcal{C}^A = \mathcal{C}$ and $\mathcal{Q}^A$ such that:

(S1) $\mathcal{C}^A$ is a sincere generalized standard family of components;
(S2) $\mathrm{Hom}_A(\mathcal{Q}^A, \mathcal{P}^A) = 0$, $\mathrm{Hom}_A(\mathcal{Q}^A, \mathcal{C}^A) = 0$, $\mathrm{Hom}_A(\mathcal{C}^A, \mathcal{P}^A) = 0$;
(S3) any homomorphism from $\mathcal{P}^A$ to $\mathcal{Q}^A$ factors through the additive category $\mathrm{add}\,\mathcal{C}^A$ generated by $\mathcal{C}^A$.

Moreover, a separating family $\mathcal{C}^A = (\mathcal{C}_i^A)_{i\in I}$ is *strongly separating* if

(S4) any homomorphism from $\mathcal{P}^A$ to $\mathcal{Q}^A$ factors trough the additive category $\mathrm{add}\,\mathcal{C}_i^A$, for any $i \in I$.

Let Λ be a canonical algebra in the sense of Ringel [**18**] (and [**19**]). Then the quiver Q_Λ of Λ has a unique sink and a unique source. Denote by Q_Λ^* the quiver obtained from Q_Λ by removing the unique source of Q_Λ and the arrows attached to it. Then Λ is said to be a *canonical algebra of Euclidean type* (respectively, of *tubular type*) if Q_Λ^* is a Dynkin quiver (respectively, a Euclidean quiver). The general shape of the Auslander–Reiten quiver Γ_Λ of Λ is as follows:

$$\Gamma_\Lambda = \mathcal{P}^\Lambda \vee \mathcal{T}^\Lambda \vee \mathcal{Q}^\Lambda,$$

where $\mathcal{P}^\Lambda$ is a family of components containing a unique postprojective component $\mathcal{P}(\Lambda)$ and all indecomposable projective Λ-modules, $\mathcal{Q}^\Lambda$ is a family of components containing a unique preinjective component $\mathcal{Q}(\Lambda)$ and all indecomposable injective Λ-modules, and $\mathcal{T}^\Lambda$ is an infinite family of pairwise orthogonal, generalized standard, faithful stable tubes, separating $\mathcal{P}^\Lambda$ from $\mathcal{Q}^\Lambda$, and with all but finitely many stable tubes of rank one. An algebra C of the form $\mathrm{End}_\Lambda(T)$, where T is a multiplicity-free tilting module from the additive category $\mathrm{add}(\mathcal{P}^\Lambda)$ of $\mathcal{P}^\Lambda$ is said to be a *concealed canonical algebra* of type Λ ([**12**]). More generally, an algebra B of the form $\mathrm{End}_\Lambda(T)$, where T is a multiplicity-free tilting module from the additive category $\mathrm{add}(\mathcal{P}^\Lambda \cup \mathcal{T}^\Lambda)$ of $\mathcal{P}^\Lambda \cup \mathcal{T}^\Lambda$ is said to be an *almost concealed canonical algebra of type* Λ [**13**]. An almost concealed canonical algebra B of tubular type is called a *tubular algebra*. Recall that a tubular algebra B may be obtained as a tubular branch extension of a concealed algebra of Euclidean type (see [**18**] and [**19**]). Moreover, an almost concealed canonical algebra of Euclidean type is a representation-infinite, tilted algebra of Euclidean type for which the preinjective component contains all indecomposable injective modules (see [**18**]).

Let A be a self-injective algebra. Recall that an algebra A is self-injective if and only if $A \cong \mathrm{D}(A)$ in $\mathrm{mod}\,A$. Let $\{e_i \mid 1 \le i \le s\}$ be a complete set of pairwise orthogonal primitive idempotents of A whose sum is the identity 1_A of A. We denote by $\nu = \nu_A$ the *Nakayama automorphism* of A inducing an $A-A$-bimodule isomorphism $A \cong \mathrm{D}(A)_\nu$, where $\mathrm{D}(A)_\nu$ denotes the right A-module obtained from $\mathrm{D}(A)$ by changing the right operation of A as follows: $f \cdot a = f\nu(a)$ for each $a \in A$ and $f \in \mathrm{D}(A)$. Hence we have $\mathrm{soc}(\nu(e_i)A) \cong \mathrm{top}(e_iA)$ $(= e_iA/\mathrm{rad}(e_iA))$ as right A-modules for all $i \in \{1,\dots,s\}$. Since $\{\nu(e_i)A \mid 1 \le i \le s\}$ is a complete set of representatives of indecomposable projective right A-modules, there is a (*Nakayama*) *permutation* of $\{1,\dots,s\}$, denoted again by ν, such that $\nu(e_i)A \cong e_{\nu(i)}A$ for all $i \in \{1,\dots,s\}$. Invoking the Krull-Schmidt theorem, we may assume that $\nu(e_iA) = \nu(e_i)A = e_{\nu(i)}A$ for all $i \in \{1,\dots,s\}$.

Let B be an algebra and $1_B = e_1 + \dots + e_n$ a decomposition of the identity of

B into the sum of pairwise orthogonal primitive idempotents of B. We associate to B a self-injective, locally bounded k-category $\widehat{B}$, called the *repetitive category* of B. The objects of $\widehat{B}$ are $e_{m,i}$, $m \in \mathbb{Z}$, $i \in \{1, \dots, n\}$ and the morphism spaces are defined in the following way

$$\widehat{B}(e_{m,i}, e_{r,j}) = \begin{cases} e_j B e_i, & r = m, \\ D(e_i B e_j), & r = m+1, \\ 0, & \text{otherwise.} \end{cases}$$

Observe that $e_j B e_i = \operatorname{Hom}_B(e_i B, e_j B)$, $\mathrm{D}(e_i B e_j) = e_j \,\mathrm{D}(B) e_i$ and

$$\bigoplus_{(r,i) \in \mathbb{Z} \times \{1,\dots,n\}} \widehat{B}(e_{m,i}, e_{r,j}) = e_j B \oplus \mathrm{D}(B e_j),$$

for any $m \in \mathbb{Z}$ and $j \in \{1, \dots, n\}$. We denote by $\nu_{\widehat{B}}$ the *Nakayama automorphism* of $\widehat{B}$ defined by

$$\nu_{\widehat{B}}(e_{m,i}) = e_{m+1,i}, \text{ for any } (m,i) \in \mathbb{Z} \times \{1, \dots, n\}.$$

An automorphism φ of the k-category $\widehat{B}$ is said to be:

- *positive* if for each pair $(m,i) \in \mathbb{Z} \times \{1, \dots, n\}$ we have $\varphi(e_{m,i}) = e_{p,j}$ for some $p \geq m$ and $j \in \{1, \dots, n\}$;
- *rigid* if for each pair $(m,i) \in \mathbb{Z} \times \{1, \dots, n\}$ there exists $j \in \{1, \dots, n\}$ such that $\varphi(e_{m,i}) = e_{m,j}$;
- *strictly positive* if φ is positive but not rigid.

A group G of automorphisms of $\widehat{B}$ is said to be *admissible* if G acts freely on the set of objects of $\widehat{B}$ and has finitely many orbits. Then we may consider the orbit category $\widehat{B}/G$ of $\widehat{B}$ with respect to G whose objects are G-orbits of objects in $\widehat{B}$, and the morphism spaces are given by

$$(\widehat{B}/G)(a,b) = \left\{ f_{y,x} \in \prod_{(x,y) \in a \times b} \widehat{B}(x,y) \mid \forall_{g \in G, (x,y) \in a \times b}\ g f_{y,x} = f_{gy,gx} \right\}$$

for all objects a, b of $\widehat{B}/G$. Since $\widehat{B}/G$ has finitely many objects and the morphism spaces in $\widehat{B}/G$ are finitely generated, we have the associated self-injective artin algebra $\bigoplus(\widehat{B}/G)$ which is the direct sum of all morphism spaces in $\widehat{B}/G$, called the *orbit algebra* of $\widehat{B}$ with respect to G. For example, the infinite cyclic group $(\nu_{\widehat{B}})$ generated by $\nu_{\widehat{B}}$ is admissible and $\widehat{B}/(\nu_{\widehat{B}})$ is the trivial extension $B \ltimes \mathrm{D}(B)$ of B by $\mathrm{D}(B)$.

We refer to [**28**] and [**30**] for criteria on a self-injective algebra A to be so that there exist an algebra B and a strictly positive automorphism φ of $\widehat{B}$, such that A is isomorphic to $\widehat{B}/(\varphi \nu_{\widehat{B}})$.

3. Tubular algebras.

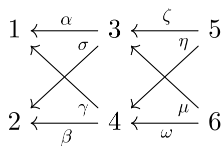
Let B be an algebra. Moreover, let $1_B = e_1 + \cdots + e_n$ be a decomposition of the identity of B into the sum of pairwise orthogonal, primitive idempotents. We say that an algebra automorphism φ of B is *rigid* if $\varphi(\{e_1, \ldots, e_n\}) = \{e_1, \ldots, e_n\}$. Note that, if we consider an algebra B as a category, whose objects are idempotents $e_1, \ldots, e_n$, then an algebra rigid automorphism φ is a rigid automorphism of the category B.

From now on we assume that an automorphism of an algebra B is rigid. Let $1_B = e_1 + \cdots + e_n$ be a decomposition of the identity of B into the sum of pairwise orthogonal, primitive idempotents.

The aim of this section is to show that for a tubular algebra B and a rigid automorphism φ of B, if B is not an exceptional algebra and φ a distinguished automorphism, then φ admits a fixed point, that is φ induces an automorphism φ of the ordinary quiver Q_B of the algebra B such that there is a vertex e for which $\varphi(e) = e$. This will be a consequence of Lemma 3.1 and Proposition 3.4.

Let B be a tubular algebra which is a tubular branch extension of a concealed algebra of Euclidean type $\widetilde{\mathbb{A}}_n$. If φ is an automorphism of the ordinary quiver Q_B of B, then φ acts freely on the set of vertices of Q_B only if Q_B is the ordinary quiver of the exceptional tubular algebra B_{ex} (see [**22**, Section 3]). Therefore, we start with the following lemma describing the case of an exceptional tubular algebra B_{ex}.

Lemma 3.1. *Let B be a tubular algebra over a field k given by the quiver*



with relations $\alpha\zeta = \gamma\eta$, $\alpha\mu = \gamma\omega$, $\sigma\zeta = \beta\eta$ and $\sigma\mu = x\beta\omega$, where $x \in k \setminus \{0,1\}$. Let φ be an automorphism of B. Then φ acts freely on vertices of Q_B if and only if B is exceptional and φ a distinguished automorphism of B.

Proof. Let φ be an automorphism of the algebra B acting freely on vertices of Q_B. Then $\varphi(\gamma) = a\sigma$, $\varphi(\sigma) = b\gamma$, $\varphi(\beta) = c\alpha$, $\varphi(\alpha) = d\beta$, $\varphi(\mu) = e\eta$, $\varphi(\eta) = r\mu$, $\varphi(\omega) = u\zeta$ and $\varphi(\zeta) = v\omega$ for a, b, c, d, e, r, u, $v \in k \setminus \{0\}$. Taking the values of the automorphism φ of B on the above equalities we get $dv\beta\omega = ar\sigma\mu$, $de\beta\eta = au\sigma\zeta$, $bv\gamma\omega = cr\alpha\mu$, $be\gamma\eta = xcu\alpha\zeta$, and hence the equalities $dv = xar$, $de = au$, $bv = cr$ and $be = xcu$. Therefore, combining those equalities, we get $xaer = dev = auv$ and $cer = bev = xcuv$, which implies $xer = uv$, $er = xuv$, hence $x^2 = 1$. Because $x \neq 0, 1$, we get a contradiction if and only if $x \neq -1$ and k is not a field of characteristic 2. Therefore, φ does not fix a point if $x = -1 \neq 1$, that is B is the exceptional tubular algebra and φ a distinguished automorphism of B. □

Remark 3.2. The above lemma corrects [**22**, Lemma 3.5].

By *small Euclidean graphs* we understand the following Euclidean graphs: $\widetilde{\mathbb{A}}_{11}$, $\widetilde{\mathbb{A}}_{12}$,

$\widetilde{\mathbb{B}}_m$, $\widetilde{\mathbb{C}}_m$, $\widetilde{\mathbb{BC}}_m$, for $m = 2, \dots, 5$, $\widetilde{\mathbb{BD}}_m$, $\widetilde{\mathbb{CD}}_m$, for $m = 3, 4, 5$, $\widetilde{\mathbb{F}}_{41}$, $\widetilde{\mathbb{F}}_{42}$, $\widetilde{\mathbb{G}}_{21}$ and $\widetilde{\mathbb{G}}_{22}$ (see [**3**]).

Let H be a representation-infinite hereditary artin algebra. A *concealed domain* $\mathcal{DP}(H)$ of H is a subquiver of the postprojective component $\mathcal{P}(H)$ of Γ_H with the following properties:

(d1) $\mathcal{DP}(H)$ is a finite full translation subquiver of $\mathcal{P}(H)$ which is closed under the predecessors in $\mathcal{P}(H)$,

(d2) for any multiplicity-free postprojective tilting H-module $T = T_1 \oplus \dots \oplus T_n$, there exists a postprojective tilting module $T' = T'_1 \oplus \dots \oplus T'_n$ such that the H-modules $T'_1, \dots, T'_n$ are indecomposable, pairwise non-isomorphic, lie in $\mathcal{DP}(H)$, and there is an isomorphism of algebras

$$\operatorname{End} T_H \cong \operatorname{End} T'_H.$$

We set $\mathcal{DP}(H) := \{\tau_H^{-r} P(a) \mid r \in \{1, \dots, r_\Delta\} \text{ and } a \in \Delta_0\}$, where r_Δ is a minimal positive integer such that $\operatorname{Hom}_H(P(a), \tau_H^{-r} P(b)) \neq 0$ for all $r \geq r_\Delta$ and $a, b \in \Delta_0$. Note, that an integer r_Δ exists because the postprojective component $\mathcal{P}(H)$ of a hereditary algebra H, contains only finitely many non-sincere modules. Therefore, in order to compute the ordinary quivers of concealed algebras, it is sufficient to calculate the ordinary quivers of tilted algebras $\operatorname{End}_H(T)$, for which the tilting module T has all direct summands from $\mathcal{DP}(H)$.

Remark 3.3. Note that for a given hereditary algebra H of Euclidean type, different from $\widetilde{\mathbb{A}}_n$, the concealed domain $\mathcal{DP}(H)$ of H admits a tilting module T such that the quiver of $B = \operatorname{End}_H(T)$ has the same underlying graph as Δ and an arbitrary orientation of arrows. Moreover, there is an equivalence of categories $\mathcal{F}(T) := \{Y \in \operatorname{mod} B \mid \operatorname{Ext}^1_B(U, \mathrm{D}(T)) = 0\}$ in $\operatorname{mod} H$, containing $\mathcal{DP}(H)$, and $\mathcal{X}(T) = \{X \in \operatorname{mod} B \mid \operatorname{Hom}_B(X, \mathrm{D}(T)) = 0\}$ in $\operatorname{mod} B$.

Proposition 3.4. *Let B be a non-exceptional tubular algebra. Then any automorphism $\varphi \in \operatorname{End}_k(B)$ fixes a point of Q_B.*

Proof. Let C be a tame concealed algebra of Euclidean type Δ such that B is a branch extension of C of tubular type.

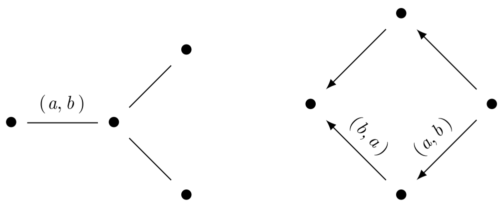
Clearly, for any automorphism $\varphi\colon B \to B$ its restriction to C is an automorphism of C. Therefore, to prove that φ fixes a point in B, it suffices to show that the restriction $\varphi\mid_C$ of φ to C fixes a point. In order to show this, we need shapes of ordinary quivers of tame concealed algebras.

The classification of concealed algebras of Euclidean type Δ, where Δ is one of Euclidean graphs $\widetilde{\mathbb{A}}_n$, for $n \geq 2$, $\widetilde{\mathbb{D}}_n$, for $n \geq 4$, $\widetilde{\mathbb{E}}_6, \widetilde{\mathbb{E}}_7, \widetilde{\mathbb{E}}_8$, in terms of quivers with relations was given by Bongartz and Happel-Vossieck [**8**] (see also [**20**, Section XIV]). Moreover, a simple inspection of the Bongartz-Happel-Vossieck list shows that every automorphism of a concealed algebra of Euclidean type, different from $\widetilde{\mathbb{A}}_n$, fixes a point.

Now, because tubular algebras also arise from tilting modules over canonical algebras of tubular type, then it follows that the Grothendieck group has a small rank. Therefore, we will provide possible shapes of ordinary quivers of concealed algebras of Euclidean type of the form $C = \mathrm{End}_H(T)$, where T is a multiplicity-free tilting module and H is a hereditary algebra, whose ordinary quiver is an orientation of one of the small Euclidean graphs.

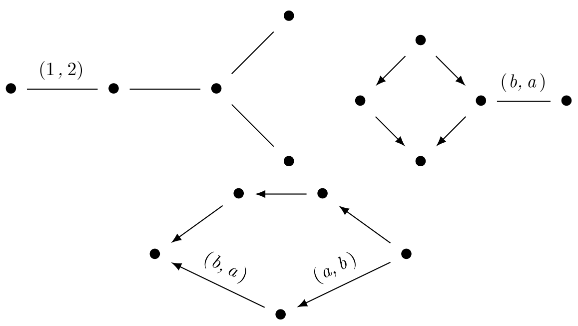
By Remark 3.3 we may consider only hereditary algebras whose ordinary quiver, denoted by $\Delta(G)$, where G is one of the small Euclidean graphs, is equipped with an orientation of arrows from the right hand side to the left hand side. Computation of tilting modules in the concealed domains of such hereditary algebras is rather an easy task. We list here only the shapes (frames) of ordinary quivers of these concealed algebras. In the list of frames below, an unoriented arrow may have an arbitrary orientation.

1. For a hereditary algebra H whose ordinary quiver is $\Delta(\widetilde{\mathbb{B}}_2)$ (respectively, $\Delta(\widetilde{\mathbb{A}}_{11})$, $\Delta(\widetilde{\mathbb{A}}_{12})$, $\Delta(\widetilde{\mathbb{B}}_3)$, $\Delta(\widetilde{\mathbb{B}}_4)$, $\Delta(\widetilde{\mathbb{BC}}_2)$, $\Delta(\widetilde{\mathbb{BC}}_3)$, $\Delta(\widetilde{\mathbb{BC}}_4)$, $\Delta(\widetilde{\mathbb{C}}_2)$, $\Delta(\widetilde{\mathbb{C}}_3)$ and $\Delta(\widetilde{\mathbb{C}}_4)$) we get the frame $\widetilde{\mathbb{B}}_2$ (respectively, $\widetilde{\mathbb{A}}_{11}$, $\widetilde{\mathbb{A}}_{12}$, $\widetilde{\mathbb{B}}_3$, $\widetilde{\mathbb{B}}_4$, $\widetilde{\mathbb{BC}}_2$, $\widetilde{\mathbb{BC}}_3$, $\widetilde{\mathbb{BC}}_4$, $\widetilde{\mathbb{C}}_2$, $\widetilde{\mathbb{C}}_3$ and $\widetilde{\mathbb{C}}_4$).
2. For a hereditary algebra H whose ordinary quiver is $\Delta(\widetilde{\widetilde{\mathbb{BD}}}_3)$ (respectively, $\Delta(\widetilde{\widetilde{\mathbb{CD}}}_3)$) we have the following frames:



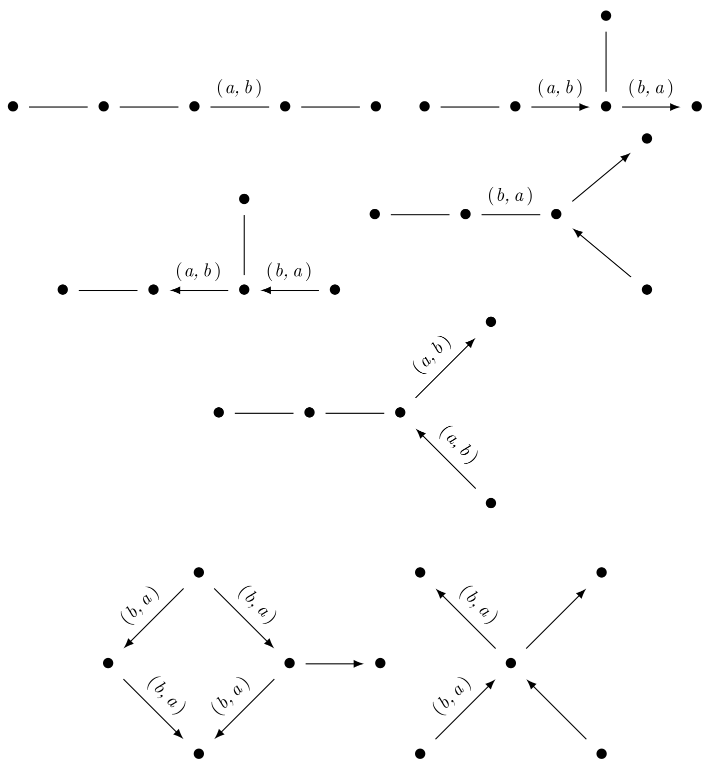
,

where $(a, b) = (1, 2)$ (respectively, $(a, b) = (2, 1)$).
3. For a hereditary algebra H whose ordinary quiver is $\Delta = \widetilde{\widetilde{\mathbb{BD}}}_4$ (respectively, $\widetilde{\widetilde{\mathbb{CD}}}_4$) we have the following frames:



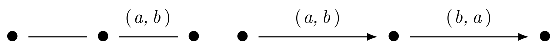
,

where $(a, b) = (1, 2)$ (respectively, $(a, b) = (2, 1)$).
4. For a hereditary algebra H whose ordinary quiver is $\Delta(\widetilde{\mathbb{F}}_{41})$ (respectively, $\Delta(\widetilde{\mathbb{F}}_{42})$) we have the following frames:



where $(a,b)=(1,2)$ (respectively, $(a,b)=(2,1)$).

5. For a hereditary algebra H whose ordinary quiver is $\Delta(\widetilde{\mathbb{G}}_{21})$ (respectively, $\Delta(\widetilde{\mathbb{G}}_{22})$) we have the following two frames:



where $(a,b)=(1,3)$ (respectively, $(a,b)=(3,1)$).

Now, a simple inspection of the above listed frames shows that every automorphism of a concealed algebra of Euclidean type Δ, where Δ is one of the small Euclidean graphs, has a fixed point. □

4. Proof of the main theorem.

Let A be a self-injective artin algebra of infinite representation type such that the component quiver Σ_A of A contains no short cycles. Then the Auslander–Reiten quiver Γ_A of A consists of modules which do not lie on infinite short cycles and all components in Γ_A are generalized standard. In particular, for any indecomposable A-module M, we have $\mathrm{rad}_A^{\infty}(M,M)=0$.

Given a module M in $\mathrm{mod}\,A$, we denote by $[M]$ the image of M in the Grothendieck group $K_0(A)$ of A. Thus $[M]=[N]$ if and only if the modules M and N have the same composition factors including the multiplicities. We also mention that, by a result proved in [**17**], every indecomposable module M in $\operatorname{mod} A$ which does not lie on a short cycle is uniquely determined by $[M]$ (up to isomorphism). In addition, recall that a family $\mathcal{C} = (\mathcal{C}_i)_{i \in I}$ of components of Γ_A is said to have *common composition factors*, if, for each pair i and j in I, there are modules $X_i \in \mathcal{C}_i$ and $X_j \in \mathcal{C}_j$ with $[X_i] = [X_j]$. Moreover, $\mathcal{C}$ is *closed under composition factors* if, for every indecomposable modules M and N in $\operatorname{mod} A$ with $[M] = [N]$, $M \in \mathcal{C}$ forces $N \in \mathcal{C}$.

We start with proving the following proposition which, for an algebraically closed field, was proved by Ringel in [**18**, (5.2)]. We denote by $|V|$ the length of a k-module V.

Proposition 4.1. *Let B be a tubular algebra with the canonical decomposition*

$$\Gamma_B = \mathcal{P} \vee \mathcal{T}_0 \vee \bigvee_{q \in \mathbb{Q}^+} \mathcal{T}_q \vee \mathcal{T}_\infty \vee \mathcal{Q}$$

of the Auslander–Reiten quiver. Then, for any $q \in \mathbb{Q}^+ \cup \{0, \infty\}$, the family $\mathcal{T}_q$ of tubes is closed under composition factors.

Proof. Let M be a B-module from $\mathcal{T}_p^B$ and N be a B-module from $\mathcal{T}_q^B$, p, $q \in \mathbb{Q}^+ \cup \{0, \infty\}$. Assume that $[M] = [N]$. We will show that $p = q$. Assume to the contrary that $p < q$. Take some $s \in \mathbb{Q}^+$ with $p < s < q$. Since the family of stable tubes $\mathcal{T}_s^B = (\mathcal{T}_{s,x}^B)_{x \in \mathbb{X}_s}$ is infinite, there is $x \in \mathbb{X}_s$ such that $\mathcal{T}_{s,x}^B$ is a stable tube of rank one. Take the unique mouth module X in $\mathcal{T}_{s,x}^B$. Clearly, $X = \tau_B X$. We know that the family $\mathcal{T}_s^B$ strongly separates $\mathcal{X} = \mathcal{P}^B \vee \bigvee_{l<s} \mathcal{T}_l^B$ from $\mathcal{Y} = \bigvee_{l>s} \mathcal{T}_l^B \vee \mathcal{Q}^B$, that is, every homomorphism f from $\operatorname{add} \mathcal{X}$ to $\operatorname{add} \mathcal{Y}$ factors through $\operatorname{add} \mathcal{T}_{s,y}^B$, for every $y \in \mathbb{X}_s$. Observe that the injective hull $E_B(M)$ of M is in $\operatorname{add}(\mathcal{T}_\infty \vee \mathcal{Q})$ and the projective cover $P_B(N)$ of N is in $\operatorname{add}(\mathcal{P} \vee \mathcal{T}_0)$. Therefore, $\operatorname{Hom}_B(M, \mathcal{T}_{s,x}^B) \neq 0$ and $\operatorname{Hom}_B(\mathcal{T}_{s,x}^B, N) \neq 0$. Hence, applying [**26**, Lemma 3.9] $\operatorname{Hom}_B(M, X) \neq 0$ and $\operatorname{Hom}_B(X, N) \neq 0$, because $\mathcal{C}_{s,x}^B$ is of rank one. Next, since $\mathcal{T}_s^B$ separates $\mathcal{X}$ from $\mathcal{Y}$, we have $\operatorname{Hom}_B(X, M) = 0$ and $\operatorname{Hom}_B(N, X) = 0$. Further, since $[M] = [N]$, applying [**26**, Proposition 4.1] we have the equalities

$$\begin{aligned} |\operatorname{Hom}_B(X, M)| - |\operatorname{Hom}_B(M, X)| &= |\operatorname{Hom}_B(X, M)| - |\operatorname{Hom}_B(M, \tau_B X)| \\ &= |\operatorname{Hom}_B(X, N)| - |\operatorname{Hom}_B(N, \tau_B X)| \\ &= |\operatorname{Hom}_B(X, N)| - |\operatorname{Hom}_B(N, X)|. \end{aligned}$$

Then $\operatorname{Hom}_B(X, M) = 0$ and $\operatorname{Hom}_B(N, X) = 0$ leads to a contradiction

$$0 > -|\operatorname{Hom}_B(M, X)| = |\operatorname{Hom}_B(X, N)| > 0. \qquad \square$$

Proposition 4.2. *Let A be a basic, connected self-injective algebra of infinite representation type such that the component quiver Σ_A of A contains no short cycles. Then the Auslander–Reiten quiver Γ_A of A admits a family $\mathcal{C} = (\mathcal{C})_{x \in \mathbb{X}}$ of quasi-tubes having common composition factors, closed under composition factors and consisting of modules which do not lie on infinite short cycles in* $\operatorname{mod} A$.

PROOF. Because Σ_A contains no short cycles, then every component in Γ_A is generalized standard. Therefore, since A is of infinite representation type, applying [**25**, Corollary 4.4], we conclude that there exists an ideal I in A such that $A' = A/I$ is tame concealed. Thus, there exists a family of stable tubes $\mathcal{T}^{A'} = (\mathcal{T}_x)_{x\in\mathbb{X}}$ in $\Gamma_{A'}$ with common composition factors. In addition (see [**11**, Section 2]), there is an infinite family $\mathcal{C} = (\mathcal{C}_x)_{x\in\mathbb{X}}$ of quasi-tubes in Γ_A such that, for any $x \in \mathbb{X}$, $\mathcal{T}_x \subseteq \mathcal{C}_x$ and the equality holds for almost all $x \in \mathbb{X}$. Obviously, because $\mathcal{T}^{A'}$ is a family of stable tubes in $\Gamma_{A'}$ with common composition factors, the family $\mathcal{C}$ is a family of quasi-tubes with common composition factors. We claim that $\mathcal{C}$ is closed under composition factors.

Let N be a module in Γ_A and M a module in $\mathcal{C} = (\mathcal{C}_x)_{x\in\mathbb{X}}$. Assume that $[M] = [N]$. We will show that N belongs to $\mathcal{C}$. Let $\mathcal{C}_y$, for some $y \in \mathbb{X}$, be the quasi-tube in the family $\mathcal{C}$ containing M. Let $1_A = e + f$ be a decomposition of 1_A into a sum of two idempotents such that all direct summands of $eA/\operatorname{rad}(eA)$ are isomorphic to the composition factors of modules in $\mathcal{C}$, but the module $fA/\operatorname{rad}(fA)$ has no such direct summands. Consider the quotient algebra $B = A/AfA$. Then $\mathcal{C}_y$ is a component in Γ_B. Moreover, the A-module N is also a module over B.

Since $\mathcal{C}_y$ is a generalized standard quasi-tube without external short paths, applying [**14**, Theorem A], we conclude that B is a quasi-tube enlargement of a concealed canonical algebra C and there is a separating family $\mathcal{C}^B$ of quasi-tubes in Γ_B containing the quasi-tube $\mathcal{C}_y$. In particular, we have a decomposition $\Gamma_B = \mathcal{P}^B \vee \mathcal{C}^B \vee \mathcal{Q}^B$ with $\mathcal{C}^B$ separating $\mathcal{P}^B$ from $\mathcal{Q}^B$.

Therefore, by dual arguments, we may assume that N belongs to $\mathcal{P}^B$.

From [**14**, Theorem C] there is a unique maximal tubular coextension B_l of C inside B and a generalized standard family $\mathcal{C}^{B_l}$ of coray tubes of Γ_{B_l} such that B is obtained from B_l (respectively, $\mathcal{C}^B$ is obtained from $\mathcal{C}^{B_l}$) by a sequence of admissible operations of types $(ad\,1)$ and $(ad\,2)$ (see [**14**]), using modules from $\mathcal{C}^{B_l}$. Moreover, $\mathcal{P}^B = \mathcal{P}^{B_l}$. Hence N is also a B_l-module and therefore, because B_l is a quotient algebra of B by an ideal BhB for an idempotent h, M is also a B_l-module. Further, since every component in Γ_{B_l} is generalized standard, we infer from [**9**] that B_l is an almost concealed canonical algebra of Euclidean or tubular type.

Assume first that B_l is of Euclidean type. Then, because every module from $\mathcal{P}^{B_l}$ is uniquely determined by its composition factors, N belongs to $\mathcal{C}^{B_l}$ in Γ_{B_l}. Thus N is a module from the family $\mathcal{C}$ in Γ_A.

Assume that B_l is of tubular type. Then $\mathcal{P}^{B_l}$ consists of all indecomposable modules which precede the family $\mathcal{C}^{B_l}$ of coray tubes of Γ_{B_l}. Hence, applying Proposition 4.1, we conclude that N belongs to $\mathcal{C}^{B_l}$. Thus N is a module from the family $\mathcal{C}$ in Γ_A.

Summing up, the family $\mathcal{C}^A$ consists of quasi-tubes having common composition factors, is closed under composition factors and, from our assumptions on Σ_A, consists of modules which do not lie on infinite short cycles. □

Recall the following characterization of self-injective algebras proved in [**11**, Theorem 1.1].

THEOREM 4.3. *Let A be a basic, connected, self-injective artin algebra. The following statements are equivalent.*

(i) Γ_A *admits a nonempty family* $\mathcal{C} = (\mathcal{C}_i)_{i\in I}$ *of quasi-tubes having common composition factors, closed on composition factors, and consisting of modules which do not lie on infinite short cycles in* $\operatorname{mod} A$.

(ii) *there exists an almost concealed canonical algebra* B *and an infinite cyclic group of automorphisms of* $\widehat{B}$ *such that* A *is isomorphic to an orbit algebra* $\widehat{B}/G$ *and moreover*:

(a) *either there exists a strictly positive automorphism* φ *of* $\widehat{B}$ *such that* $G = (\varphi\nu^2_{\widehat{B}})$, *or*

(b) B *is a tubular algebra and there exists a rigid automorphism* φ *of* $\widehat{B}$ *such that* $G = (\varphi\nu^2_{\widehat{B}})$, *or*

(c) B *is of Euclidean or wild type and there exists a rigid automorphism* φ *of* $\widehat{B}$ *acting freely on the nonstable tubes of the unique separating family* $\mathcal{T}^B$ *of ray tubes of* Γ_B, *such that* $G = (\varphi\nu^2_{\widehat{B}})$.

It follows now from Lemma 4.2 and Theorem 4.3 that the algebra A is of the form $\widehat{B}/(\varphi\nu^2_{\widehat{B}})$, where B is an almost concealed canonical algebra and φ is a positive automorphism of $\widehat{B}$. Moreover, because Σ_A contains no short cycles, we infer from [**9**] that B is either a tilted algebra of Euclidean type or a tubular algebra. Thus, in order to prove Theorem 1.1, it remains to show that φ is a strictly positive automorphism of $\widehat{B}$ if B is not an exceptional tubular algebra. This will be a consequence of Propositions 4.5 and 4.6.

Note that, by [**11**, Corollary 1.4], k is a field, and therefore A is a finite-dimensional algebra over a field.

We need the following general result which is a consequence of results proved in [**1**], [**4**], [**5**], [**15**] and [**22**].

Theorem 4.4. *Let* B *be a non-exceptional tubular algebra or tilted algebra of Euclidean type,* G *an admissible torsion-free group of automorphisms of* $\widehat{B}$, *and* $A = \widehat{B}/G$ *the associated orbit algebra. Then the following statements hold.*

(i) G *is an infinite cyclic group generated by a strictly positive automorphism* ψ *of* $\widehat{B}$.

(ii) *The push-down functor* $F_\lambda : \operatorname{mod}\widehat{B} \to \operatorname{mod} A$ *associated to the Galois covering* $F\colon \widehat{B} \to \widehat{B}/G = A$ *with Galois group* G *is dense.*

(iii) *The Auslander–Reiten quiver* Γ_A *of* A *is isomorphic to the orbit quiver* $\Gamma_{\widehat{B}}/G$ *of the Auslander–Reiten quiver* $\Gamma_{\widehat{B}}$ *of* $\widehat{B}$ *with respect to the induced action of* G *on* $\Gamma_{\widehat{B}}$.

Proof. The statement (i) is a direct consequence of the assumption imposed on a group G and [**22**, Lemma 2.8 and Lemma 3.5] as well as the results from the Section 3.

Let B be a triangular algebra and $e_1, \dots, e_n$ be pairwise orthogonal primitive idempotents of B with $1_B = e_1 + \cdots + e_n$. For a sink i of Q_B, the reflection S_i^+B of B at i is the algebra $(1-e_i)T_i^+B(1-e_i)$, where T_i^+B is the one-point extension $B[I_B(i)]$ of B by the indecomposable injective B-module $I_B(i)$ at the vertex i. Moreover, identifying B with the full subcategory of the repetitive category $\widehat{B}$ given by the objects $e_{0,j}$, $1 \le j \le n$, S_i^+B is the full subcategory of $\widehat{B}$ given by the objects $e_{0,j}$, for $j \in \{1, \dots, n\} \setminus \{i\}$, and

$e_{1,i} = \nu_{\widehat{B}}(e_{0,i})$. Then the ordinary quiver $Q_{S_i^+ B}$ of $S_i^+ B$ is the reflection $\sigma_i^+ Q_B$ of Q_B at i. From [**1**] and [**15**] there is a sequence of reflections $i_1, \ldots, i_n$, where n is the rank of the Grothendieck group $K_0(B)$ of B, such that $S_{i_n}^+ \cdots S_{i_1}^+ B = \nu_{\widehat{B}}(B)$ and the full subcategory $T_{i_n}^+ \cdots T_{i_2}^+ T_{i_1}^+ B$ of $\widehat{B}$ is exactly the full subcategory form by objects of B and $\nu_{\widehat{B}}(B)$. Moreover, the support of every indecomposable module in $\operatorname{mod} \widehat{B}$ is contained in one of the full subcategories $B_{m,n+1} = \nu_{\widehat{B}}^m (T_{i_n}^+ \cdots T_{i_2}^+ T_{i_1}^+ B)$, for $m \in \mathbb{Z}$, of $\widehat{B}$. Therefore, $\widehat{B}$ is locally support finite (see also [**22**]) and from [**4**], the push-down functor $F_\lambda\colon \operatorname{mod} \widehat{B} \to \operatorname{mod} A$ is dense. Hence (ii).

Finally, the statement (iii) follows from (ii) and [**5**]. □

Proposition 4.5. *Let B be a non-exceptional tubular algebra, G an infinite cyclic admissible group of automorphisms of $\widehat{B}$, and $A = \widehat{B}/G$. Then the component quiver Σ_A of A has no short cycles if and only if there exists a strictly positive automorphism φ of $\widehat{B}$ such that $G = (\varphi \nu_{\widehat{B}}^2)$.*

Proof. It follows from the results established in [**6**], [**7**], [**15**] and [**22**] that the Auslander–Reiten quiver $\Gamma_{\widehat{B}}$ of $\widehat{B}$ has a decomposition

$$\Gamma_{\widehat{B}} = \bigvee_{q \in \mathbb{Q}} \mathcal{C}_q^{\widehat{B}} = \bigvee_{q \in \mathbb{Q}} \bigvee_{x \in \mathbb{X}_q} \mathcal{C}_{q,x}^{\widehat{B}}$$

such that

(1) For each $q \in \mathbb{Z}$, $\mathcal{C}_q^{\widehat{B}}$ is an infinite family $\mathcal{C}_{q,\lambda}^{\widehat{B}}$, $\lambda \in \mathbb{X}_q$, of quasi-tubes containing at least one projective module.

(2) For each $q \in \mathbb{Q} \setminus \mathbb{Z}$, $\mathcal{C}_q^{\widehat{B}}$ is an infinite family $\mathcal{C}_{q,x}^{\widehat{B}}$, $x \in \mathbb{X}_q$, of stable tubes.

(3) For each $q \in \mathbb{Q}$, $\mathcal{C}_q^{\widehat{B}}$ is a family of pairwise orthogonal generalized standard quasi-tubes with common composition factors, closed under composition factors, and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod} \widehat{B}$.

(4) There is a positive integer m such that $3 \le m \le \operatorname{rk} K_0(B)$ and $\nu_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+m}^{\widehat{B}}$ for any $q \in \mathbb{Q}$.

(5) $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{C}_r^{\widehat{B}}) = 0$ for all $q > r$ in $\mathbb{Q}$.

(6) $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{C}_r^{\widehat{B}}) = 0$ for all $r > q + m$ in $\mathbb{Q}$.

(7) For $q \in \mathbb{Q}$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{C}_{q+m}^{\widehat{B}}) \neq 0$ if and only if $q \in \mathbb{Z}$.

(8) For $p < q$ in $\mathbb{Q}$ with $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_r^{\widehat{B}}) \neq 0$ and $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_r^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$ for any $r \in \mathbb{Q}$ with $p \le r \le q$.

(9) For all $p \in \mathbb{Q} \backslash \mathbb{Z}$ and all $q \in \mathbb{Q}$ with $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{p,x}^{\widehat{B}}, \mathcal{C}_{q,y}^{\widehat{B}}) \neq 0$ for all $x \in \mathbb{X}_p$ and $y \in \mathbb{X}_q$.

(10) For all $p \in \mathbb{Q}$ and all $q \in \mathbb{Q} \backslash \mathbb{Z}$ with $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_p^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}}) \neq 0$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{p,x}^{\widehat{B}}, \mathcal{C}_{q,y}^{\widehat{B}}) \neq 0$ for all $x \in \mathbb{X}_p$ and $y \in \mathbb{X}_q$.

We know also from Theorem 4.4 (i) that G is generated by a strictly positive automorphism g of $\widehat{B}$. Consider the canonical Galois covering $F : \widehat{B} \to \widehat{B}/G = A$ and the associated push-down functor $F_\lambda : \operatorname{mod} \widehat{B} \to \operatorname{mod} A$. Since F_λ is dense, we obtain natural isomorphisms of k-modules

$$\bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(X, {}^{g^i}Y) \xrightarrow{\sim} \mathrm{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

$$\bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\widehat{B}}({}^{g^i}X, Y) \xrightarrow{\sim} \mathrm{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

for all indecomposable modules X and Y in $\operatorname{mod}\widehat{B}$.

Assume first that $g = \varphi\nu_{\widehat{B}}^2$ for some strictly positive automorphism φ of $\widehat{B}$. Then it follows from (4) that there is a positive integer $l > 2m$ such that $g(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+l}^{\widehat{B}}$ for any $q \in \mathbb{Q}$. Since $g = \varphi\nu_{\widehat{B}}^2 = (\varphi\nu_{\widehat{B}})\nu_{\widehat{B}}$ with $\varphi\nu_{\widehat{B}}$ a strictly positive automorphism of $\widehat{B}$, invoking the knowledge of the supports of indecomposable modules in $\operatorname{mod}\widehat{B}$ (see [**15**, Section 3]), we conclude that the images $F_\lambda(S)$ and $F_\lambda(T)$ of any non-isomorphic simple $\widehat{B}$-modules S and T which occur as composition factors of modules in a fixed family $\mathcal{C}_q^{\widehat{B}}$ are non-isomorphic simple A-modules. Therefore, it follows from Theorem 4.4 and properties (1)-(4), that, for each $q \in \mathbb{Q}$, $\mathcal{C}_q^A = F_\lambda(\mathcal{C}_q^{\widehat{B}}) = (\mathcal{C}_{q,x}^A)_{x\in\mathbb{X}_q}$, where $\mathcal{C}_{q,x}^A = F_\lambda(\mathcal{C}_{q,x}^{\widehat{B}})$, $x \in \mathbb{X}_q$, is an infinite family of quasi-tubes of Γ_A with common composition factors and closed under composition factors. Take now $p \in \mathbb{Q}$. We claim that $\mathcal{C}_{p,x}^A$, for any $x \in \mathbb{X}_p$, is a quasi-tube without external short paths in $\operatorname{mod} A$. Observe first that, for two indecomposable modules M and N in $\mathcal{C}_p^A$, we have $M = F_\lambda(X)$ and $L = F_\lambda(Y)$, for some indecomposable modules X and Y in $\mathcal{C}_p^{\widehat{B}}$, and F_λ induces an isomorphism of k-modules $\mathrm{Hom}_A(M,N) \xrightarrow{\sim} \mathrm{Hom}_{\widehat{B}}(X,Y)$, by (5), (6) and the inequalities $q + l > q + 2m > q + m$. Suppose now that there is an external short path $M \to L \to N$ in $\operatorname{mod} A$ with M and N in $\mathcal{C}_{p,x}^A$, for some $x \in \mathbb{X}_p$, and L not in $\mathcal{C}_{p,x}^A$. Observe that L is not in $\mathcal{C}_p^A$ because by (3) different quasi-tubes in $\mathcal{C}_p^A$ are orthogonal. Therefore, $M = F_\lambda(X)$, $N = F_\lambda(Y)$ for some X and Y in $\mathcal{C}_{p,x}^{\widehat{B}}$ and $L = F_\lambda(Z)$ for some Z in $\mathcal{C}_r^{\widehat{B}}$ with $r > p$. We have an isomorphism of k-modules, induced by F_λ,

$$\mathrm{Hom}_A(M, L) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(X, {}^{g^i}Z).$$

Since $\mathrm{Hom}_A(M,L) \neq 0$, we may choose, invoking (5), a minimal $r > p$ and $Z \in \mathcal{C}_r^{\widehat{B}}$ such that $L = F_\lambda(Z)$ and $\mathrm{Hom}_{\widehat{B}}(X,Z) \neq 0$. Since X lies in $\mathcal{C}_p^{\widehat{B}}$, applying (6) and (7), we infer that $p < r \leq p + m$. Further, we have also an isomorphism of k-modules, induced by F_λ,

$$\mathrm{Hom}_A(L, N) \xrightarrow{\sim} \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(Z, {}^{g^i}Y).$$

Observe that, for each $i \in \mathbb{Z}$, ${}^{g^i}Y$ is an indecomposable module from $\mathcal{C}_{p+li}^{\widehat{B}}$, and clearly with $F_\lambda({}^{g^i}Y) = F_\lambda(Y) = N$. Since $\mathrm{Hom}_A(L,N) \neq 0$, $L = F_\lambda(Z)$ for $Z \in \mathcal{C}_r^{\widehat{B}}$ with $r > p$, and $Y \in \mathcal{C}_p^{\widehat{B}}$, applying (5), we conclude that $\mathrm{Hom}_{\widehat{B}}(Z, {}^{g^i}Y) \neq 0$, for some $i \geq 1$. But then $p + li \geq p + l > p + 2m \geq r + m$, because $r \leq p + m$, and we obtain a contradiction with (6).

Summing up, we have proved that all quasi-tubes in Γ_A are generalized standard and consist of modules which do not lie on external short paths in $\operatorname{mod} A$. Thus, the component quiver Σ_A of A has no short cycles.

Assume that the component quiver Σ_A has no short cycles. Then, by Lemma 4.2, Γ_A admits a family $\mathcal{C} = (\mathcal{C}_x)_{x\in\mathbb{X}}$ of quasi-tubes with common composition factors, closed under composition factors and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod} A$. We know from property (3) that, for each $q \in \mathbb{Q}$, $\mathcal{C}^A_q = F_\lambda(\mathcal{C}^{\widehat{B}}_q)$ is a family $\mathcal{C}^A_{q,x} = F_\lambda(\mathcal{C}^{\widehat{B}}_{q,x})$, $x \in \mathbb{X}_q$, of quasi-tubes with common composition factors. Moreover, the push-down functor F_λ induces an isomorphism of translation quivers $\Gamma_{\widehat{B}}/G \xrightarrow{\sim} \Gamma_A$ (see Theorem 4.4), and hence every component of Γ_A is a quasi-tube of the form $\mathcal{C}^A_{q,x} = F_\lambda(\mathcal{C}^{\widehat{B}}_{q,x})$ for some $q \in \mathbb{Q}$ and $x \in \mathbb{X}_q$. Then, since the family $\mathcal{C}$ is closed under composition factors, we conclude that there is $r \in \mathbb{Q}$ such that $\mathcal{C}$ contains all quasi-tubes $\mathcal{C}^A_{r,x}$, $x \in \mathbb{X}_r$, of $\mathcal{C}^A_r$. This forces, by [**11**, Proposition 6.4], g to be of the form $g = \varphi\nu^2_{\widehat{B}}$ for some positive automorphism φ of $\widehat{B}$. Suppose that φ is a rigid automorphism of $\widehat{B}$. Then, from Proposition 3.4, we know that the restriction of φ to B fixes an indecomposable projective module, that is there is an indecomposable projective module P such that $\varphi(P) = P$. Thus, let $\mathcal{C}_{p,x}$, for some $p \in \mathbb{Z}$ and $x \in \mathbb{X}_p$, be the quasi-tube, in $\Gamma_{\widehat{B}}$, containing P. Without loss of generality, we may assume that $p = 0$. We have a short cycle of modules in $\operatorname{mod}\widehat{B}$ of the form $P \xrightarrow{f} \nu_{\widehat{B}}(P) \xrightarrow{g} \nu^2_{\widehat{B}}(P)$, where f and g are the following compositions of homomorphisms

$$P \to \operatorname{top}(P) \xrightarrow{\sim} \operatorname{soc}(\nu_{\widehat{B}}(P)) \to \nu_{\widehat{B}}(P),$$

and

$$\nu_{\widehat{B}}(P) \to \operatorname{top}(\nu_{\widehat{B}}(P)) \xrightarrow{\sim} \operatorname{soc}(\nu^2_{\widehat{B}}(P)) \to \nu^2_{\widehat{B}}(P).$$

Consequently, we obtain a short cycle

$$F_\lambda(\mathcal{C}_{0,x}) \to F_\lambda(\mathcal{C}_{m,y}) \to F_\lambda(\mathcal{C}_{0,x})$$

in Σ_A, because $\operatorname{rad}^\infty_A(F_\lambda(\mathcal{C}_{0,x}), F_\lambda(\mathcal{C}_{m,y})) \neq 0$ and $\operatorname{rad}^\infty_A(F_\lambda(\mathcal{C}_{m,y}), F_\lambda(\mathcal{C}_{2m,x})) = \operatorname{rad}^\infty_A(F_\lambda(\mathcal{C}_{m,y}), F_\lambda(\mathcal{C}_{0,x})) \neq 0$, where $\nu_{\widehat{B}}(P) \in \mathcal{C}_{m,\mu}$, for some $y \in \mathbb{X}_m$, which contradicts our assumption. □

Proposition 4.6. *Let B be a tilted algebra of Euclidean type, G an infinite cyclic admissible group of automorphisms of $\widehat{B}$, and $A = \widehat{B}/G$. Then the component quiver Σ_A of A has no short cycle if and only if there exists a strictly positive automorphism φ of $\widehat{B}$ such that $G = (\varphi\nu^2_{\widehat{B}})$.*

Proof. It follows from [**1**], [**2**] and [**22**] that the Auslander–Reiten quiver $\Gamma_{\widehat{B}}$ of $\widehat{B}$ has a decomposition

$$\Gamma_{\widehat{B}} = \bigvee_{q\in\mathbb{Z}} (\mathcal{C}_q^{\widehat{B}} \vee \mathcal{X}_q^{\widehat{B}})$$

such that

(1) For each $q \in \mathbb{Z}$, $\mathcal{X}_q^{\widehat{B}}$ is an acyclic component of Euclidean type.
(2) For each $q \in \mathbb{Z}$, $\mathcal{C}_q^{\widehat{B}}$ is a family $\mathcal{C}_{q,x}^{\widehat{B}}$, $x \in \mathbb{X}_q$, of pairwise orthogonal generalized standard quasi-tubes with common composition factors, closed under composition factors, and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod}\widehat{B}$.
(3) For each $q \in \mathbb{Z}$, we have $\nu_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+2}^{\widehat{B}}$ and $\nu_{\widehat{B}}(\mathcal{X}_q^{\widehat{B}}) = \mathcal{X}_{q+2}^{\widehat{B}}$.
(4) For each $q \in \mathbb{Z}$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{X}_q^{\widehat{B}}, \mathcal{C}_q^{\widehat{B}} \vee \bigvee_{r<q}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$ and $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \bigvee_{r<q}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$.
(5) For each $q \in \mathbb{Z}$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_q^{\widehat{B}}, \mathcal{X}_{q+2}^{\widehat{B}} \vee \bigvee_{r>q+2}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$ and $\operatorname{Hom}_{\widehat{B}}(\mathcal{X}_q^{\widehat{B}}, \bigvee_{r>q+2}(\mathcal{C}_r^{\widehat{B}} \vee \mathcal{X}_r^{\widehat{B}})) = 0$.
(6) For $q \in \mathbb{Z}$ and $x, y \in \mathbb{X}_q$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{q,x}^{\widehat{B}}, \mathcal{C}_{q+2,y}^{\widehat{B}}) \neq 0$ if and only if the quasi-tube $\mathcal{C}_{q,x}^{\widehat{B}}$ is non-stable and $\nu_{\widehat{B}}(\mathcal{C}_{q,y}^{\widehat{B}}) = \mathcal{C}_{q+2,x}^{\widehat{B}}$.
(7) For all $q \in \mathbb{Z}$ and $x, y \in \mathbb{X}_q$, we have $\operatorname{Hom}_{\widehat{B}}(\mathcal{C}_{q,x}^{\widehat{B}}, \mathcal{C}_{q+1,y}^{\widehat{B}}) \neq 0$.
(8) Each each $r \in \mathbb{Z}$, $\mathcal{X}_r$ contains at least one projective module.

We know also from Theorem 4.4 that G is generated by a strictly positive automorphism g of $\widehat{B}$. Hence there exists a positive integer l such that $g(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+l}^{\widehat{B}}$ and $g(\mathcal{X}_q^{\widehat{B}}) = \mathcal{X}_{q+l}^{\widehat{B}}$ for any $q \in \mathbb{Z}$. Consider the canonical Galois covering $F : \widehat{B} \to \widehat{B}/G = A$ and the associated push-down functor $F_\lambda : \operatorname{mod}\widehat{B} \to \operatorname{mod} A$. Since F_λ is dense, we obtain natural isomorphisms of k-modules

$$\bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}(X, {}^{g^i}Y) \xrightarrow{\sim} \operatorname{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

$$\bigoplus_{i\in\mathbb{Z}} \operatorname{Hom}_{\widehat{B}}({}^{g^i}X, Y) \xrightarrow{\sim} \operatorname{Hom}_A(F_\lambda(X), F_\lambda(Y)),$$

for all indecomposable modules X and Y in $\operatorname{mod}\widehat{B}$.

Assume first that the component quiver Σ_A has no short cycles. Then, by Lemma 4.2, Γ_A admits a family $\mathcal{C} = (\mathcal{C}_\lambda)_{\lambda\in\mathbb{X}}$ of quasi-tubes with common composition factors, closed under composition factors and consisting of modules which do not lie on infinite short cycles in $\operatorname{mod} A$. Then it follows from [**11**, Proposition 6.5] that $g = \varphi\nu_{\widehat{B}}^2$ for some positive automorphism φ of $\widehat{B}$. We claim that φ is a strictly positive automorphism of $\widehat{B}$.

Assume that φ is a rigid automorphism of $\widehat{B}$. Take $q = 0$ and, invoking (8), some projective-injective module P in $\mathcal{X}_0$. Let f and g be the following compositions of homomorphisms

$$P \to \operatorname{top}(P) \xrightarrow{\sim} \operatorname{soc}(\nu_{\widehat{B}}(P)) \to \nu_{\widehat{B}}(P)$$

and

$$\nu_{\widehat{B}}(P) \to \mathrm{top}(\nu_{\widehat{B}}(P)) \xrightarrow{\sim} \mathrm{soc}(\nu_{\widehat{B}}^2(P)) \to \nu_{\widehat{B}}^2(P),$$

respectively. Then we have a short path of indecomposable modules

$$P \xrightarrow{f} \nu_{\widehat{B}}(P) \xrightarrow{g} \nu_{\widehat{B}}^2(P)$$

in $\operatorname{mod} \widehat{B}$, where, by (3), $P \in \mathcal{X}_0$, $\nu_{\widehat{B}}(P) \in \mathcal{X}_2$ and $\nu_{\widehat{B}}^2(P) \in \mathcal{X}_4$. Thus, it follows, from Theorem 4.4, that we have a short path of indecomposable modules $F_\lambda(P) \to F_\lambda(\nu_{\widehat{B}}(P)) \to F_\lambda(\nu_{\widehat{B}}^2(P))$ in $\operatorname{mod} A$. Because φ is a rigid automorphism of $\widehat{B}$ we conclude that $F_\lambda(P)$ and $F_\lambda(\nu_{\widehat{B}}^2(P))$ belong to the same component $F_\lambda(\mathcal{X}_0)$. Obviously, $\mathrm{rad}_A^\infty(F_\lambda(P), F_\lambda(\nu_{\widehat{B}}(P)) \neq 0$ and $\mathrm{rad}_A^\infty(F_\lambda(\nu_{\widehat{B}}(P), F_\lambda(\nu_{\widehat{B}}^2(P)) \neq 0$. Therefore, the component quiver Σ_A contains a short cycle $F_\lambda(\mathcal{X}_0) \to F_\lambda(\mathcal{X}_2) \to F_\lambda(\mathcal{X}_0)$, and we get a contradiction.

Assume now that there exists a strictly positive automorphism φ of $\widehat{B}$ such that $G = (\varphi\nu_{\widehat{B}}^2)$. In particular, we have $g = \varphi\nu_{\widehat{B}}^2$ for a strictly positive automorphism φ of $\widehat{B}$. Then it follows from (3) that there is a positive integer $l > 4$ such that $g(\mathcal{C}_q^{\widehat{B}}) = \mathcal{C}_{q+l}^{\widehat{B}}$ and $g(\mathcal{X}_q^{\widehat{B}}) = \mathcal{X}_{q+l}^{\widehat{B}}$ for any $q \in \mathbb{Z}$. By (2) and Theorem 4.4, in order to show that Σ_A has no short cycles, we must show, that every component in Γ_A is generalized standard and has no external short paths. From property (2) and [**29**, Theorem 3] every component in Γ_A is generalized standard. Assume that there is a component $\mathcal{C}$ in Γ_A and an external short path $M \to N \to L$ with M and L in $\mathcal{C}$ but N not in $\mathcal{C}$. By Theorem 4.4, there are indecomposable modules X, Y and Z in $\operatorname{mod} \widehat{B}$ such that $M = F_\lambda(X)$, $N = F_\lambda(Y)$ and $L = F_\lambda(Z)$. Moreover, X belongs to $\mathcal{C}_{p,x}$, for some $p \in \mathbb{Z}$ and $x \in \mathbb{X}_p$, or X belongs to $\mathcal{X}_p$, for some $p \in \mathbb{Z}$. Then $\mathcal{C} = F_\lambda(\mathcal{C}_{p,x})$ or $\mathcal{C} = F_\lambda(\mathcal{X}_p)$. Without loss of generality, we may assume that $p = 0$. Therefore, we have two cases to consider.

Assume that $X \in \mathcal{C}_{0,x}$. We have an isomorphism of k-modules, induced by F_λ,

$$\mathrm{Hom}_A(M, N) \xrightarrow{\sim} \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(X, {}^{g^i}Y).$$

Since $\mathrm{Hom}_A(M, N) \neq 0$, invoking (5), we conclude that Y belongs to

$$\mathcal{X}_0 \vee \mathcal{C}_1 \vee \mathcal{X}_1 \vee \mathcal{C}_2.$$

We have also an isomorphism of k-modules

$$\mathrm{Hom}_A(N, L) \xrightarrow{\sim} \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(Y, {}^{g^i}Z).$$

Again, since $\mathrm{Hom}_A(N, L) \neq 0$, we conclude from (5) that Z belongs to

$$\mathcal{X}_2 \vee \mathcal{C}_2 \vee \mathcal{X}_3 \vee \mathcal{C}_3 \vee \mathcal{X}_4 \vee \mathcal{C}_4.$$

On the other hand, by property (4) and our assumption on φ, we have that $Z \in \mathcal{C}_{l,\lambda}$, for some $l > 4$, a contradiction.

Assume that $X \in \mathcal{X}_0$. We have an isomorphism of k-modules, induced by F_λ,

$$\mathrm{Hom}_A(M, N) \xrightarrow{\sim} \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(X, {}^{g^i}Y).$$

Since $\mathrm{Hom}_A(M, N) \neq 0$, invoking (5), we infer that Y belongs to

$$\mathcal{C}_1 \vee \mathcal{X}_1 \vee \mathcal{C}_2 \vee \mathcal{X}_2.$$

We have also an isomorphism of k-modules,

$$\mathrm{Hom}_A(N, L) \xrightarrow{\sim} \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\widehat{B}}(Y, {}^{g^i}Z).$$

Again, since $\mathrm{Hom}_A(N, L) \neq 0$, we conclude from (5) that Z belongs to

$$\mathcal{X}_1 \vee \mathcal{X}_2 \vee \mathcal{X}_3 \vee \mathcal{X}_4.$$

On the other hand, by property (4) and our assumption on φ, we obtain that $Z \in \mathcal{X}_l$, for some $l > 4$, a contradiction. □

Acknowledgements. The project was supported by the Polish National Science Center grant awarded on the basis of the decision number DEC-2011/01/N/ST1/02064.

References

[1] I. Assem, J. Nehring and A. Skowroński, Domestic trivial extensions of simply connected algebras, Tsukuba J. Math., **13** (1989), 31–72.

[2] I. Assem and A. Skowroński, Iterated tilted algebras of type $\tilde{\mathbb{A}}_n$, Math. Z., **195** (1987), 269–290.

[3] V. Dlab and C. M. Ringel, Indecomposable representations of graphs and algebras, Mem. Amer. Math. Soc., **6** (1976), no. 173.

[4] P. Dowbor and A. Skowroński, Galois coverings of representation-infinite algebras, Comment. Math. Helv., **62** (1987), 311–337.

[5] P. Gabriel, The universal cover of a representation-finite algebra, Representations of Algebras, Lecture Notes in Math., **903**, Springer-Verlag, Berlin-Heidelberg, 1981, 68–105.

[6] W. Geigle and H. Lenzing, A class of weighted projective curves arising in representation theory of finite dimensional algebras, Singularities, Representation of Algebras and Vector Bundles, Lecture Notes in Math., **1273**, Springer-Verlag, Berlin-Heidelberg, 1987, 265–297.

[7] D. Happel and C. M. Ringel, The derived category of a tubular algebra, Representation Theory I, Finite Dimensional Algebras, Lecture Notes in Math., **1177**, Springer-Verlag, Berlin-Heidelberg, 1986, 156–180.

[8] D. Happel and D. Vossieck, Minimal algebras of infinite representation type with preprojective component, Manuscr. Math., **42** (1983), 221–243.

[9] A. Jaworska, P. Malicki and A. Skowroński, On Auslander–Reiten components of algebras without external short paths, J. London Math. Soc., **85** (2012), 245–268.

[10] M. Karpicz, On selfinjective algebras without short cycles in the component quiver, Colloq. Math., **123** (2011), 219–232.

[11] M. Karpicz, A. Skowroński and K. Yamagata, On selfinjective artin algebras having generalized

standard quasitubes, J. Pure Appl. Algebra, **215** (2011), 2738–2760.

[12] H. Lenzing and J. A. de la Peña, Concealed-canonical algebras and separating tubular families, Proc. Lond. Math. Soc., **78** (1999), 513–540.

[13] H. Lenzing and A. Skowroński, Quasi-tilted algebras of canonical type, Colloq. Math., **71** (1996), 161–181.

[14] P. Malicki and A. Skowroński, Algebras with separating almost cyclic coherent Auslander–Reiten components, J. Algebra, **291** (2005), 208–237.

[15] J. Nehring and A. Skowroński, Polynomial growth trivial extensions of simply connected algebras, Fund. Math., **132** (1989), 117–134.

[16] I. Reiten and A. Skowroński, Sincere stable tubes, J. Algebra, **232** (2000), 64–75.

[17] I. Reiten, A. Skowroński and S. O. Smalø, Short chains and short cycles of modules, Proc. Amer. Math. Soc., **117** (1993), 343–354.

[18] C. M. Ringel, Tame Algebras and Integral Quadratic Forms, Lecture Notes in Math., **1099**, Springer-Verlag, Berlin-Heidelberg, 1984.

[19] C. M. Ringel, The canonical algebras, with an appendix by W. Crawley-Boevey, Topics in Algebra. Part 1: Rings and Representations of Algebras, Banach Center Publications, **26**, PWN, Warsaw, 1990, 407–432.

[20] D. Simson and A. Skowroński, Elements of the Representation Theory of Associative Algebras 2: Tubes and Concealed Algebras of Euclidean Type, London Mathematical Society Student Texts, **71**, Cambridge University Press, Cambridge, 2007.

[21] D. Simson and A. Skowroński, Elements of the Representation Theory of Associative Algebras 3: Representation-Infinite Tilted Algebras, London Mathematical Society Student Texts, **72**, Cambridge University Press, Cambridge, 2007.

[22] A. Skowroński, Selfinjective algebras of polynomial growth, Math. Ann., **285** (1989), 177–199.

[23] A. Skowroński, Cycles in module categories, Finite Dimensional Algebras and Related Topics, NATO ASI Series, Series C: Math. and Phys. Sciences, **424**, Kluwer Acad. Pub., Dordrecht, 1994, 309–345.

[24] A. Skowroński, Generalized standard Auslander–Reiten components, J. Math. Soc. Japan, **46** (1994), 517–543.

[25] A. Skowroński, Minimal representation-infinite artin algebras, Math. Proc. Cambridge Phil. Soc., **116** (1994), 229–243.

[26] A. Skowroński, On the composition factors of periodic modules, J. London Math. Soc., **49** (1994), 477–492.

[27] A. Skowroński, A construction of complex syzygy periodic modules over symmetric algebras, Colloq. Math., **103** (2005), 61–69.

[28] A. Skowroński and K. Yamagata, Galois coverings of selfinjective algebras by repetitive algebras, Trans. Amer. Math. Soc., **351** (1999), 715–734.

[29] A. Skowroński and K. Yamagata, On selfinjective artin algebras having nonperiodic generalized standard Auslander–Reiten components, Colloq. Math., **96** (2003), 235–244.

[30] A. Skowroński and K. Yamagata, Positive Galois coverings of selfinjective algebras, Adv. Math., **194** (2005), 398–436.

[31] A. Skowroński and K. Yamagata, Frobenius Algebras I: Basic Representation Theory, EMS Textbooks in Mathematics, European Mathematical Society, 2011.

Maciej Karpicz
Faculty of Mathematics and Computer Science
Nicolaus Copernicus University
Chopina 12/18
87-100 Toruń, Poland
E-mail: mkarpicz@mat.umk.pl