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Erratum to "Indivisibility of class numbers of totally imaginary quadratic extensions and their Iwasawa invariants"

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The author was kindly informed by Professor M. Ohta that there is the following mistake in the proof of Proposition in §4, which we needed only to verify numerical examples. Namely, in p. 192 l. 38, $\mathfrak{o}_k$ does not appear in Ω_0, if some of $\mathfrak{r}_1, \cdots, \mathfrak{r}_s$ split completely in k/F. Therefore we could not prove $\mathrm{tr}.\mathfrak{T}((N)) \not\equiv 0 \bmod l^{e_F+1}$ in p. 193 l. 11.

The author could not recover the proof of it. Consequently, Proposition is not proved. But we recover numerical examples.

Let l and p be primes such that $3 \leqq l \leqq 73$, $p \leqq 17389$ and $p \equiv 1 \bmod 4$. We put $F = \boldsymbol{Q}(\sqrt{p})$. We verify that F has infinitely many totally imaginary quadratic extensions whose relative class numbers are not divisible by l, even if l divides $w_F \zeta_F(-1)$.

Let $q \geqq 5$ be a prime. We denote by $h(-q)$ (resp. $h(-pq)$) the class number of $\boldsymbol{Q}(\sqrt{-q})$ (resp. $\boldsymbol{Q}(\sqrt{-pq})$). We can search by using UBASIC86 written by Y. Kida a prime $q \neq p$, l and an element α in the ring $\mathfrak{o}_q$ of integers of $\boldsymbol{Q}(\sqrt{-q})$ with the following properties:

(1) $h(-q)$ and $h(-pq)$ are prime to l,

(2) (α) is a prime ideal in $\mathfrak{o}_q$,

(3) $\boldsymbol{Z}[\alpha] = \mathfrak{o}_q$,

(4) $N = \alpha\bar{\alpha}$ remains prime in $F/\boldsymbol{Q}$ and $N \neq l$,

(5) $\alpha^2 + \alpha\bar{\alpha} + \bar{\alpha}^2$ is prime to l in the case of $l \neq 3$.

In the above, $\bar{\alpha}$ stands for the complex conjugation of α.

We put $k = F(\sqrt{-q})$. Let $\mathfrak{o}_F$ (resp. $\mathfrak{o}_k$) be the ring of integers of F(resp. k). We take a division quaternion algebra B/F satisfying (i), (iii), (iv), (v) as in p. 192 and

(ii)′ $\mathfrak{p}_1', \cdots, \mathfrak{p}_t'$ are ramified in B/F.

We see that only the orders of k do appear in Ω_0. Since the discriminant of F and that of $\boldsymbol{Q}(\sqrt{-q})$ are prime to each other, we get $\mathfrak{o}_k = \mathfrak{o}_F \cdot \mathfrak{o}_q$ by Satz 88 in Zahlbericht of D. Hilbert (Gesammelte Abhandlungen I, Chelsea). Thus we get $\mathfrak{o}_F[\alpha]=\mathfrak{o}_k$ by (3). Hence $\Omega_0=\{\mathfrak{o}_k\}$. (N) is a prime ideal in F because of (2) and (4). We get

$$\begin{aligned}\mathrm{tr}.\,\mathfrak{T}((N)) = &-\varepsilon\frac{[E:E^+]}{2}h_k\prod_{\mathfrak{p}\mid D(B/F)}\left(1-\left(\frac{\mathfrak{o}_k}{\mathfrak{p}}\right)\right)\\ &\times\mathrm{tr}.\,\Psi(\alpha)\frac{\zeta_\alpha{}^{n+1}-\eta_\alpha{}^{n+1}}{\zeta_\alpha-\eta_\alpha}N^{-n/2},\end{aligned}$$

where we put $\varepsilon=2$ (resp. 1), if N splits completely (resp. ramifies) in $\boldsymbol{Q}(\sqrt{-q})/\boldsymbol{Q}$.

We see $\mathrm{tr}.\,\Psi(\alpha)(\zeta_\alpha{}^{n+1}-\eta_\alpha{}^{n+1})/(\zeta_\alpha-\eta_\alpha)N^{-n/2}=(\alpha^{n+1}-\bar{\alpha}^{n+1})^2(\alpha-\bar{\alpha})^{-2}N^{-n}$. We put $n=2$ (resp. $n=4$) for $l\neq 3$ (resp. $l=3$). For $l\neq 3$, we see by (4) that $\mathrm{tr}.\,\Psi(\alpha)=(\alpha^3-\bar{\alpha}^3)^2(\alpha-\bar{\alpha})^{-2}N^{-2}$ is prime to l. For $l=3$, we also see that $\mathrm{tr}.\,\Psi(\alpha)=(\alpha^5-\bar{\alpha}^5)^2(\alpha-\bar{\alpha})^{-2}N^{-4}=((\alpha+\bar{\alpha})^4-3(\alpha+\bar{\alpha})^2\alpha\bar{\alpha}+(\alpha\bar{\alpha})^2)^2N^{-4}$ is prime to 3. In the present case, we have $[E:E^+]=2$. We see that the group of units of k is equal to that of F, because the prime ideals lying over (q) are ramified in k/F and k contains no root of unity other than ± 1. So we get $h_k=h_F\cdot h(-q)\cdot h(-pq)/2$. Then we get $\mathrm{tr}.\,\mathfrak{T}((N))\equiv 0 \bmod l^{e_F}$ and $\mathrm{tr}.\,\mathfrak{T}((N))\not\equiv 0 \bmod l^{e_F+1}$. By the same argument in that paper, we proved that F has infinitely many imaginary quadratic extensions whose relative class numbers are prime to l.

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