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A stochastic approach to a Liouville property for plurisubharmonic functions

By Hiroshi KANEKO

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1. Introduction.

It is well known that any plurisubharmonic function u on $\boldsymbol{C}^n$ increasing slower than logarithmic order at infinity must be constant. This kind of the Liouville property of the plurisubharmonic functions has been extended to certain complex manifolds. The purpose of this paper is to prove assertions of this type by making use of weakly recurrent holomorphic diffusion processes associated with plurisubharmonic exhaustion functions on the complex manifolds. As an application, we give a refinement of a recent result due to Takegoshi [**16**] concerning Kähler manifolds with poles.

In the case of the complex plane $\boldsymbol{C}$ any subharmonic function u defined on $\boldsymbol{C}$ does not exceed the harmonic function $v_R(z)=\sup_{|w|\leqq 1}u(w)(1-\log|z|/\log R)+m(u, R)\log|z|/\log R$ on $\{1<|z|<R\}$, where $m(u, R)=\sup_{|z|\leqq R}u(z)$. Letting $R\to\infty$, we have $\sup_{|w|\leqq 1}u(w)\geqq u(z)$ over $\boldsymbol{C}$ provided that $\lim_{R\to\infty}m(u, R)/\log R=0$. Therefore u becomes constant by the maximum principle. We observe in this argument that $\log|z|/\log R$ is just the probability that the standard complex Brownian motion exits from the circle $\{|z|=R\}$ before hitting the inner circle $\{|z|=1\}$. This observation suggests a probabilistic method to work with plurisubharmonic functions on complex manifolds which admit holomorphic diffusions $\boldsymbol{M}=\{Z_t, \zeta, \mathcal{F}_t, P_z\}$ enjoying a kind of recurrence property. In fact any plurisubharmonic function becomes $\boldsymbol{M}$-subharmonic in the sense of Dynkin [**4**] for every holomorphic diffusion $\boldsymbol{M}$. The term "holomorphic" is due to the property of $\boldsymbol{M}$ that the composite $f(Z_t)$ of the sample path Z_t with any holomorphic function f is a local martingale.

In § 2, we present a general property of subharmonic functions with respect to a weakly recurrent diffusion, generalizing the above mentioned argument for the Brownian motion. It will then be shown in § 3 that a Liouville type theorem for plurisubharmonic functions holds on a complex manifold of dimension n possessing a plurisubharmonic exhaustion function Ψ such that $(dd^c\Psi)^n$ tends to zero in a certain sense as $\Psi\to\infty$ (Theorem 1). By making use of this exhaustion function Ψ, we can construct a Dirichlet form following Fukushima-

Okada [8] and the associated holomorphic diffusion $\boldsymbol{M}$ with a weakly recurrence property. We note that there exists Ψ for which our decay condition on $(dd^c\Psi)^n$ is satisfied when the underlying manifold is parabolic in the sense of Stoll [**14**] ($(dd^c\Psi)^n=0$ outside a compact set in this case).

In § 4, we give an application of Theorem 1 to Kähler manifolds with poles. Indeed, we can prove the non-existence of non-constant bounded plurisubharmonic function even in some case that there is no function $K(x)$ such that |the radial curvature$|\leqq K$(distance from the pole) with $\int_0^\infty xK(x)dx<\infty$ (Corollary of Theorem 2). We note a remarkable fact that if the radial curvature is non-positive and if there exists $K(x)$ as above, then the Kähler manifold is biholomorphic to $\boldsymbol{C}^n$ ([**10**]) and therefore the present assertion is trivial in this case. See [**11**], [**12**], [**13**] and [**17**] for other type of gap theorems for complex manifolds.

Corollary of Theorem 2 is an extension of a part of a recent work [**16**] where some estimates of energy integrals are utilized in proving that, under the same conditions as in the corollary, there is no non-constant bounded harmonic function nor continuous strictly plurisubharmonic function.

Our general setting on the existence of an exhaustion function Ψ is quite similar to the one appeard in [**16**]. However the present probabilistic approach enables us to remove the smoothness condition on the relevant plurisubharmonic functions.

2. Weakly recurrent diffusion and the associated subharmonic functions.

Let us consider a diffusion process $\boldsymbol{M}=\{Z_t, \zeta, F_t, P_z\}$ on a manifold M. $\boldsymbol{M}$ is said to be *weakly recurrent*, if there exists a compact set Γ which satisfies $P_z(\sigma_\Gamma<\infty)=1$ for a.a. z. Here and in the sequel, $\sigma_E=\inf\{t>0\,;\, Z_t\in E\}$ stands for the hitting time to any set $E\subset M$ and "*a.a.*" means "except a set of Lebesgue measure zero with respect to the complex local coordinate system" on each complex coordinate neighbourhood.

We say a $[-\infty, \infty)$-valued Borel function u defined on an open subset $\Omega\subset M$ to be $\boldsymbol{M}$*-subharmonic*, if u is locally upper bounded and $\boldsymbol{M}$-finely upper semi-continuous and the submean value property $E_z[u(Z_{\tau_D})\,;\, \tau_D<\zeta]\geqq u(z)$ holds on D for any open set D with closure being a compact subset of Ω. Here and in what follows we employ the notation $\tau_E=\inf\{t>0\,;\, Z_t\notin E\}$ for the exit time from $E\subset M$. Let us show that any $\boldsymbol{M}$-subharmonic function attains the essential supremum on a compact set, provided that $\boldsymbol{M}$ is weakly recurrent and that the function is of slow growth.

PROPOSITION 1. *Let $\boldsymbol{M}=\{Z_t, \zeta, \mathcal{F}_t, P_z\}$ be a weakly recurrent diffusion on a manifold M with a compact set Γ satisfying $P_z(\sigma_\Gamma<\infty)=1$ a.a. z and u be an*

$\boldsymbol{M}$-subharmonic function on M. Suppose that there exists an increasing sequence of relatively compact domains $\{\Omega_k\}_{k=1}^{\infty}$ satisfying $\bigcup_{k=1}^{\infty}\Omega_k=M$. If, for $\mathcal{M}(u, \Omega_K)=\sup_{z\in\bar{\Omega}_k}u(z)$,

$$\lim_{k\to\infty}\mathcal{M}(u, \Omega_k)P_z(\sigma_\Gamma>\tau_{\Omega_k})=0 \qquad a.a.\ z$$

holds, then we have

$$u(z)\leqq\sup_{z\in\Gamma}u(z) \qquad a.a.\ z.$$

PROOF. We may assume $\Gamma\subset\Omega_k$ for all k. The $\boldsymbol{M}$-subharmonicity implies that

$$\begin{aligned}u(z)&\leqq E_z[u(Z_{\sigma_\Gamma\wedge\tau_{\Omega_k}})]\\&\leqq\mathcal{M}(u, \Omega_k)P_z(\sigma_\Gamma>\tau_{\Omega_k})+\sup_{z\in\Gamma}u(z)P_z(\tau_{\Omega_k}>\sigma_\Gamma)\,.\end{aligned}$$

The desired inequality is immediately seen by letting $k\to\infty$. q.e.d.

3. A Liouville type theorem.

Let M be a connected complex manifold of dimension $n\geqq 2$ which possesses an unbounded plurisubharmonic exhaustion function Ψ. Here Ψ is said to be an exhaustion function if Ψ is a continuous proper map. Define a closed positive current θ of bidegree $n-1$ by the exterior power $(dd^c\Psi)^{n-1}$, where $d=\partial+\bar{\partial}$ and $d^c=i(\bar{\partial}-\partial)$. θ is well defined because Ψ is locally bounded ([**2**] and [**9**]). A plurisubharmonic function p defined on a complex manifold is said to be strictly plurisubharmonic in distribution sense, if, for each complex local chart, there exists some $\delta>0$ such that $p-\delta|z|^2$ is plurisubharmonic on the complex coordinate neighbourhood. Here $|z|^2$ is the function associated with the local coordinate system. In order to construct a weakly recurrent holomorphic diffusion on M, we assume in this section that the following conditions are fulfilled for some compact set $K\subset M$:

(C.1) $$(dd^c\Psi)^n=\theta\wedge dd^c\Psi\leqq\rho(\Psi)\theta\wedge d\Psi\wedge d^c\Psi$$

outside K for some function $\rho\in\mathcal{A}$, where $\mathcal{A}$ denotes the set of all non-negative continuous functions $\rho(x)$ on $[\inf\Psi, \infty)$ such that

$$\int_c^x\exp\left(-\int_c^\eta\rho(\xi)d\xi\right)d\eta\longrightarrow\infty$$

as $x\to\infty$ for any $c\in[\inf\Psi, \infty)$.

(C.2) *There exists a locally bounded strictly plurisubharmonic function p in distribution sense defined on $M-K$ such that, for each complex chart with coordinate neighbourhood included in $M-K$, the associated Lebesgue measure is absolutely continuous with respect to the positive Radon measure $\mu=\theta\wedge dd^cp$ on the neigh-*

bourhood.

For convenience of descriptions, we introduce some notations. Let

$$\begin{aligned} &M(s) = \{\Psi < s\}, \\ &M[s] = \{\Psi \leqq s\}, \\ &M(s, R) = \{s < \Psi < R\}, \qquad \text{and} \\ &M_* = M - K. \end{aligned}$$

Take $s_0 > \inf \Psi$ such that $K \subset M(s_0)$ and set $g_\rho(x) = \int_{s_0}^{x} \exp\left(-\int_{s_0}^{\eta} \rho(\xi) d\xi\right) d\eta$. This function appears in the above definition of the family $\mathcal{A}$ (with $c = s_0$).

We will employ the Dirichlet space method related to plurisubharmonic functions in [8] and [9] and further use the terms in these papers. It is assured that the real symmetric bilinear form

$$\mathcal{E}_0(\phi_1, \phi_2) = \int_{M_*} d\phi_1 \wedge d^c \phi_2 \wedge \theta\,, \qquad \phi_1, \phi_2 \in C_0^\infty(M_*)$$

is closable in $L^2(M_*, \mu)$ by condition (C.2), because $\theta \wedge dd^c |z|^2$ is dominated by $\mu = \theta \wedge dd^c p$ on each complex coordinate neighbourhood up to a constant factor. There exists then holomorphic diffusion $\boldsymbol{M}_\theta = \{Z_t, \zeta, \mathcal{F}_t, P_z\}$ on M_* such that the transition function of $\boldsymbol{M}_\theta$ is a realization of the L^2-semigroup generated by the Dirichlet space $(\mathcal{E}, \mathcal{F})$ the smallest closed extension of $\mathcal{E}_0$ in $L^2(M_*, \mu)$. We call the diffusion $\boldsymbol{M}_\theta$ holomorphic because, for every holomorphic function f, $\mathrm{Re}\, f(Z_{t \wedge \eta})$ is a $(P_z, \mathcal{F}_{t \wedge \eta})$-local martingale for every starting point z in the defining domain D of f and every stopping time $\eta < \zeta \wedge \tau_D$. It is known that any $\mathcal{E}$-subharmonic function is $\boldsymbol{M}_\theta$-submean valued (see [**9**; Appendix]). The following lemma and proposition concern preliminary properties of $\boldsymbol{M}_\theta$.

LEMMA 1. *For any $R > s_0$, let $\tau_R = \tau_{M(R)} \wedge \sigma_{M[s_0]}$. Then*

$$P_z(\tau_R < \infty) = 1\,, \qquad \textit{q.e. } z \textit{ in } M(s_0, R)\,.$$

PROOF. Because $p \in \mathcal{F}_{\mathrm{loc}}$, we can clearly see that

$$E[p(Z_{\tau_R \wedge t})] - p(z) = E_z[\tau_R \wedge t] \qquad \text{q.e. } z$$

as in the proof of Lemma 6 in [8]. Therefore we have

$$E_z[\tau_R] \leqq 2 \sup_{z \in M(s_0, R)} |p(z)| \qquad \text{q.e. } z$$

by letting $t \to \infty$. This completes the proof. q.e.d.

PROPOSITION 2. *If $R > s_0$, then*

$$P_z(\tau_{M(R)} < \sigma_{M[s_0]}) \leqq g_\rho(\Psi(z)) / g_\rho(R)$$

holds for q.e. z on $M(s_0, R)$. Especially there exists a weakly recurrent holomorphic diffusion $\boldsymbol{M}$ on M which coincides with $\boldsymbol{M}_\theta$ on $M[s_0]$.

PROOF. Since $g_\rho(x)$ has bounded derivatives, Theorem 5.4.3 in [5] implies that $g_\rho(\Psi)\in\mathcal{F}_{\mathrm{loc}}$. It is easy to see that

$$E(\phi, g_\rho(\Psi)) = -\int_{M_*}\phi g'_\rho(\Psi)[dd^c\Psi-\rho(\Psi)d\Psi\wedge d^c\Psi]\wedge\theta \geqq 0\,,$$

for every non-negative $\phi\in C_0^\infty(M_*)$. In fact, it is enough to establish this equality by assuming that the support of ϕ is included in some complex coordinate neighbourhood and employing the smooth approximating procedure for the plurisubharmonic function Ψ on the neighbourhood. Hence, we know that $g_\rho(\Psi)$ is $\boldsymbol{M}_\theta$-supermean valued and consequently

$$E_z[g_\rho(\Psi)(Z_{\tau_{M(R)}\wedge\sigma_{M[s_0]}})] \leqq g_\rho(\Psi)(z)$$

for q.e. $z\in M(s_0, R)$. This expression leads us to the estimate in the proposition. Let us extend the state space of $\boldsymbol{M}_\theta$ by setting each point $z\in M[s_0]$ to be the trap (i.e., $P_z(Z_t=z$, for all $t\geqq 0)=1$). Then the extended diffusion $\boldsymbol{M}$ is holomorphic and weakly recurrent, because any $\mathcal{E}$-capacity zero set has Lebesgue measure zero on every complex coordinate neighbourhood on account of (C.2). q.e.d.

THEOREM 1. *Let M be a connected complex manifold of dimension $n\geqq 2$ which possesses an unbounded plurisubharmonic exhaustion function Ψ satisfying conditions* (C.1) *and* (C.2). *Let u be a plurisubharmonic function on M. If $\lim_{R\to\infty}m_\Psi(u, R)/g_\rho(R)=0$, for $m_\Psi(u, R)=\sup_{\Psi(z)\leqq R}u(z)$, then u is constant. In particular, M admits no non-constant bounded plurisubharmonic function.*

PROOF. By virtue of Proposition 1 in [7], u is M-subharmonic. Applying our Proposition 1 to $\Gamma=M[s_0]$ and $\Omega_k=M(s_0+k)$, $k=1, 2, 3, \cdots$, and using Proposition 2, we have

$$u(z)\leqq \sup_{\Psi(z)\leqq s_0} u(z) \qquad \text{a.a. } z\,.$$

Because the coordinate spherical submean value property for u is always satisfied on every complex coordinate neighbourhood, the above inequality is valid for all $z\in M$. By the maximum principle, u must be constant. q.e.d.

We are confronted with the problem of constructing a suitable plurisubharmonic exhaustion function Ψ satisfying properties (i) and (ii). In the next section, we consider the case that Ψ is a function of the distance from a point of M.

We close this section by giving a simple application of Theorem 1 to a parabolic manifold. A complex manifold N of dimension n with a non-negative smooth unbounded exhaustion function ϕ is called parabolic, if $\log\phi$ is a plurisubharmonic function on $\{\phi>0\}$ satisfying $(dd^c \log\phi)^n=0$ and $(dd^c \log\phi)^{n-1}\not\equiv 0$ on $\{\phi>0\}$.

COROLLARY. *Let (N, ϕ) be a parabolic manifold of dimension $n\geqq 2$ and ϕ be strictly plurisubharmonic outside a compact subset of N. Then any plurisubharmonic function u on N satisfying $\lim_{R\to\infty} m_{\log\phi}(u, R)/\log R=0$ for $m_{\log\phi}(u, R)=\sup_{\log\phi(z)\leqq R} u(z)$ is constant.*

PROOF. An elementary calculation shows that $\Psi=\log\phi\vee 0$ is plurisubharmonic. Since $(dd^c\Psi)^n\geqq 0$, it is clear that $(dd^c\phi)^n\geqq n\phi^{-1}(dd^c\phi)^{n-1}\wedge d\phi\wedge d^c\phi$. Hence, we obtain

$$\theta\wedge dd^c\phi \geqq (n\phi^{n-1})^{-1}(dd^c\phi)^n .$$

Therefore, the unbounded plurisubharmonic exhaustion function Ψ satisfies condition (C.1) for $\rho=0$ and (C.2) for $p=\phi$. The result follows from Theorem 1. q. e. d.

By looking into the proof, we find that this corollary remains valid even in the case that ϕ is strictly plurisubharmonic in distribution sense. It is well-known that if (N, ϕ) is a parabolic manifold of dimension n and ϕ is smooth strictly plurisubharmonic on N, then there exists a biholomorphic map $f: \boldsymbol{C}^n\to N$ enjoying $f^*\phi(z)=|z|^2$ on $\boldsymbol{C}^n$ ([3] and [**14**]), thus our assertion becomes trivial in that case.

4. An application to a hermitian manifold.

THEOREM 2. *Let (M, g) be a hermitian manifold of dimension $n\geqq 2$ and let r be the distance from a point $o\in M$. Suppose that r^2 is a smooth strictly plurisubharmonic function outside a compact subset of M and that there exists a non-decreasing function $\lambda(x)$ defined on $[0, \infty)$ satisfying the following conditions:*

(i) *$\lambda(0)=0$ and further $\lambda'(x)$ is positive and differentiable for large x,*
(ii) *$\Psi=\lambda(r)$ is an unbounded plurisubharmonic exhaustion function on M,*
(iii) *for some $\rho\in\mathcal{A}$, the inequality*

$$0<\omega\leqq \{r\lambda'(r)\rho(\Psi)/n-r\lambda''(r)/\lambda'(r)+1\}\omega_M/2$$

holds outside a compact subset of M, where $\omega=dd^c r^2/4$ and ω_M denotes the fundamental 2-form associated with the metric g.

Then M does not admit a non-constant plurisubharmonic function u enjoying $\lim_{R\to\infty} m_\Psi(u, R)/g_\rho(R)=0$ for $m_\Psi(u, R)=\sup_{\Psi(z)\leqq R} u(z)$.

PROOF. Since $dd^c\Psi=2\lambda'(r)\omega/r+r(\lambda'(r)/r)'dr\wedge d^c r$, a simple computation shows that the condition (C.1) of §3 is satisfied provided that we get

$$\omega^n/\alpha n \leqq \omega^{n-1}\wedge dr\wedge d^c r$$

for $\alpha=\{r\lambda'(r)\rho(\Psi)/n-r\lambda''(r)/\lambda'(r)+1\}/2$ with a suitably choosen $\rho\in\mathcal{A}$. This inequality actually follows from condition (iii):

$$\omega^{n-1}\wedge dr\wedge d^c r/\omega^n \geqq \omega_M^{n-1}\wedge dr\wedge d^c r/\alpha\omega_M^n \geqq (n\alpha)^{-1}.$$

Condition (C.2) can be seen to be fulfilled by taking $p=r^2$ similarly to the proof of Corollary of Theorem 1. Hence Theorem 2 follows from Theorem 1. q.e.d.

Finally we state an application of Theorem 2.

COROLLARY. *Let M be a Kähler manifold with a pole of dimension $n\geqq 2$ and r be the distance from the pole. If the radial curvature k satisfies*

$$|k| \leqq \delta/(r+a)^2\log(r+a) \qquad on\ \{r>0\}$$

for some $\delta<1/(9n-2)$ and $a>\exp\{2(1+2\delta)\}$, then there exists no non-constant plurisubharmonic function u on M such that

$$\lim_{s\to\infty} m_r(u, s)/\log(\log s) = 0,$$

for $m_r(u, s)=\sup_{r(z)\leqq s}u(z)$.

PROOF. The proof is based on the estimate concerning $\omega=dd^c r^2/4$ in [**16**]. For the sake of completeness, we state the procedure to compare ω on M with the one on models. Set

$$\begin{aligned} k_0(s) &= 2\delta\{(s+a)^2\log(s+a)\}^{-1} \\ k_1(s) &= k_0(s)\{1+2a/s-(1+2\delta)/\log(s+a)\} \\ k_2(s) &= -k_0(s)\{1-1/\log(s+a)\} \end{aligned}$$

and consider the solutions $f_i\,(i=1, 2)$ of the Jacobi equations

$$\begin{cases} f_i''(s)+k_i(s)f_i(s)=0 & (s>0), \\ f_i'(0)=1, \quad f_i(0)=0, & i=1, 2. \end{cases}$$

In what follows, we use the explicit expression

$$f_1(s) = s\{\log a/\log(s+a)\}^{2\delta}$$

of the solution for $i=1$.

Let $\lambda(x)=\int_a^{x\vee a} ds/f_1(s)$, which enjoys condition (i) in Theorem 2. Because $1/2<1-(1+2\delta)/\log a$, the bound $k_2(r)\leqq k\leqq k_1(r)$ holds everywhere on $\{r>0\}$. Accordingly we can obtain estimates for $\omega=dd^c r^2/4$ by comparing it with the one on models. In fact, the Hessian comparison theorem (Theorem A and Proposition 2.20) and Lemma 1.13 in [**10**] first assures condition (ii) of Theorem 2 for $\Psi=\lambda(r)$ and secondly lead us to the relation $0<\omega\leqq(rf_2'(r)/f_2(r)\vee 1)\omega_M$ concerning the Kähler form ω_M. We can then check the final condition (iii) from the last relation in the following manner.

Because $\phi_2(s)=f_2(s)/f_2'(s)$ is the solution of $\phi_2'(s)=1+k_2(s)\phi_2^2(s)$, we have

$$\phi_2(s)\leqq s$$

and further we know

$$\phi_2(s)\geqq s-2\delta s/\log(s+a).$$

This expression gives

$$s/\phi_2(s)\leqq 1+4\delta/\log(s+a).$$

Therefore, by an easy computation we have that

$$r/\phi_2(r)\leqq\{r\lambda'(r)/n\lambda(r)-r\lambda''(r)/\lambda'(r)+1\}/2$$

for sufficiently large r. Condition (iii) for $\rho(x)=1/x$ follows from the above estimate. The proof is complete by Theorem 2. q. e. d.

References

[1] E. Bedford and B. A. Taylor, Variational properties of the complex Monge-Ampère equation I, Dirichlet principle, Duke Math. J., **45** (1978), 375-403.

[2] E. Bedford and B. A. Taylor, A new capacity for plurisubharmonic functions, Acta Math., **149** (1982), 1-44.

[3] D. Burns, Curvature of Monge-Ampère foliation and parabolic manifolds, Ann. Math., **115** (1982), 349-373.

[4] E. D. Dynkin, Markov Processes, Springer, 1965.

[5] M. Fukushima, Dirichlet forms and Markov processes, North-Holland and Kodansha, 1980.

[6] M. Fukushima, On holomorphic diffusions and plurisubharmonic functions, in "Geometry of Random Motion" Contemporary Mathematics, to appear.

[7] M. Fukushima, On the continuity of plurisubharmonic functions along conformal diffusions, Osaka J. Math., **23** (1986), 69-75.

[8] M. Fukushima and M. Okada, On conformal martingale diffusions and pluripolar set, J. Funct. Anal., **55** (1984), 377-388.

[9] M. Fukushima and M. Okada, On Dirichlet forms for plurisubharmonic functions, Acta Math., **159** (1988), 171-214.

[10] R. H. Greene and H. Wu, Function theory on manifolds which possesses a pole, Lecture Notes in Math., **699**, Springer, 1979.

[11] R. H. Greene and H. Wu, Gap theorems for non-compact Riemannian manifolds, Duke Math. J., **49** (1982), 731-756.

[12] N. Mok, Y. T. Siu and S. T. Yau, The Poincaré-Lelong equation on complete Kähler manifolds, Comp. Math., **44** (1981), 183-218.

[13] Y. T. Siu and S. T. Yau, Complete Kähler manifolds with non-positive curvature of faster than quadratic decay, Ann. Math., **105** (1977), 235-264.

[14] W. Stoll, The Ahlfors-Weyl theorem of meromorphic maps on parabolic manifolds, Lecture Notes in Math., **981**, Springer, 1983.

[15] K. Takegoshi, A non-existence theorem for plurisubharmonic maps of finite energy, Math. Z., **192** (1986), 21-27.
[16] K. Takegoshi, Energy estimates and Liouville theorems for harmonic maps, Max-Planck-Institut für Mathematik, preprint.
[17] H. Wu, On a problem concerning the intrinsic characterization of $\boldsymbol{C}^n$, Math. Ann., **246** (1979), 15-22.

Hiroshi KANEKO
Department of Mathematical Sciences
College of Engineering
University of Osaka Prefecture
Mozu-Umemachi, Sakai, Osaka 591
Japan