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Spaces of discrete shape and c-refinable maps that induce shape equivalences

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Introduction.

In [15], following a Cantor completion process, the authors give a complete, non-Archimedean metric (or ultrametric) on the set of shape morphisms between two unpointed compacta (compact metric spaces) X, Y, written $Sh(X, Y)$. The ultrametric spaces so constructed allow to rediscover some of the more important invariants in shape theory and to introduce many others. It is clear that the construction given in [15] can be translated to the pointed case, consequently, as a particular case, we obtain a complete ultrametric that induces a norm on the shape groups of a compactum Y.

Let (X, x_0) and (Y, y_0) be pointed compacta. We will assume Y to be embedded in the Hilbert cube Q. Let $i_\varepsilon: Y \rightarrow B(Y, \varepsilon)$ be the inclusion. For any pair $f, g:(X, x_0) \rightarrow (Q, y_0)$ of maps, take $F(f, g)=\inf\{\varepsilon>0: f \cong g \text{ in } B(Y, \varepsilon)=Y_\varepsilon\}$ ($\cong$ means the pointed homotopy relation).

It is clear that (pointed) approximative maps (see [3]) $\{f_k\}:(X, x_0) \rightarrow (Y, y_0)$ correspond with F-Cauchy sequences and that (pointed) homotopic approximative maps are equivalent F-Cauchy sequences.

Given $\alpha, \beta \in Sh((X, x_0), (Y, y_0))$ and F-Cauchy sequences $\{f_k\}$, $\{g_k\}$ in the classes of α, β respectively, the formula $d(\alpha, \beta)=\lim_{k\to\infty} F(f_k, g_k)$ produces a well defined complete, non-Archimedean metric in $Sh((X, x_0), (Y, y_0))$ such that the composition of pointed shape morphisms induces uniformly continuous maps between the spaces involved. This fact provides many new pointed shape invariants (see [15] for details in the unpointed case).

PROPOSITION 1 ([15]). *Given $\alpha, \beta \in Sh((X, x_0), (Y, y_0))$, $d(\alpha, \beta)<\varepsilon$ if and only if $S(i_\varepsilon)\circ\alpha=S(i_\varepsilon)\circ\beta$, as pointed morphisms ($S$ denotes the shape functor).*

In order to simplify notation *we suppress base points consistently* until section 2.

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When we consider the special cases $X=S^n$, $n\in \boldsymbol{N}$, we obtain an ultrametric on the shape groups $\check{\Pi}_n(Y)$ of a pointed compactum Y. If, for any $\alpha\in\check{\Pi}_n(Y)$, we define $\|\alpha\|=d(\alpha, 1)$ we have a norm such that

i) $\|\alpha\beta\alpha^{-1}\|=\|\beta\|$ for any α, $\beta\in\check{\Pi}_n(Y)$.

ii) $\|\alpha\|=\|\alpha^{-1}\|$ for any $\alpha\in\check{\Pi}_n(Y)$.

iii) $\|\cdot\|$ gives rise to a left and right invariant complete ultrametric in $\check{\Pi}_n(Y)$ given by $d(\alpha, \beta)=\|\alpha\beta^{-1}\|$.

If X, Y are arbitrary topological spaces, let $\boldsymbol{p}: X\to \boldsymbol{X}=(X_\lambda, p_{\lambda\lambda'}, A)$ and $\boldsymbol{q}: Y\to\boldsymbol{Y}=(Y_\mu, q_{\mu\mu'}, M)$ be HPol-expansions of X and Y respectively.

Take $\boldsymbol{Sh}(Z, X)=(Sh(Z, X_\lambda), p^*_{\lambda\lambda'}, A)$ and $\boldsymbol{Sh}(Z, Y)=(Sh(Z, Y_\mu), q^*_{\mu\mu'}, M)$, for any space Z. In [16] we generalize the construction for arbitrary spaces, by giving to $Sh(X, Y)$ the inverse limit topology as inverse limit in Top of $\{Sh(X, Y_\lambda)\}_{\lambda\in A}$ where $Sh(X, Y_\lambda)$ is assumed to have the discrete topology for any $\lambda\in A$. Using these spaces, we will show in section 2 a generalization of a theorem of Kato ([11], [12]). We prove that any c-refinable map $f: X\to Y$ is a shape equivalence provided the induced morphism $S(f)\in Sh(X, Y)$ is isolated. It is not difficult to see that $S(f)$ is isolated if Y is calm or AWNR (because $Sh(X, Y)$ is discrete) see [4], [2] and [27].

Returning to the compact framework, it is well known that out of pointed (compact) connected polyhedra there is a countable set $\{P_n : n\in\boldsymbol{N}\}$ containing one of each pointed homotopy type. Consider the inverse system $\{P_n, p_n, n\in\boldsymbol{N}\}$ where $p_n: P_{n+1}\to P_n$ is the constant (pointed) map. Let $(W, *)$ be the pointed internally movable connected space obtained by applying the star-construction, see [21] or [20] page 185, to the above inverse sequence.

The space W is useful because the *uniform topological type* of $Sh(W, X)$ characterizes the shape of X, provided X is pointed movable. More precisely, in [18] it is shown that a shape morphism $F: X\to Y$ between connected pointed compacta is a shape equivalence if and only if the induced map $F^*: Sh(W, X)\to Sh(W, Y)$ is a bi-uniform homeomorphism. Similar results can be obtained, in the unpointed case, by using the spaces introduced in [17].

Above considerations raise naturally what we are going to study here. The reader is referred to the text of [7] and [20] for information about shape theory.

1. Spaces of discrete shape.

DEFINITION 1. A pointed compactum X has discrete shape if $Sh(W, X)$ is uniformly discrete, i.e. there is $\varepsilon>0$ such that if α, $\beta\in Sh(W, X)$ and $d(\alpha, \beta)<\varepsilon$ then $\alpha=\beta$.

PROPOSITION 2. *Let X, Y be pointed compacta. If $Sh(X)\leqq Sh(Y)$ and Y has discrete shape then X has discrete shape. Consequently, the property of having discrete shape is a shape invariant.*

PROOF. It is a consequence of the fact that if $Sh(X)\leqq Sh(Y)$ then $Sh(W, X)$ is a uniform retract of $Sh(W, Y)$. □

PROPOSITION 3. *Let X be a pointed compactum, then X has discrete shape provided X is calm or $X\in AWNR$.*

PROPOSITION 4. *Let X be a pointed movable compactum. X is a pointed FANR if and only if X has discrete shape.*

PROOF. It suffices to show that any pointed movable compactum X having discrete shape is calm. Take any $\delta>0$. Let $0<\varepsilon<\delta$ as in Definition 1. For any $0<\varepsilon_1<\varepsilon$ we consider $\varepsilon_2<\varepsilon_1$ such that $S(i_{B(X,\varepsilon_2),B(X,\varepsilon_1)})=S(i_{X,B(X,\varepsilon_1)})\circ r$ for some shape morphism $r: B(X, \varepsilon_2)\to X$.

Let K be any polyhedron and let $f, g: K\to B(X, \varepsilon_2)$ be pointed H-maps such that $S(i_{B(X,\varepsilon_2),B(X,\varepsilon)})\circ f=S(i_{B(X,\varepsilon_2),B(X,\varepsilon)})\circ g$. Consider H-maps $\alpha: K\to W$ and β: $W\to K$ such that $\beta\circ\alpha=1_K$.

We have that

$$\begin{aligned}&S(i_{B(X,\varepsilon_1),B(X,\varepsilon)})\circ S(i_{B(X,\varepsilon_2),B(X,\varepsilon_1)})\circ f\circ\beta\\ &=S(i_{B(X,\varepsilon_1),B(X,\varepsilon)})\circ S(i_{B(X,\varepsilon_2),B(X,\varepsilon_1)})\circ g\circ\beta .\end{aligned}$$

Then,

$$\begin{aligned}&S(i_{B(X,\varepsilon_1),B(X,\varepsilon)})\circ S(i_{X,B(X,\varepsilon_1)})\circ r\circ f\circ\beta\\ &=S(i_{B(X,\varepsilon_1),B(X,\varepsilon)})\circ S(i_{X,B(X,\varepsilon_1)})\circ r\circ g\circ\beta .\end{aligned}$$

Consequently, $d(r\circ f\circ\beta, r\circ g\circ\beta)<\varepsilon$ and $r\circ f\circ\beta=r\circ g\circ\beta$.
It follows that $r\circ f=r\circ f\circ\beta\circ\alpha=r\circ g\circ\beta\circ\alpha=r\circ g$.
Therefore,

$$\begin{aligned}S(i_{B(X,\varepsilon_2),B(X,\varepsilon_1)})\circ f &= S(i_{X,B(X,\varepsilon_1)})\circ r\circ f = S(i_{X,B(X,\varepsilon_1)})\circ r\circ g\\ &= S(i_{B(X,\varepsilon_2),B(X,\varepsilon_1)})\circ g .\end{aligned}$$

□

REMARKS. $Sh(W, X)$ contains isometric copies of all shape groups $\check{\Pi}_n(X)$, $n\in\boldsymbol{N}$. Then if X has discrete shape it follows that $\check{\Pi}_n(X)$, $n\in\boldsymbol{N}$ are uniformly discrete topological groups such that $\varepsilon>0$ as in Definition 1 does not depend on $n\in\boldsymbol{N}$. Using Baire's Theorem and the homogeneity of these groups we have that they are discrete if and only if they are countable. Therefore, if $sd(X)<\infty$, the assumption of $Sh(W, X)$ to be discrete is very strong and can be much weakened ([**20**]).

Since $Sh(X, Y)$ is separable we have that if $Sh(X, Y)$ is discrete then it is countable. As we said before, the converse is also true for the shape groups. A natural question is whether $Sh(X, Y)$ countable implies $Sh(X, Y)$ discrete. In the unpointed case there are very easy examples showing that this implication does not hold. In fact $Sh(*, X)=\square X$, the space of components of X, ([**15**]). Then, if $X=\{1/n : n\in \boldsymbol{N}\}\cup\{0\}$, $Sh(W, X)$ is countable but it is not discrete. However, in the pointed case, it seems more difficult to find examples. Anyway, Corollary 3 will provide one of them. It will be given a pointed movable compactum X such that $Sh(W, X)$ is countable but X has not discrete shape.

THEOREM 1. *Let T be the Taylor's compactum,* [**26**]. *It follows that $Sh(W, T)=*$. In particular T has discrete shape.*

The proof of above theorem depends on two previous results.

THEOREM 2. *Let $F: X\to Y$ be a shape morphism that is a weak shape equivalence; then, for any compact connected pointed polyhedron P, F induces an isomorphism $\boldsymbol{Sh}(P, X)\to\boldsymbol{Sh}(P, Y)$ in pro-Top (pro-Set).*

PROOF. We assume F to be represented by a level preserving morphism (f_λ).

Let P be a compact connected polyhedron, $\dim P=m<n$. Using Lemma 1.4 in [**10**], see also [**14**], for any λ there exists $\theta(\lambda, n)\geqq n$ and a map h making the following diagram commutative, up to pointed homotopy ($M(f)$ denotes the reduced mapping cylinder of f)

$$\begin{array}{ccccc}
X_\lambda & \xleftarrow{\;h\;} & M(f_{\theta(\lambda,n)})^n\cup X_{\theta(\lambda,n)} & \xleftarrow{\;i\;} & X_{\theta(\lambda,n)} \\
{\scriptstyle f_n}\downarrow & & \downarrow{\scriptstyle j} & & \downarrow{\scriptstyle f_{\theta(\lambda,n)}} \\
Y_\lambda & \xleftarrow{\;q\;} & M(f_{\theta(\lambda,n)}) & \xleftarrow{\;k\;} & Y_{\theta(\lambda,n)}.
\end{array}$$

By the cellular approximation theorem,

$$j^*: Sh(P, M(f_{\theta(\lambda,n)})^n\cup X_{\theta(\lambda,n)}) \longrightarrow Sh(P, M(f_{\theta(\lambda,n)}))$$

is bijective.

Then, we have a map $g_\lambda^*: Sh(P, Y_{\theta(\lambda,n)})\to Sh(P, X_\lambda)$ such that the diagram

$$\begin{array}{ccc}
Sh(P, X_\lambda) & \xleftarrow{\;p^*_{\lambda\theta(\lambda,n)}\;} & Sh(P, X_{\theta(\lambda,n)}) \\
{\scriptstyle f_\lambda^*}\downarrow & \overset{g_\lambda^*}{\nwarrow} & \downarrow{\scriptstyle f^*_{\theta(\lambda,n)}} \\
Sh(P, Y_\lambda) & \xleftarrow{\;q^*_{\lambda\theta(\lambda,n)}\;} & Sh(P, Y_{\theta(\lambda,n)})
\end{array}$$

commutes.

Now, from Morita's characterization of isomorphisms in pro-categories, [**14**], we have that F induces an isomorphism $\boldsymbol{Sh}(P, X) \to \boldsymbol{Sh}(P, Y)$. □

PROPOSITION 5. *Let $F: X \to Y$ be a shape morphism between connected pointed compacta such that $F^*: Sh(P, X) \to Sh(P, Y)$ in injective, for every connected compact pointed polyhedron P; then, $F^*: Sh(W, X) \to Sh(W, Y)$ is injective.*

PROOF. Given $\varepsilon > 0$, using the local contractibility of $B(X, \varepsilon)$, it is easy to check that if $a, b: W \to X$ are shape morphism such that $F \circ a = F \circ b$ then $d(a, b) < \varepsilon$. □

PROOF OF THEOREM 1. Using [**26**], we have a CE-map $f: T \to Q$. Consequently, $S(f)$ is a weak shape equivalence. From Theorem 2 and Proposition 5, we have that $S(f)$ induces an injective map

$$S(f)^*: Sh(W, T) \longrightarrow Sh(W, Q) = *.$$ □

Next corollaries point out that even though $Sh(W, X)$ is uniformly discrete X does not need being an AWNR neither a calm space.

COROLLARY 1. *Let T be the Taylor's compactum. T is not AWNR but $Sh(W, T) = *$.*

COROLLARY 2. *Consider $\{T_j, j \in \boldsymbol{N}\}$ to be a family of copies of the Taylor's compactum. Then, $\prod_{j \in N} T_j$ is a non calm compactum such that $Sh(W, \prod_{j \in N} T_j) = *$.*

Theorem 1 also allows to state the next corollary.

COROLLARY 3. *There exists a pointed movable compactum T' such that $Sh(W, T')$ is countable but T' has not discrete shape.*

PROOF. It suffices to take the space T' obtained by applying the star-construction of Overton-Segal [**21**] to the inverse sequence associated with T. □

Note that from Theorem 2 and Proposition 5 we have,

COROLLARY 4. *For any pointed compactum X, pro-$\prod_k(X) = *$ for every $k \in \boldsymbol{N}$, implies $Sh(W, X) = *$.*

2. c-refinable maps that induce shape equivalences.

In this section we will work with (unpointed) arbitrary topological spaces.

In [**11**] ([**12**]) H. Kato proved that any refinable map $r: X \to Y$ between compacta induces a shape equivalence $S(r): X \to Y$ provided $Y \in FANR$ (Y is calm) (S denotes the shape functor). Recently J. M. R. Sanjurjo [**22**], gave an intrinsic description of the shape category of compacta by using upper-semicontinuous multivalued maps. This approach allowed him to give an alternative proof of the result of Kato. The authors in [19] extended the upper-semicontinuous multivalued maps approach to shape to the class of paracompacta by means of resolutions theory. Simultaneously Z. Čerin has given, see [4], by using the *cofinite* Čech expansion and non-upper-semicontinuous multivalued maps, an intrinsic description of the shape category for arbitrary topological spaces. In this paper, we apply later useful description to prove in a short way, by general topology methods, a rather general result in the realm of arbitrary topological spaces dealing with c-refinable maps (see [13]).

In order to do this section as self-contained as possible we point out some of the notions we will handle.

A *normal* covering of a topological space Y is an open covering ω which admits a partition of the unity subordinated to ω. Normal coverings can also be characterized as those admitting a sequence of open coverings $\omega \overset{*}{\leqq} \omega_1 \overset{*}{\leqq} \omega_2 \overset{*}{\leqq} \omega_3 \cdots$ where the symbol $\overset{*}{\leqq}$ stands for the star-refinement relation [1].

Two open coverings of Y are said to be *equivalent* if they refine each other. $\hat{Y}$ will denote the collection of all normal coverings classes of a topological space Y. By $\tilde{Y}$ we shall mean the family of all finite subsets $c \subset \hat{Y}$ having, respect the refinement relation, a maximal element $\bar{c} \in \hat{Y}$.

Let X, Y be topological spaces and $\alpha \in \hat{X}$, $\beta \in \hat{Y}$. A multivalued map $F: X \to Y$ is said to be *(α, β)-small* if for any $U \in \alpha$ there is a $V \in \beta$ such that $F(U) \subset V$. We will say that F is *β-small* if there exists $\alpha \in \hat{X}$ such that F is (α, β)-small.

Two multivalued maps $F, G: X \to Y$ are said to be *β-homotopic*, written $F \overset{\beta}{\approx} G$, if there is a β-small map $H: X \times I \to Y$ such that $F \subset H(\cdot, 0)$ and $G \subset H(\cdot, 1)$. Note that $F \overset{\beta_1}{\approx} G$ and $G \overset{\beta_1}{\approx} T$ imply $F \overset{\beta}{\approx} T$ provided $\beta_1 \overset{*}{\geqq} \beta$.

A *multinet* $F: X \to Y$ is a collection $F = \{F_c\}_{c \in \tilde{Y}}$ of multivalued functions $F_c: X \to Y$ such that for every $\gamma \in \hat{Y}$ there is $c \in \tilde{Y}$ with $F_c \overset{\gamma}{\approx} F_d$ for any $d > c$. Two multinets $F = \{F_c\}$, $G = \{G_c\}: X \to Y$ are *homotopic* if for every $\gamma \in \hat{Y}$ there is a $c \in \tilde{Y}$ with $F_d \overset{\gamma}{\approx} G_d$ for any $d > c$.

In [5], Čerin defined the composition of homotopy classes of multinets producing a category isomorphic to the shape category.

Given $[F] \in Sh(X, Y)$ and $\gamma \in \hat{Y}$ let $B([F], \gamma) = \{[G] \in Sh(X, Y):$ there exists $c \in \tilde{Y}$ with $F_d \overset{\gamma}{\approx} G_d$ for any $d > c\}$. It is readily seen that the family $\{B([F], \gamma)\}_{\gamma \in \hat{Y}}$ is a neighborhood system for the shape morphism $[F]: X \to Y$. We will consider $Sh(X, Y)$ endowed with the induced topology. This topology coincide with the topology obtained by giving to $Sh(X, Y)$ the inverse limit topology as inverse limit in Top of $\{Sh(X, Y_\lambda)\}_{\lambda \in A}$ where $\{Y_\lambda, q_{\lambda \lambda'}, A\}$ is any HPol-expansion of Y and $Sh(X, Y_\lambda)$ is assumed to have the discrete topology for any $\lambda \in A$, see [16].

Before of stating our result we recall that a surjective map $r: X \to Y$ is said to be *c-refinable* if for any normal coverings α, β of X and Y respectively, there is a closed and onto (α, β)*-refinement* $s: X \to Y$, of r; i.e. s and r are β-near and for any $y \in Y$ there is $U_y \in \alpha$ such that $s^{-1}(y) \subset U_y$.

THEOREM 3. *Let X, Y be topological spaces and let $r: X \to Y$ be a c-refinable map. Then $S(r)$ is a shape equivalence provided $S(r)$ is an isolated point in $Sh(X, Y)$.*

PROOF. Let $\gamma_0 \in \hat{Y}$ such that $B(S(r), \gamma_0) = \{S(r)\}$. Take $\gamma_1 \in \hat{Y}$ such that $\gamma_1 \overset{*}{\geqq} \gamma_0$.

Let $c \in \tilde{X}$ and $\bar{c} \in \hat{X}$. Consider $\bar{d} \in \hat{X}$ to be a 3-star-refinement of $\bar{c}$. More precisely, choose normal coverings $\bar{d} \overset{*}{\geqq} \bar{d}_2 \overset{*}{\geqq} \bar{d}_1 \overset{*}{\geqq} \bar{c}$. Take $s: X \to Y$ be any $(\bar{d}, \gamma_1)$-refinement of r. We define $F_c: Y \to X$ by $F_c(y) = s^{-1}(y)$. Since s is closed $\{Y \backslash s(X \backslash U)\}_{U \in \bar{d}}$ is a normal covering of Y; hence F_c is a $\bar{d}$-small multivalued map.

The base of the proof is the following fact:

CLAIM. If we start from different $(\bar{d}, \gamma_1)$-refinements of r we obtain $\bar{c}$-homotopic multivalued maps.

Indeed, let $s_1, s_2: X \to Y$ be two $(\bar{d}, \gamma_1)$-refinements of r and denote by F_c^1 and F_c^2 the corresponding $\bar{d}$-small multivalued maps obtained from s_1 and s_2 respectively.

Since $B(S(r), \gamma_0) = \{S(r)\}$ we have that for any (single-valued) map $f: X \to Y$ γ_1-near to r, $r \overset{\mu}{\approx} f$ for every $\mu \in \hat{Y}$. Consequently, $F_c^1 \circ r \overset{\bar{d}}{\approx} F_c^1 \circ s_1 \supset Id_X$. A similar argument shows that $F_c^2 \circ r \overset{\bar{d}}{\approx} F_c^2 \circ s_2 \supset Id_X$. Then, $F_c^1 \circ r \overset{\bar{d}_2}{\approx} F_c^2 \circ r$.

Let $H: X \times I \to X$ be a $(\alpha, \bar{d}_2)$-small homotopy connecting $F_c^1 \circ r$ and $F_c^2 \circ r$. Choose a normal covering $\bar{\alpha} \in \hat{X}$ such that there is a stacking function, in the sense of [6] (page 358), $\bar{\alpha} \to \{1, 2, 3, \cdots\}$ producing a refinement of $\alpha \in \widehat{X \times I}$. Let $\beta \in \hat{Y}$ be a refinement of both $\{Y \backslash s_1(X \backslash U)\}_{U \in \bar{d}}$ and $\{Y \backslash s_2(X \backslash U)\}_{U \in \bar{d}}$ and take $\beta_1 \in \hat{Y}$ such that $\beta_1 \overset{*}{\geqq} \beta$.

Let $s': X \to Y$ any $(\bar{\alpha}, \beta_1)$-refinement of r. Define a $\bar{\alpha}$-small map $G: Y \to X$ by $G(y) = s'^{-1}(y)$. It follows that $r \circ G \overset{\beta}{\approx} Id_Y$. Therefore, $F_c^1 \overset{\bar{d}_2}{\approx} F_c^1 \circ r \circ G \overset{\bar{d}_2}{\approx} F_c^2 \circ r \circ G \overset{\bar{d}_2}{\approx} F_c^2$. Consequently, $F_c^1 \overset{\bar{c}}{\approx} F_c^2$. This proves the claim.

Now it is a routine to check that $F = \{F_c\}: Y \to X$ is a multinet such that $S(r) \circ [F] = Id_Y$ and $[F] \circ S(r) = Id_X$. □

REMARKS. The assumption of r to be isolated in $Sh(X, Y)$ holds, in particular, when $Sh(X, Y)$ is discrete. For example, if Y is stable, for every topological space X one has that $Sh(X, Y)$ is discrete. In the non necessarily movable context the same follows if Y is calm.

Note that it is easy to produce examples showing that c-refinable maps can not be substituted by refinable maps in above theorem. In fact, if Y is any infinite trivial shape space and we denote by X the set Y endowed with the discrete topology, it is clear that $Id: X \to Y$ is a refinable map that fails to be a shape equivalence.

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References

[1] R. A. Alo and H. L. Shapiro, Normal topological spaces, Cambridge Univ. Press, 1974.
[2] S. A. Bogatyi, Approximative and fundamental retracts, Math. USSR Sbornik, **22**, 1 (1974), 91–103.
[3] K. Borsuk, Theory of shape, Monografie Matematyczne, **59** (PWN, Warsaw, 1975).
[4] Z. Čerin, Homotopy properties of locally compact spaces at infinity-calmness and smoothness, Pacific J. Math., **79**, 1 (1978), 69–91.
[5] Z. Čerin, Shape theory intrinsically, Publications Matemàtiques, **37** (1993), 317–334.
[6] A. Dold, Lectures in algebraic topology, Springer-Verlag, Berlin, 1972.
[7] J. Dydak and J. Segal, Shape theory: An Introduction, Lecture Notes in Math., **688** (Springer-Verlag, Berlin, 1978).
[8] D. A. Edwards and R. Geoghegan, Compact weak equivalent to ANR's, Fund. Math., **90** (1975), 115–124.
[9] D. A. Edwards and R. Geoghegan, Infinite-dimensional Whitehead and Vietoris theorems in shape and pro-homotopy, Trans. Amer. Math. Soc., **219** (1976), 351–360.
[10] R. Geoghegan, Elementary proofs of stability theorems in pro-homotopy and shape, Gen. Top. Appl., **8** (1978), 265–281.
[11] H. Kato, Refinable maps in the theory of shape, Fund. Math., **113** (1981), 119–129.
[12] H. Kato, A remark on refinable maps and calmness, Proc. Amer. Math. Soc., **90**, 4 (1984), 649–652.
[13] A. Koyama, Refinable maps in dimension theory, Topology and its Appl., **17** (1984), 247–255.
[14] K. Morita, The Hurewicz and the Whitehead theorems in shape theory, Sci. Rep. Tokyo Kyoiku Daigaku, Sec. A, **12** (1974), 246–258.
[15] M. A. Morón and F. R. Ruiz del Portal, Shape as a Cantor completion process, Math. Zeitschrift, 225, 1, (1997), 67–86.
[16] M. A. Morón and F. R. Ruiz del Portal, A topology on the set of shape morphisms, preprint.
[17] M. A. Morón and F. R. Ruiz del Portal, Counting shape and homotopy types among FANR's: An elementary approach, Manuscripta Math., **79** (1993), 411–414.
[18] M. A. Morón and F. R. Ruiz del Portal, Ultrametrics and infinite-dimensional Whitehead theorems in shape theory, Manuscripta Math., **89** (1996), 325–333.
[19] M. A. Morón and F. R. Ruiz del Portal, Multivalued maps and shape for paracompacta, Math. Japonica, **39**, 3 (1994), 489–500.
[20] S. Mardešić and J. Segal, Shape theory, North-Holland, Amsterdam, 1982.
[21] R. H. Overton and J. Segal, A new construction of movable compacta, Glasnik Mat., **6** (1971), 361–363.

[22] J. M. R. Sanjurjo, An intrinsic description of shape, Trans. Amer. Math. Soc., **329** (1992), 625–636.
[23] W. H. Schikof, Ultrametric calculus: An introduction to p-adic analysis, Cambridge University Press, 1984.
[24] E. Spanier, Algebraic Topology, McGraw-Hill, NY, 1966.
[25] S. Spiez, A majorant for the class of movable compacta, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astron. Phys., **21** (1973).
[27] J. L. Taylor, A counterexample in shape theory, Bull. Amer. Math. Soc., **81**, 3 (1975), 629–632.
[27] K. Tsuda, On AWNR-spaces in shape theory, Math. Japonica, **22**, 4 (1977), 471–478.

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