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Introduction of new coordinates to the Schottky space —The general case—

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0. Introduction.

In the previous paper [4], we introduced new coordinates to the Schottky space with respect to a standard system of loops Σ, and we defined the augmented Schottky space. As we shall explain in § 1, a standard system of loops is a special case of a basic system of loops.

In this paper, in § 2, we will introduce new coordinates to the Schottky space in the general case, namely, in the case where Σ is a basic system of loops. In § 3, by using these coordinates, we will define the augmented Schottky space in the general case. We will discuss, in § 4, relations between the augmented Schottky space and compact Riemann surfaces with or without nodes.

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1. Multi-suffix and examples.

1-1. Let S be a compact Riemann surface of genus $g \geqq 2$. If mutually disjoint simple loops on S, $\delta_1, \delta_2, \cdots, \delta_n$, have the following property, then we call $\Sigma=\{\delta_1, \delta_2, \cdots, \delta_n\}$ a *basic system of loops*: Each component of $S-\bigcup_{j=1}^{n}\delta_j$ (we call it a *cell*) is a sphere with three disks removed, that is, a planar and triply connected domain. We have $n=3g-3$. If, in particular, the number of nondividing loops in Σ is equal to g, we call Σ a *standard system of loops* (see [4] pp. 155-157, more in detail).

Let $G^{(0)}$ be a fixed marked Schottky group generated by $A_1^{(0)}, A_2^{(0)}, \cdots, A_g^{(0)}$: $G^{(0)}=\langle A_1^{(0)}, A_2^{(0)}, \cdots, A_g^{(0)}\rangle$. Let $C_1, C_{g+1}; C_2, C_{g+2}; \cdots; C_g, C_{2g}$ be defining curves of $A_1^{(0)}, A_2^{(0)}, \cdots, A_g^{(0)}$, respectively, namely, they are mutually disjoint Jordan

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curves on the Riemann sphere which comprize the boundary of a $2g$-ply connected region ω (we call it a standard fundamental domain for $G^{(0)}$) and $A_j^{(0)}$ maps C_j onto C_{g+j} and $A_j^{(0)}(\omega)\cap\omega=\emptyset$ for each $j=1, 2, \cdots, g$. If mutually disjoint Jordan curves on $\hat{C}$, $C_1, \cdots, C_{2g}, \tilde{\gamma}_1, \cdots, \tilde{\gamma}_{2g-3}$ have the following properties, then we call $\tilde{\Sigma}=\{C_1, \cdots, C_{2g}; \tilde{\gamma}_1, \cdots, \tilde{\gamma}_{2g-3}\}$ a *basic system of Jordan curves for* $G^{(0)}$: (1) $\tilde{\gamma}_j$ $(j=1, 2, \cdots, 2g-3)$ lie in ω. (2) Each component of $\omega-\bigcup_{j=1}^{2g-3}\tilde{\gamma}_j$ (we call it a *cell* again) is a triply connected domain.

REMARK. We denote by $\alpha_i (i=1, 2, \cdots, g)$ and $\gamma_j (j=1, 2, \cdots, 2g-3)$ the images of C_i and $\tilde{\gamma}_j$, respectively, under the natural projection $\Pi: \Omega(G^{(0)})\to\Omega(G^{(0)})/G^{(0)}=S^{(0)}$. Then the set $\Sigma=\{\alpha_1, \cdots, \alpha_g; \gamma_1, \cdots, \gamma_{2g-3}\}$ is a basic system of loops on $S^{(0)}$. Then Σ is called the *basic system of loops associated with* $\tilde{\Sigma}$.

1-2. Let $\tilde{\sigma}_0$ be the component of $\omega-\bigcup_{j=1}^{2g-3}\tilde{\gamma}_j$ one of whose boundary curves is C_1. Let $\tilde{\delta}$ be an arbitrary boundary curve of $\tilde{\sigma}_0$ other than C_1. We denote by $H(\tilde{\delta})$ the union of closures of all cells which lie in the opposite part of $\tilde{\sigma}_0$ with respect to $\tilde{\delta}$.

We let $i(\tilde{\delta})$ be the following number: If $H(\tilde{\delta})\neq\emptyset$, $i(\tilde{\delta})$ is the smallest value of i with $C_i\subset H(\tilde{\delta})$ or $C_{g+i}\subset H(\tilde{\delta})$; if $H(\tilde{\delta})=\emptyset$ (then $\tilde{\delta}$ should be one of $C_2, C_3, \cdots, C_{2g}$), $i(\tilde{\delta})$ is the i with $\tilde{\delta}=C_i$ or $\tilde{\delta}=C_{g+i}$.

Now we denote the boundary curves of $\tilde{\sigma}_0$ other than C_1 by $\tilde{\gamma}(0)$ and $\tilde{\gamma}(1)$ according with the following rule: We should have $i(\tilde{\gamma}(0))\leqq i(\tilde{\gamma}(1))$ and, if $i(\tilde{\gamma}(0))=i(\tilde{\gamma}(1))$, then $C_{i(\tilde{\gamma}(0))}\subset H(\tilde{\gamma}(0))$ and $C_{g+i(\tilde{\gamma}(1))}\subset H(\tilde{\gamma}(1))$.

1-3. For $i_0=0$ and 1, we denote by $(\tilde{\sigma}_0: \tilde{\gamma}(i_0))$ the cell which lies in the opposite side of $\tilde{\sigma}_0$ with respect to $\tilde{\gamma}(i_0)$ and one of whose boundary curves is $\tilde{\gamma}(i_0)$. For the sake of simplicity, we write $\tilde{\sigma}(i_0)$ for $(\tilde{\sigma}_0: \tilde{\gamma}(i_0))$. We shall denote the boundary curves of $\tilde{\sigma}(i_0)$ other than $\tilde{\gamma}(i_0)$ by $\tilde{\gamma}(i_0, 0)$ and $\tilde{\gamma}(i_0, 1)$ in the same way as above.

Let $\tilde{\delta}$ be an arbitrary boundary curve of $\tilde{\sigma}(i_0)$ other than $\tilde{\gamma}(i_0)$. We denote by $H(\tilde{\delta})$ the union of closures of all cells which lie in the opposite part of $\tilde{\sigma}(i_0)$ with respect to $\tilde{\delta}$.

We let $i(\tilde{\delta})$ be the following number: If $H(\tilde{\delta})\neq\emptyset$, $i(\tilde{\delta})$ is the smallest value of i with $C_i\subset H(\tilde{\delta})$ or $C_{g+i}\subset H(\tilde{\delta})$; if $H(\tilde{\delta})=\emptyset$, $i(\tilde{\delta})$ is the i with $\tilde{\delta}=C_i$ or $\tilde{\delta}=C_{g+i}$.

Now we denote the boundary curves of $\tilde{\sigma}(i_0)$ other than $\tilde{\gamma}(i_0)$ by $\tilde{\gamma}(i_0, 0)$ and $\tilde{\gamma}(i_0, 1)$ according with the following rule: We should have $i(\tilde{\gamma}(i_0, 0))\leqq i(\tilde{\gamma}(i_0, 1))$ and, if $i(\tilde{\gamma}(i_0, 0))=i(\tilde{\gamma}(i_0, 1))$, then $C_{i(\tilde{\gamma}(i_0,0))}\subset H(\tilde{\gamma}(i_0, 0))$ and $C_{g+i(\tilde{\gamma}(i_0,1))}\subset H(\tilde{\gamma}(i_0, 1))$.

1-4. For $i_\nu=0$ and $1 (\nu=0, 1)$, we denote by $(\tilde{\sigma}(i_0): \tilde{\gamma}(i_0, i_1))$ the cell which lies in the opposite side of $\tilde{\sigma}(i_0)$ with respect to $\tilde{\gamma}(i_0, i_1)$ and one of whose boundary curves is $\tilde{\gamma}(i_0, i_1)$. For the sake of simplicity, we write $\tilde{\sigma}(i_0, i_1)$ for $(\tilde{\sigma}(i_0);$

$\tilde{\gamma}(i_0, i_1)$). We shall denote the boundary curves of $\tilde{\sigma}(i_0, i_1)$ other than $\tilde{\gamma}(i_0, i_1)$ by $\tilde{\gamma}(i_0, i_1, 0)$ and $\tilde{\gamma}(i_0, i_1, 1)$ in the same way as above.

Let $\tilde{\delta}$ be an arbitrary boundary curve of $\tilde{\sigma}(i_0, i_1)$ other than $\tilde{\gamma}(i_0, i_1)$. We denote by $H(\tilde{\delta})$ the union of closures of all cells which lie in the opposite part of $\tilde{\sigma}(i_0, i_1)$ with respect to $\tilde{\delta}$.

We let $i(\tilde{\delta})$ be the following number: If $H(\tilde{\delta})\neq\emptyset$, $i(\tilde{\delta})$ is the smallest value of i with $C_i\subset H(\tilde{\delta})$ or $C_{g+i}\subset H(\tilde{\delta})$, if $H(\tilde{\delta})=\emptyset$, $i(\tilde{\delta})$ is the i with $\tilde{\delta}=C_i$ or $\tilde{\delta}=C_{g+i}$.

Now we denote the boundary curves of $\tilde{\sigma}(i_0, i_1)$ other than $\tilde{\gamma}(i_0, i_1)$ by $\tilde{\gamma}(i_0, i_1, 0)$ and $\tilde{\gamma}(i_0, i_1, 1)$ according with the following rule: We should have $i(\tilde{\gamma}(i_0, i_1, 0))\leqq i(\tilde{\gamma}(i_0, i_1, 1))$ and, if $i(\tilde{\gamma}(i_0, i_1, 0))=i(\tilde{\gamma}(i_0, i_1, 1))$ then $C_{i(\tilde{\gamma}(i_0, i_1, 1))}\subset H(\tilde{\gamma}(i_0, i_1, 0))$ and $C_{g+i(\tilde{\gamma}(i_0, i_1, 1))}\subset H(\tilde{\gamma}(i_0, i_1, 1))$.

1-5. The above process is repeated. In general, for $i_\nu=0$ and 1 ($\nu=0, 1, \cdots, \mu$), suppose $\tilde{\gamma}(i_0), \tilde{\gamma}(i_0, i_1), \cdots, \tilde{\gamma}(i_0, i_1, \cdots, i_\mu)$ have determined. We denote by $(\tilde{\sigma}(i_0, i_1, \cdots, i_{\mu-1})\colon \tilde{\gamma}(i_0, i_1, \cdots, i_\mu))$ the cell which lies in the opposite side of $\tilde{\sigma}(i_0, i_1, \cdots, i_{\mu-1})$ with respect to $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu)$ and one of whose boundary curves is $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu)$. For the sake of simplicity, we write $\tilde{\sigma}(i_0, i_1, \cdots, i_\mu)$ for $(\tilde{\sigma}(i_0, i_1, \cdots, i_{\mu-1})\colon \tilde{\gamma}(i_0, i_1, \cdots, i_\mu))$. We shall denote the boundary curves of $\tilde{\sigma}(i_0, i_1, \cdots, i_\mu)$ other than $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu)$ by $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 0)$ and $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 1)$ in the same way as above.

Let $\tilde{\delta}$ be an arbitrary boundary curve of $\tilde{\sigma}(i_0, i_1, \cdots, i_\mu)$ other than $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu)$. We denote by $H(\tilde{\delta})$ the union of closures of all cells which lie in the opposite part of $\tilde{\sigma}(i_0, i_1, \cdots, i_\mu)$ with respect to $\tilde{\delta}$.

We let $i(\tilde{\delta})$ be the following number: If $H(\tilde{\delta})\neq\emptyset$, $i(\tilde{\delta})$ is the smallest value of i with $C_i\subset H(\tilde{\delta})$ or $C_{g+i}\subset H(\tilde{\delta})$; if $H(\tilde{\delta})=\emptyset$, $i(\tilde{\delta})$ is the i with $\tilde{\delta}=C_i$ or $\tilde{\delta}=C_{g+i}$.

Now we denote the boundary curves of $\tilde{\sigma}(i_0, i_1, \cdots, i_\mu)$ other than $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu)$ by $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 0)$ and $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 1)$ according with the following rule: We should have $i(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 0))\leqq i(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 1))$ and, if $i(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 0))=i(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 1))$ then $C_{i(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 0))}\subset H(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 0))$ and $C_{g+i(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 1))}\subset H(\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 1))$.

1-6. Examples. Here we present two illustrative examples.

EXAMPLE 1. Let $G^{(0)}=\langle A_1^{(0)}, A_2^{(0)}, A_3^{(0)}\rangle$ be a marked Schottky group. Let defining curves $C_1, C_2, \cdots, C_6$ and curves $\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3$ be as in Fig. 1. Then we have a Riemann surface $S^{(0)}$ and loops $\alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2, \gamma_3$ as in Fig. 2, where $\alpha_i=\Pi(C_i)$ and $\gamma_j=\Pi(\tilde{\gamma}_j)$ and $\Pi\colon \Omega(G^{(0)})\rightarrow S^{(0)}$ is the natural projection. We express the Fig. 1 as a tree in Fig. 3. Here every white circle ○ denotes a cell and every segment denotes an element of $\tilde{\Sigma}=\{C_1, \cdots, C_6; \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3\}$. If we represent the cells and elements of $\tilde{\Sigma}$ in Fig. 3 by using multi-suffixes, we have

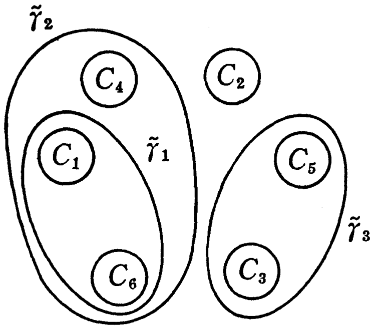

Figure 1.

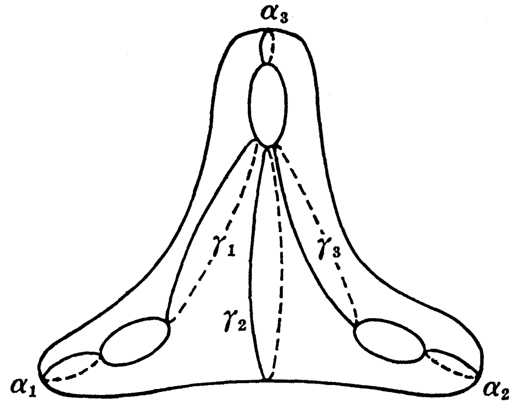

Figure 2.

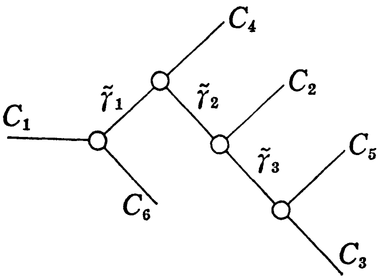

Figure 3.

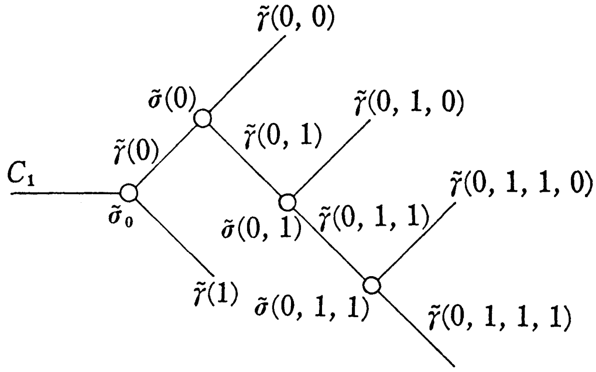

Figure 4.

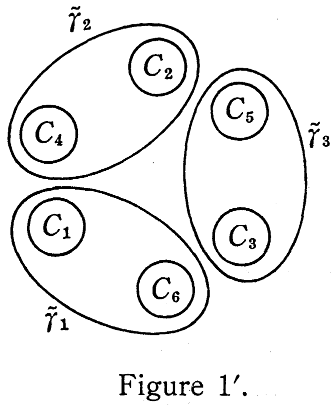

Figure 1′.

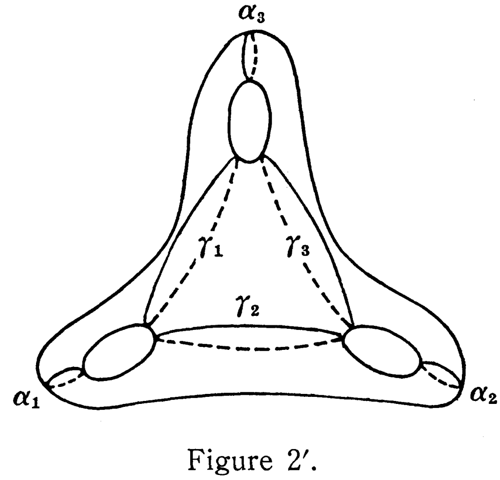

Figure 2′.

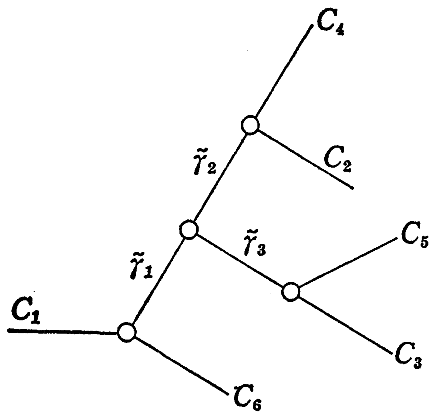

Figure 3′.

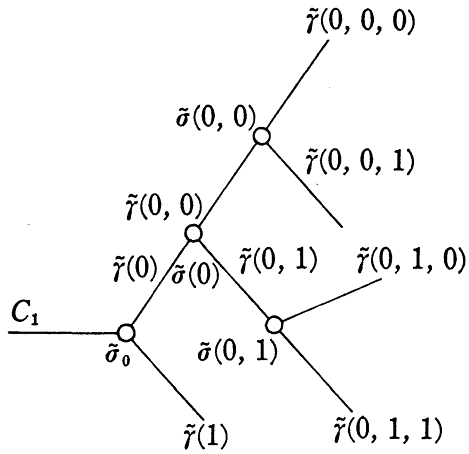

Figure 4′.

Fig. 4. We have the following: $\tilde{\gamma}(0)=\tilde{\gamma}_1$, $\tilde{\gamma}(1)=C_6$, $\tilde{\gamma}(0,0)=C_4$, $\tilde{\gamma}(0,1)=\tilde{\gamma}_2$, $\tilde{\gamma}(0,1,0)=C_2$, $\tilde{\gamma}(0, 1, 1)=\tilde{\gamma}_3$, $\tilde{\gamma}(0, 1, 1, 0)=C_5$ and $\tilde{\gamma}(0, 1, 1, 1)=C_3$.

EXAMPLE 2. Let $G^{(0)}$ be the same marked Schottky group as in Example 1. Let defining curves $C_1, C_2, \cdots, C_6$ and curves $\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3$ be as in Fig. 1′. Corresponding to them, we have the following Fig. 2′, Fig. 3′ and Fig. 4′. Observe $\tilde{\gamma}(0)=\tilde{\gamma}_1$, $\tilde{\gamma}(1)=C_6$, $\tilde{\gamma}(0, 0)=\tilde{\gamma}_2$, $\tilde{\gamma}(0, 1)=\tilde{\gamma}_3$, $\tilde{\gamma}(0, 0, 0)=C_4$, $\tilde{\gamma}(0, 0, 1)=C_2$, $\tilde{\gamma}(0, 1, 0)=C_5$ and $\tilde{\gamma}(0, 1, 1)=C_3$.

2. Introduction of new coordinates.

2-1. We fix a marked Schottky group $G^{(0)}=\langle A_1^{(0)}, \cdots, A_g^{(0)}\rangle$. Let $\tilde{\Sigma}=\{C_1, \cdots, C_{2g};\ \tilde{\gamma}_1, \cdots, \tilde{\gamma}_{2g-3}\}$ be a fixed basic system of Jordan curves for $G^{(0)}$. In this section, we will introduce new coordinates to the Schottky space with respect to $\tilde{\Sigma}$.

Let $G=\langle A_1, A_2, \cdots, A_g\rangle$ be a marked Schottky group. Let λ_j $(|\lambda_j|>1)$, p_j and p_{g+j} be the multiplier, the repelling and the attracting fixed points of A_j, respectively. We normalize G by setting $p_1=0$, $p_{g+1}=\infty$ and $p_2=1$. Then a point in the Schottky space $\mathfrak{S}_g$ is identified with

$$\tilde{\tau}=(\lambda_1, \cdots, \lambda_g, p_{g+2}, p_3, p_{g+3}, \cdots, p_g, p_{2g})\in \boldsymbol{C}^{3g-3}.$$

Now we will introduce new coordinates with respect to $\tilde{\Sigma}$:

$$\tau=(t_1, \cdots, t_g, \rho_1, \cdots, \rho_{2g-3})\in \boldsymbol{C}^{3g-3}.$$

First define t_i by setting $t_i=1/\lambda_i\,(i=1, 2, \cdots, g)$. Thus $t_i\in D^*=\{z\,|\,0<|z|<1\}$.

Next in order to define ρ_j associated with $\tilde{\gamma}_j=\tilde{\gamma}(i_0, i_1, \cdots, i_\mu)\in\tilde{\Sigma}$ $(j=1, 2, \cdots, 2g-3)$, we determine integers $k(j)$, $l(j)$, $m(j)$ and $n(j)$, which are $\geqq 1$ and $\leqq 2g$ as follows: $k(j)=1$, $C_{l(j)}=\tilde{\gamma}(i_0, i_1, \cdots\cdots i_{\mu-1}, 1-i_\mu, 0, \cdots, 0)$, $C_{m(j)}=\tilde{\gamma}(i_0, i_1, \cdots, i_\mu,$

$0, \cdots, 0)$ and $C_{n(j)}=\tilde{\gamma}(i_0, i_1, \cdots, i_\mu, 1, 0, \cdots, 0)$.

For each $j=1, 2, \cdots, 2g-3$, the coordinate ρ_j is now defined as follows: We determine $T_j \in$ Möb by $T_j(p_{k(j)})=0$, $T_j(p_{l(j)})=\infty$ and $T_j(p_{m(j)})=1$ and set $\rho_j=T_j(p_{n(j)})$.

REMARK. Let $\{C_1, \cdots, C_{2g}; \tilde{\gamma}'_1, \cdots, \tilde{\gamma}'_{2g-3}\}$ be a basic system of Jordan curves satisfying the following condition: For each $j=1, 2, \cdots, 2g-3$, $\tilde{\gamma}'_j$ is homotopic to $\tilde{\gamma}_j$ in the standard fundamental domain ω. Let ρ'_j be the coordinate associated with $\tilde{\gamma}'_j$. Then $\rho_j=\rho'_j$.

By the same method as in the proof of Proposition 4 in [4], we have the following.

PROPOSITION. *Two equivalent marked Schottky groups* $G=\langle A_1, \cdots, A_g\rangle$ *and* $\hat{G}=\langle \hat{A}_1, \cdots, \hat{A}_g\rangle$, *that is,* $\hat{A}_k=UA_kU^{-1}$, $U\in$ Möb, *have the same coordinates* t_i *and* ρ_j.

Thus we can define a mapping φ of $\mathfrak{S}_g$ into $D^{*g}\times(\boldsymbol{C}\setminus\{0, 1\})^{2g-3}$ by setting $\varphi([G])=(t_1, \cdots, t_g, \rho_1, \cdots, \rho_{2g-3})$, where $[G]$ denotes the equivalence class of G, that is, a point in $\mathfrak{S}_g$. We denote by $\mathfrak{S}_g(\tilde{\Sigma})$ the image of $\mathfrak{S}_g$ under the mapping φ.

2-2. Next we consider the converse. Let $G^{(0)}$ and $\tilde{\Sigma}$ be as in § 2-1. Since $C_j\in\tilde{\Sigma}$ $(j=1, 2, \cdots, 2g)$ are represented as $\tilde{\gamma}(i_0, i_1, \cdots, i_\mu)$, we may write $p(i_0, i_1, \cdots, i_\mu)$ for the fixed points p_j. Furthermore $\tilde{\gamma}_j\in\tilde{\Sigma}$ $(j=1, 2, \cdots, 2g-3)$ are represented as $\tilde{\gamma}_j(i_0, i_1, \cdots, i_\nu)$ and so we may write $\rho(i_0, i_1, \cdots, i_\nu)$ for ρ_j.

We will show that λ_j, p_j and p_{g+j} $(j=1, 2, \cdots, g)$ are uniquely determined by a given point

$$\tau=(t_1, \cdots, t_g, \rho_1, \cdots, \rho_{2g-3})\in D^{*g}\times(\boldsymbol{C}\setminus\{0, 1\})^{2g-3}$$

under the normalized condition $p_1=0$, $p(0, 0, \cdots, 0)=\infty$ and $p(1, 0, 0, \cdots, 0)=1$.

The first step. We determine $p(0, 1, 0, \cdots, 0)$ and $p(1, 1, 0, \cdots, 0)$ by the process opposite to the above: We determine $T\in$ Möb by $T(0)=0$, $T(1)=\infty$ and $T(\infty)=1$ and set $p(0, 1, 0, \cdots, 0)=T^{-1}(\rho(0))$ and $p(1, 1, 0, \cdots, 0)=\rho(1)$.

The second step. Suppose $p(i_0, i_1, \cdots, i_{\mu-1}, 0, \cdots, 0)$ and $p(i_0, i_1, \cdots, i_{\mu-1}, 1, 0, \cdots, 0)$ are determined. Then by the process opposite to the above, we determine $p(i_0, i_1, \cdots, i_{\mu-1}, 0, 1, 0, \cdots, 0)$ and $p(i_0, i_1, \cdots, i_{\mu-1}, 1, 1, 0, \cdots, 0)$: We determine $T\in$ Möb by $T(0)=0$, $T(p(i_0, i_1, \cdots, i_{\mu-1}, 1, 0, \cdots, 0))=\infty$, $T(p(i_0, i_1, \cdots, i_{\mu-1}, 0, 0, \cdots, 0))=1$ and set $p(i_0, i_1, \cdots, i_{\mu-1}, 0, 1, 0, \cdots, 0)=T^{-1}(\rho(i_0, i_1, \cdots, i_{\mu-1}, 0))$; we determine $T\in$ Möb by $T(0)=0$, $T(p(i_0, i_1, \cdots, i_{\mu-1}, 0, \cdots, 0))=\infty$, $T(p(i_0, i_1, \cdots, i_{\mu-1}, 1, 0, \cdots, 0))=1$ and set $p(i_0, i_1, \cdots, i_{\mu-1}, 1, 1, 0, \cdots, 0)=T^{-1}(\rho(i_0, i_1, \cdots, i_{\mu-1}, 1))$.

By the induction, we determine $p_1, p_{g+1}, \cdots, p_g, p_{2g}$.

The third step. We define λ_i $(i=1, 2, \cdots, g)$ by setting $\lambda_i=1/t_i$.

By the above, we determine $A_j(\tau)\in$ Möb by τ as follows: The multiplier, the repelling and the attracting fixed points of $A_j(\tau)$ are λ_j, p_j and p_{g+j}, respectively. Thus we obtain a mapping ψ of $D^{*g}\times(\boldsymbol{C}\setminus\{0, 1\})^{2g-3}$ into Möb^g by setting $\psi(\tau)=\langle A_1(\tau), A_2(\tau), \cdots, A_g(\tau)\rangle$ (we denote it by $G(\tau)$).

2-3. THEOREM 1. *Let the mapping* $\varphi: \mathfrak{S}_g\to D^{*g}\times\{\boldsymbol{C}\setminus\{0, 1\}\}^{2g-3}$ *and* ψ: $D^{*g}\times(\boldsymbol{C}\setminus\{0, 1\})^{2g-3}\to\text{Möb}^g$ *be as above. Then* $\psi\varphi=\text{id.}$ *and* $\varphi\psi|\mathfrak{S}_g(\Sigma)=\text{id.}$, *where* id. *and* $\psi|\mathfrak{S}_g(\Sigma)$ *denote the identity mapping and the restriction of the mapping* ψ *to the set* $\mathfrak{S}_g(\Sigma)$, *respectively.*

3. Augmented Schottky spaces.

3-1. Let $G^{(0)}=\langle A_1^{(0)}, A_2^{(0)}, \cdots, A_g^{(0)}\rangle$, $\tilde{\Sigma}=\{C_1, \cdots, C_{2g}; \tilde{\gamma}_1, \cdots, \tilde{\gamma}_{2g-3}\}$ and $\Sigma=\{\alpha_1, \cdots, \alpha_g; \gamma_1, \cdots, \gamma_{2g-3}\}$ be a fixed marked Schottky group, a basic system of Jordan curves for $G^{(0)}$ and the basic system of loops associated with $\tilde{\Sigma}$ as in § 2, respectively. By identifying C_i and C_{g+i} $(i=1, 2, \cdots, g)$, we have different figures from ones in § 1, namely we have figures for Σ. For example, we have the following Fig. 5 and Fig. 5′ instead of Fig. 3 and Fig. 3′, respectively.

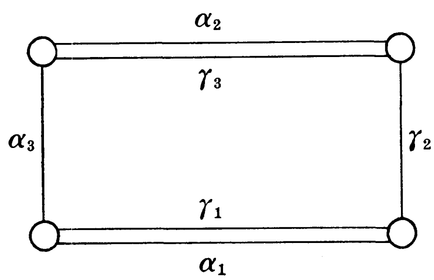

Figure 5.

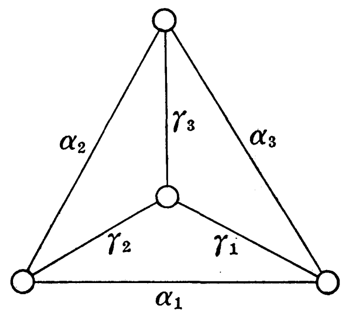

Figure 5′.

Let $\tilde{\gamma}_{i1}, \tilde{\gamma}_{i2}, \cdots, \tilde{\gamma}_{il(i)}$ be a sequence of $\tilde{\gamma}_j$ in $\tilde{\Sigma}$ as follows: They separate p_i from p_{g+i} and they are arranged from p_i to p_{g+i}. We say the sequence of elements in Σ, $(\alpha_i; \gamma_{i1}, \gamma_{i2}, \cdots, \gamma_{il(i)})$, the "cycle" containing α_i, and denote it by L_i. Obviously there are g "cycles" L_i $(i=1, 2, \cdots, g)$ and each L_i corresponds to the generator $A_i^{(0)}$ of $G^{(0)}$. For example, in Fig. 5, there are three "cycles" $(\alpha_1; \gamma_1)$, $(\alpha_2; \gamma_3)$ and $(\alpha_3; \gamma_3, \gamma_2, \gamma_1)$ corresponding to $A_1^{(0)}$, $A_2^{(0)}$ and $A_3^{(0)}$, respectively, and in Fig. 5′, there are three "cycles" $(\alpha_1; \gamma_1, \gamma_2)$, $(\alpha_2; \gamma_2, \gamma_3)$ and $(\alpha_3; \gamma_3, \gamma_1)$ corresponding to $A_1^{(0)}$, $A_2^{(0)}$ and $A_3^{(0)}$, respectively.

Let $I\subset\{1, 2, \cdots, g\}$, $J\subset\{1, 2, \cdots, 2g-3\}$, $|I|$=number of elements in I and $|J|$=number of elements in J. We define a subset $I(J)$ of $\{1, 2, \cdots, g\}$ as follows.

Let $J=\{j_1, \cdots, j_m\}$. For each $i=1, 2, \cdots, m$, let $L_{j_i,1}, \cdots, L_{j_i,k(j_i)}$ be the "cycles" containing γ_{j_i}. Then we define $I(J)$ by setting

$I(J)=\{i\in\{1, 2, \cdots, g\}\,|\,\alpha_i\in\Sigma$ is contained in a "cycle" $L_{j,l}$ for some l $(1\leqq l\leqq k(j))$ and for some $j\in J\}$.

REMARK. The set $I(J)$ may be empty. If all γ_j, $j\in J$, are dividing loops, then $I(J)=\emptyset$. Thus if Σ is a standard system of loops, then $I(J)=\emptyset$ for all J.

3-2. We will define subsets $\delta^{I,J}\mathfrak{S}_g(\tilde{\Sigma})$ of $\overline{\mathfrak{S}_g(\tilde{\Sigma})}=\mathfrak{S}_g(\tilde{\Sigma})\cup\partial\mathfrak{S}_g(\tilde{\Sigma})\subset\bar{D}^g\times\hat{C}^{2g-3}$, where $D=\{z\,|\,|z|<1\}$. We set $X=\delta^{I,J}\mathfrak{S}_g(\Sigma)$. From now on we assume that $I(J)$ is a subset of I.

(1) When $I=\emptyset$ and $J=\emptyset$, we define X as $\mathfrak{S}_g(\tilde{\Sigma})$, the Schottky space.

(2) When $I\neq\emptyset$ and $J=\emptyset$, we define X by the same method as in [4], and we denote it by $\delta^I\mathfrak{S}_g(\tilde{\Sigma})$.

(3) When $J\neq\emptyset$ and $I=\emptyset$ (hence $I(J)=\emptyset$), we define X by the same method as in [4], and we denote it by $\hat{\delta}^J\mathfrak{S}_g(\tilde{\Sigma})$.

(4) When $J\neq\emptyset$, $I(J)=I\neq\emptyset$, we will define X as the set of all points $\tau=(t_1, \cdots, t_g, \rho_1, \cdots, \rho_{2g-3})$ satisfying the properties (i)-(v) which will be described in § 3-5, and denote it by $\delta^{I(J),J}\mathfrak{S}_g(\tilde{\Sigma})$.

(5) When $J\neq\emptyset$, $I(J)\neq\emptyset$ and $I\setminus I(J)\neq\emptyset$, we define $\delta^{I,J}\mathfrak{S}_g(\tilde{\Sigma})$ by combining the methods of the above (2), (3) and (4) altogether.

3-3. Now we will define sets $\delta^{I(J),J}\mathfrak{S}_g(\tilde{\Sigma})$. Let $J=\{j_1, j_2, \cdots, j_m\}$ $(j_1<j_2<\cdots<j_m)$ and $I(J)=\{i_1, i_2, \cdots, i_s\}$ $(i_1<i_2<\cdots<i_s)$.

The figure of tree defined in § 1 is divided into $|J|+1$ $(=m+1)$ parts by cutting along m Jordan curves $\tilde{\gamma}_j$ $(j\in J)$.

The first step. We consider the following sequences:

Case 1. $\rho(i_0)\neq1$, $\rho(i_0, i_1)\neq1, \cdots, \rho(i_0, i_1, \cdots, i_{\mu_0})\neq1$, and $\tilde{\gamma}(i_0, i_1, \cdots, i_{\mu_0+1})$ is one of the defining curves $C_2, C_3, \cdots, C_{2g}$.

Case 2. $\rho(i_0)\neq1$, $\rho(i_0, i_1)\neq1, \cdots, \rho(i_0, i_1, \cdots, i_{\mu_0})\neq1$, $\rho(i_0, i_1, \cdots, i_{\mu_0+1})=1$.

We perform the same process as in the previous section by using 0, ∞ and 1 instead of p_1, $p(0, 0, \cdots, 0)$ and $p(1, 0, \cdots, 0)$, respectively. Then we can determine a number for each $\rho(i_0, i_1, \cdots, i_{\nu_0})$ $(0\leqq\nu_0\leqq\mu_0)$. Namely, we get the following:

1) Suppose $\rho(i_0, i_1, \cdots, i_{\nu_0}, 1)\neq1$, $\rho(i_0, i_1, \cdots, i_{\nu_0}, 1, 0)\neq1, \cdots, \rho(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)\neq1$ $(\nu_0\leqq\mu_0)$ and $\tilde{\gamma}(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)$ is one of the defining curves $C_2, C_3, \cdots, C_{2g}$. Then we can determine $p(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)$ from $\rho(i_0, i_1, \cdots, i_{\nu_0})$ by the same way as in § 2.

2) Suppose $\rho(i_0, i_1, \cdots, i_{\nu_0}, 1)\neq1$, $\rho(i_0, i_1, \cdots, i_{\nu_0}, 1, 0)\neq1, \cdots, \rho(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)\neq1$, $\rho(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)=1$. Then we determine a number from $\rho(i_0, i_1, \cdots, i_{\nu_0})$ by the same way as in § 2. We denote the number by $p^+(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)$ and call it the *right distinguished point associated with* $\tilde{\gamma}(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)$.

In particular, if $\rho(0)\neq1$, $\rho(0, 0)\neq1, \cdots, \rho(\underbrace{0, 0, \cdots, 0}_{h})\neq1$ and $\tilde{\gamma}(\underbrace{0, 0, \cdots, 0}_{h+1})$ is one of the defining curves $C_2, C_3, \cdots, C_{2g}$, then we set $p(\underbrace{0, 0, \cdots, 0}_{h+1})=\infty$ under our normalization and if $\rho(0)\neq 1$, $\rho(0, 0)\neq 1, \cdots, \rho(\underbrace{0, 0, \cdots, 0}_{k})\neq 1$, $\rho(\underbrace{0, 0, \cdots, 0}_{k+1})=1$, then we set $p^+(\underbrace{0, 0, \cdots, 0}_{k+1})=\infty$. Next if $\rho(1)\neq 1$, $\rho(1, 0)\neq 1, \cdots, \rho(\underbrace{1, 0, \cdots, 0}_{h'})\neq 1$ and $\tilde{\gamma}(1, \underbrace{0, \cdots, 0}_{h'+1})$ is one of the defining curves $C_2, C_3, \cdots, C_{2g}$, then we set $p(1, \underbrace{0, \cdots, 0}_{h'+1})=1$ under our normalization, and if $\rho(1)\neq 1$, $\rho(1, 0)\neq 1, \cdots, \rho(1, \underbrace{0, \cdots, 0}_{k'})$ $\neq 1$, $\rho(1, \underbrace{0, \cdots, 0}_{k'+1})=1$, then we set $p^+(1, \underbrace{0, \cdots, 0}_{k'+1})=1$.

Suppose that there are m_0-1 numbers of sequences of Case 1. We denote p_1 and m_0-1 points determined in the above by $p_{0(1)}, p_{g+0(1)}, p_{0(2)}, p_{g+0(2)}, \cdots,$ $p_{0(g_0)}, p_{g+0(g_0)}, p_{0(2g_0+1)}, p_{0(2g_0+2)}, \cdots, p_{0(m_0)}$. Then we say $p_{0(2g_0+1)}, \cdots, p_{0(m_0)}$ *distinguished points of the first kind.*

Suppose that there are n_0+1 numbers of sequences of Case 2. We write $p^+_{0(m_0+1)}, \cdots, p^+_{0(m_0+n_0+1)}$ for the n_0+1 points $p^+(i_0, i_1, \cdots, i_{\nu_0}, 1, 0, \cdots, 0)$ determined in the above. We call them *distinguished points of the second kind.*

We set $\lambda_{0(i)}=1/t_{0(i)}$ $(i=1, 2, \cdots, g_0)$. Let $A_{0(i)}$ be the Möbius transformation whose multiplier, the repelling and the attracting fixed points are $\lambda_{0(i)}$, $p_{0(i)}$ and $p_{g+0(i)}$, respectively. We denote by $G_0(\tau)$ the group generated by $A_{0(1)}, A_{0(2)},$ $\cdots, A_{0(g_0)}$, that is $G_0(\tau)=\langle A_{0(1)}, A_{0(2)}, \cdots, A_{0(g_0)}\rangle$.

3-4. The second step. Next we consider the general case. Let $\tilde{\gamma}_{j_l}=\tilde{\gamma}(i_0,$ $i_1, \cdots, i_{\mu_l+1})$ $(l=1, 2, \cdots, m)$. We treat the following two cases:

Case 1. $\rho(i_0, \cdots, i_{\mu_l+1})=1$, $\rho(i_0, \cdots, i_{\mu_l+2})\neq 1, \cdots, \rho(i_0, \cdots, i_{\mu_{l'}})\neq 1$ and $\tilde{\gamma}(i_0,$ $\cdots, i_{\mu_{l'}+1})$ is one of the defining curves $C_2, C_3, \cdots, C_{2g}$.

Case 2. $\rho(i_0, i_1, \cdots, i_{\mu_l+1})=1$, $\rho(i_0, i_1, \cdots, i_{\mu_l+2})\neq 1, \cdots, \rho(i_0, i_1, \cdots, i_{\mu_{l'}})\neq 1$ and $\rho(i_0, i_1, \cdots, i_{\mu_{l'}+1})=1$.

We use 0, ∞ and 1 instead of p_1, $p(i_0, i_1, \cdots, i_{\mu_l+1}, 0, \cdots, 0)$ and $p(i_0, i_1, \cdots,$ $i_{\mu_l+1}, 1, 0, \cdots, 0)$, respectively (see p. 28 and use μ_l+1 instead of $\mu-1$ there). For each ν_l $(\mu_l+2\leqq\nu_l\leqq\mu_{l'})$, we determine p or a number (we call it the *right distinguished point associated with* $\tilde{\gamma}(i_0, i_1, \cdots, i_{\nu_l}, 1, 0, \cdots, 0)$ and denote by $p^+(i_0,$ $i_1, \cdots, i_{\nu_l}, 1, 0, \cdots, 0))$ from $\rho(i_0, i_1, \cdots, i_{\nu_l})$ by the same way as in p. 28.

Suppose that there are m_l numbers of sequences of Case 1. We denote the m_l points determined in the above by $p_{l(1)}, p_{g+l(1)}, \cdots, p_{l(g_l)}, p_{g+l(g_l)}, p_{l(2g_l+1)},$ $p_{l(2g_l+2)}, \cdots, p_{l(m_l)}$. Then we say $p_{l(2g_l+1)}, \cdots, p_{l(m_l)}$ *distinguished points of the first kind.* Suppose that there are n_l numbers of sequences of Case 2. We write $p^+_{l(m_l+1)}, p^+_{l(m_l+2)}, \cdots, p^+_{l(m_l+n_l)}$ for the n_l distinguished points, and we call them and $p^-_l(i_0, i_1, \cdots, i_{\mu_l+1})(=0)$ *distinguished points of the second kind.* In particular, we call $p^-_l(i_0, i_1, \cdots, i_{\mu_l+1})$ the *left distinguished points associated with* $\tilde{\gamma}(i_0, i_1, \cdots, i_{\mu_l+1})$ and write it $p^-_{\tilde{j}_l}$).

We set $\lambda_{l(i)}=1/t_{l(i)}$ $(i=1, 2, \cdots, g_l)$. Let $A_{l(i)}$ be the Möbius transformation whose multiplier, the repelling and the attracting fixed points are $\lambda_{l(i)}$, $p_{l(i)}$ and $p_{g+l(i)}$, respectively. We denote by $G_{j_l}(\tau)$ the group generated by $A_{l(1)}, \cdots, A_{l(g_l)}$, that is, $G_{j_l}(\tau)=\langle A_{l(1)}, \cdots, A_{l(g_l)}\rangle$.

3-5. By the above things, we get $m+1$ $(m=|J|)$ numbers of groups $G_0(\tau)$, $G_{j_1}(\tau), \cdots, G_{j_m}(\tau)$. Furthermore we obtain distinguished points of the first kind $p_{l(2g_l+1)}, \cdots, p_{l(m_l)}$ $(l=0, 1, \cdots, m)$, and distinguished points of the second kind $p^+_{l(m_l+1)}, \cdots, p^+_{l(m_l+n_l)}$ $(l=0, 1, \cdots, m)$, $p^-_{j_l}$ $(l=1, 2, \cdots, m)$ and $p^+_{0(n_0+1)}$.

Now we write the properties (i)-(v) as follows.

(i) $\rho_j=1$ for $j\in J$ and $\rho_j\neq 1$ for $j\notin J$.

(ii) $t_i=0$ for $i\in I(J)$ and $t_i\neq 0$ for $i\notin I(J)$.

(iii) For each $l=0, 1, \cdots, m$, $G_{j_l}(\tau)$ is a Schottky group or the trivial group, where $G_{j_0}(\tau)=G_0(\tau)$.

(iv) For each $l=0, 1, \cdots, m$, m_l-2g_l distinguished points of the first kind $p_{l(2g_l+1)}, \cdots, p_{l(m_l)}$ and n_l+1 distinguished points of the second kind $p^+_{l(m_l+1)}, \cdots, p^+_{l(m_l+n_l)}$ and $p^-_{j_l}(p^+_{0(n_0+1)}$ for $l=0)$ are distinct.

(v) For each $l=0, 1, \cdots, m$, the above $m_l-2g_l+n_l+1$ distinguished points lie in some standard fundamental domain for $G_{j_l}(\tau)$.

3-6. We give definitions of the following sets by using $\delta^{I,J}\mathfrak{S}_g(\tilde{\Sigma})$: $\mathfrak{S}^I_g(\tilde{\Sigma})=\bigcup_{K\subset I}\delta^K\mathfrak{S}_g(\tilde{\Sigma})$, $\hat{\mathfrak{S}}^J_g(\tilde{\Sigma})=\bigcup_{L\subset J}\delta^{I(L),L}\mathfrak{S}_g(\tilde{\Sigma})$, $\mathfrak{S}^{I,J}_g(\tilde{\Sigma})=\bigcup_{K\subset I,\, L\subset J}\delta^{K,L}\mathfrak{S}_g(\tilde{\Sigma})$ $(I(L)\subset K)$, $\mathfrak{S}^*_g(\tilde{\Sigma})=\mathfrak{S}^I_g(\tilde{\Sigma})$ with $I=\{1, 2, \cdots, g\}$, $\hat{\mathfrak{S}}_g(\tilde{\Sigma})=\hat{\mathfrak{S}}^J_g(\Sigma)$ with $J=\{1, 2, \cdots, 2g-3\}$, and $\hat{\mathfrak{S}}^*_g(\tilde{\Sigma})=\mathfrak{S}^{I,J}_g(\tilde{\Sigma})$ with $I=\{1, 2, \cdots, g\}$ and $J=\{1, 2, \cdots, 2g-3\}$.

DEFINITION. We call the set $\hat{\mathfrak{S}}^*_g(\tilde{\Sigma})$ the *augmented Schottky space associated with* $\tilde{\Sigma}$.

REMARK 1. If Σ is the standard system of loops associated with $\tilde{\Sigma}$, then we write $\hat{\mathfrak{S}}^*_g(\Sigma)$ for $\hat{\mathfrak{S}}^*_g(\tilde{\Sigma})$. In this case, $\hat{\mathfrak{S}}^*_g(\Sigma)$ is the augmented Schottky space in [4].

REMARK 2. If $I(J)=\emptyset$, then $\mathfrak{S}^{I,J}_g(\tilde{\Sigma})$ is a domain (see Proposition 5 in [4]), but if $I(J)\neq\emptyset$, then $\mathfrak{S}^{I,J}_g(\tilde{\Sigma})$ is not a domain.

REMARK 3. $\delta^{I(J),J}\mathfrak{S}_g(\tilde{\Sigma})$ is the intersection of $\partial\mathfrak{S}_g(\tilde{\Sigma})$ $(\subset\bar{D}^g\times\hat{C}^{2g-3})$ with $|I(J)|+|J|$ numbers of complex hyperplanes such that $t_i=0$ $(i\in I(J))$ or $\rho_j=1$ $(j\in J)$.

4. Augmented Schottky spaces and Riemann surfaces with or without nodes.

4-1. Throughout this section, let $G^{(0)}=\langle A^{(0)}_1, \cdots, A^{(0)}_g\rangle$, $\tilde{\Sigma}=\{C_1, \cdots, C_{2g}; \tilde{\gamma}_1, \cdots, \tilde{\gamma}_{2g-3}\}$ and $\Sigma=\{\alpha_1, \cdots, \alpha_g; \gamma_1, \cdots, \gamma_{2g-3}\}$ be a fixed marked Schottky group, a basic system of Jordan curves for $G^{(0)}$ and the basic system of loops on $S^{(0)}$

$=\Omega(G^{(0)})/G^{(0)}$ associated with $\tilde{\Sigma}$.

Let $\tau\in\delta^{I(J),J}\mathfrak{S}_g(\tilde{\Sigma})$ with $I(J)\neq\emptyset$. Let $J=\{j_1, \cdots, j_m\}$. By the same way as in the previous section, we have $m+1$ Schottky groups (including the trivial group) $G_0(\tau), G_{j_1}(\tau), \cdots, G_{j_m}(\tau)$. For each $l=0, 1, \cdots, m$, $\Omega(G_{j_l}(\tau))/G_{j_l}(\tau)=S_{j_l}(\tau)$ is a compact Riemann surface of genus g_l, where $G_{j_0}(\tau)=G_0(\tau)$ and $S_{j_0}(\tau)=S_0(\tau)$. Let $\Pi_{j_l}: \Omega(G_{j_l}(\tau))\rightarrow S_{j_l}(\tau)$ be the natural projection. We set $\hat{p}_{l(2g_l+k)}=\Pi_{j_l}(p_{l(2g_l+k)})$ $(k=1, 2, \cdots, m_l-2g_l)$, and we call them the *distinguished points of the first kind on* $S_{j_l}(\tau)$. Set $\hat{p}^+_{l(m_l+k)}=\Pi_{j_l}(p^+_{l(m_l+k)})$ $(k=1, 2, \cdots, n_l$ for $l=1, 2, \cdots, m$; $k=1, 2, \cdots, n_l+1$ for $l=0)$ and $\hat{p}^-_{j_l}=\Pi_{j_l}(p^-_{j_l})$ $(l=1, 2, \cdots, m)$. We call them *the right and the left distinguished points of the second kind on* $S_{j_l}(\tau)$, respectively.

For each distinguished point $p_{l(2g_l+k)}$ of the first kind with $p_{l(2g_l+k)}=p_i$ for i $(1\leqq i\leqq g)$, there are l' and k' such that $p_{g+i}=p_{l'(2g_{l'}+k')}$. By joining $\hat{p}_{l(2g_l+k)}$ and $\hat{p}_{l'(2g_{l'}+k')}$, and $\hat{p}^-_{j_l}$ and $\hat{p}^+_{j_l}$ for all distinguished points, we have a compact Riemann surface of genus $g_0+g_1+\cdots+g_m(=g)$ with $|I(J)|+|J|$ nodes, $S(\tau)$, where $\hat{p}^+_{j_l}=\pi_i(p^+_{i(m_i+k)})$ with $\gamma_{i(m_i+k)}=\gamma_{j_l}$.

For the cases of $\tau\in\mathfrak{S}_g(\tilde{\Sigma})$, $\tau\in\delta^I\mathfrak{S}_g(\tilde{\Sigma})$ and $\tau\in\delta^{I,J}\mathfrak{S}_g(\tilde{\Sigma})$ with $I(J)=\emptyset$, see [4].

Next let $\tau\in\delta^{I,J}\mathfrak{S}_g(\tilde{\Sigma})$ with $J\neq\emptyset$, $I(J)\neq\emptyset$ and $I\setminus I(J)\neq\emptyset$. By combining the method above and the method in [4], we obtain a compact Riemann surface of genus g with $|I|+|J|$ nodes, $S(\tau)$. We call $S(\tau)$ the *Riemann surface with nodes associated with* τ.

4-2. Before we consider the converse, we need some preparation.

For each $\tilde{\gamma}_j$ we let $C_{i(1)}, C_{i(2)}, \cdots, C_{i(k)}, C_{g+i'(1)}, \cdots, C_{g+i'(l)}$ and $C_{j(1)}, \cdots, C_{j(m)}, C_{g+j'(1)}, \cdots, C_{g+j'(n)}$ be the defining curves in $\tilde{\Sigma}$ in the interior and to the exterior to $\tilde{\gamma}_j$, respectively, where $i(1)<\cdots<i(k)$, $i'(1)<\cdots<i'(l)$; $j(1)<\cdots<j(m)$, $j'(1)<\cdots<j'(n)$. Then we say that the loop $\tilde{\gamma}_j$ gives a *partition* $\{i(1), \cdots, i(k), g+i'(1), \cdots, g+i'(l)\}\cup\{j(1), \cdots, j(m), g+j'(1), \cdots, g+j'(n)\}$ of $\{1, 2, \cdots, 2g\}$. From now on we use these partitions associated with $\tilde{\Sigma}$.

REMARK. Noting that each C_i contains only one fixed point p_i of $A_i^{(0)}$ among the set of $2g$ fixed points of generators in the interior, we see that $\tilde{\gamma}_j$ divides the set of the fixed points into two parts $p_{i(1)}, \cdots, p_{i(k)}, p_{g+i'(1)}, \cdots, p_{g+i'(l)}$ and $p_{j(1)}, \cdots, p_{j(m)}, p_{g+j'(1)}, \cdots, p_{g+j'(n)}$. The partition of the set $\{1, 2, \cdots, 2g\}$ by a loop $\tilde{\gamma}_j$ only depends on the $2g$ fixed points but not on a choice of defining curves.

4-3. Let S be a marked Riemann surface with nodes. We call the set $\Sigma'=\{\alpha'_1, \cdots, \alpha'_g; \gamma'_1, \cdots, \gamma'_{2g-3}\}$ of loops and nodes on S satisfying the following condition a *basic system of loops and nodes*: Each component of $S-\bigcup_{i=1}^{g}\alpha'_i-$

$\bigcup_{j=1}^{3g-3}\gamma'_j$ is a planar and triply connected region of type [3, 0], [2, 1], [1, 2] and [0, 3], where a surface of type $[m, n]$ means a sphere with m disks removed and n points deleted.

Cut the Riemann surface S along the loops and nodes α'_i $(i=1, 2, \cdots, g)$. We denote by α''_i and α''_{g+i} the resulting two topological circles or two points for each i. We note that each γ'_j divides the set $\{\alpha''_1, \alpha''_2, \cdots, \alpha''_{2g}\}$ into two parts $\{\alpha''_{i(1)}, \cdots, \alpha''_{i(k)}, \alpha''_{g+i'(1)}, \cdots, \alpha''_{g+i'(l)}\}$ and $\{\alpha''_{j(1)}, \cdots, \alpha''_{j(m)}, \alpha''_{g+j'(1)}, \cdots, \alpha''_{g+j'(n)}\}$, where $i(1)<\cdots<i(k)$, $i'(1)<\cdots<i'(l)$; $j(1)<\cdots<j(m)$, $j'(1)<\cdots<j'(n)$. We say that γ'_j gives a *partition* $\{i(1), \cdots, i(k), g+i'(1), \cdots, g+i'(l)\}\cup\{j(1), \cdots, j(m), g+j'(1), \cdots, g+j'(n)\}$ of the set $\{1, 2, \cdots, 2g\}$. We write $\Sigma''=\{\alpha''_1, \cdots, \alpha''_{2g}; \gamma'_1, \cdots, \gamma'_{2g-3}\}$ and call it the *set of Jordan curves and points induced from* Σ'.

From now on we assume that Σ'' satisfies the following condition: Each γ'_j $(j=1, 2, \cdots, 2g-3)$ gives the same partition of $\{1, 2, \cdots, 2g\}$ as $\tilde{\gamma}_j$. We say Σ'' being compatible with $\tilde{\Sigma}$ and denote by $\Sigma''\sim\tilde{\Sigma}$.

4-4. Let S, Σ' and Σ'' be as in §4-3. Furthermore we assume that Σ' has the following Property (A).

Property (A): Let $L_{j,1}, \cdots, L_{j,n(j)}$, be "cycles" containing $\gamma_j\in\Sigma$ and let $\alpha_{j,i}$ $(i=1, 2, \cdots, n(j))$ be the elements in Σ contained in $L_{j,i}$ (see §3-1 for the definitions of $L_{j,i}$, γ_j and $\alpha_{j,i}$). If $\gamma'_j\in\Sigma'$ is a node, then $\alpha'_{j,i}\in\Sigma'(i=1, 2, \cdots, n(j))$ are nodes.

By the same way as in [4], we can determine

$$\tau_S=(t_1, \cdots, t_g, \rho_1, \cdots, \rho_{2g-3})\in\hat{\mathfrak{S}}^*_g(\tilde{\Sigma})$$

such that S coincides with the Riemann surface associated with τ_S.

4-5. By collecting the above results, we have the following theorem.

THEOREM 2. *Let $G^{(0)}=\langle A_1^{(0)}, \cdots, A_g^{(0)}\rangle$ be a fixed Schottky group and $\tilde{\Sigma}$ a fixed basic system of Jordan curves for $G^{(0)}$.*

(1) *For $\tau\in\delta^{I,J}\mathfrak{S}_g(\tilde{\Sigma})$ with $I\supset I(J)$, there exists a compact Riemann surface $S(\tau)$ of genus g with $|I|+|J|$ nodes associated with τ in the sense of* §4-1.

(2) *Conversely, given a compact Riemann surface S of genus g with nodes, a basic system of loops and nodes Σ' on S, and the set of Jordan curves and points Σ'' induced from Σ' such that* (i) *$\Sigma''\sim\tilde{\Sigma}$ and* (ii) *Σ' has Property* (A). *Then there exists a $\tau\in\hat{\mathfrak{S}}^*_g(\tilde{\Sigma})$ such that S coincides with the Riemann surface associated with τ in the sense of* §4-1.

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