J. Math. Soc. Japan
Vol. 17, No. 2, 1965

Correction to "Some aspects of real-analytic manifolds and differentiable manifolds"

(This Journal, Vol. 16 (1964), 128-142)

By Kôji SHIGA

In [2; Appendix, p. 139] we stated an extension theorem of C^s-mappings which was necessary to the proof of Classification Theorem of C^s-fibre bundles $(1 \leqq s \leqq \omega)$. However, the proof given there was incorrect and our reference to [3] was not pertinent to this theorem. Now we give a proof of the following extension theorem which corrects the theorem stated in [2, p. 139].

THEOREM. *Let M and N be C^ω-manifolds, and let L be a closed C^ω-submanifold of M. Suppose that we have a C^ω-mapping φ of L into N such that φ can be extended to a C^0-mapping f of M into N. Then, for any positive family $\mathcal{E}$, there exists a C^ω-mapping ϕ from M into N having the following properties:*

(i) *ϕ gives an $\mathcal{E}$-approximation to f in order* 0.

(ii) $\phi(p) \mid L = \varphi(p)$.

Here we formulate only real-analytic case, because differentiable case is trivial [cf. 2, p. 140]. If this theorem is established, then Theorem C [2, p. 138] remains valid.

In order to prove this theorem, we need two results due to H. Cartan [1].

PROPOSITION 1. *Any C^ω-function on L can be extended to a C^ω-function on M.*

PROPOSITION 2. *L can be defined as the zero points of a non-negative C^ω-function $\mu(p)$ on $M: L = \mu^{-1}(0)$.*

Proposition 2 is usually stated that L is defined as the common zero points of a finite number of C^ω-functions μ_i on M. Then we note that $\mu = \sum \mu_i^2$ satisfies the requirements of Proposition 2.

PROOF OF THEOREM. Imbed N in a Euclidean space E^k as a closed C^ω-submanifold. Then the given map φ of L into N can be written in coordinate components of $E^k: \varphi(p) = (\varphi^1(p), \cdots, \varphi^k(p))$. Applying Proposition 1 to each $\varphi^i(p)$, we get a C^ω-mapping Φ of M into E^k such that $\Phi(p) = (\Phi^1(p), \cdots, \Phi^k(p))$, and that $\Phi(p) \mid L = \varphi(p)$. We approximate f closely by a C^ω-mapping $\Psi(p)$ of M into N. Observe that $\Phi(p)$ and $\Psi(p)$ lie near each other in E^k when p is near L. Take a small neighborhood V of L and consider a C^ω-function $1/\mu(p)$ on V^c where $\mu(p)$ is a C^ω-function stated in Proposition 2. We extend this function to a C^0-function $g(x)$ on M. Thus we have $g(p)=1/\mu(p)$, $p \notin V$; also we may assume $0 \leqq g(p)\mu(p) \leqq 1$. Let $\nu(p)$ be a C^ω-function on M which approximates $g(p)$. Set

$$\lambda(p)=\mu(p)\nu(p).$$

Then $\lambda(p)$ is a C^ω-function on M which vanishes on L. Moreover $\lambda(p)$ tends to 1 whenever p becomes distant from L.

Now set

$$\tilde{\psi}(p)=(1-\lambda(p))\Phi(p)+\lambda(p)\Psi(p).$$

Then $\tilde{\psi}(p)$ gives a C^ω-mapping of M into E^k and $\tilde{\psi}(p) \mid L=\Phi(p) \mid L=\varphi(p)$. It is easily seen that if we choose V small and take each approximation Ψ to f and ν to g well, $\tilde{\psi}$ approximates f arbitrarily closely in order 0. Hence $\tilde{\psi}$ can be regarded as a C^ω-mapping of M into a tubular neighborhood $T(N)$ of N $(T(N) \subset E^k)$. Denote the canonical projection of $T(N)$ onto N by π, and set

$$\psi(p)=\pi \circ \tilde{\psi}(p).$$

As is easily verified, ψ is a desired C^ω-mapping of M into N which completes the proof.

Finally the following misprint in [2] should be corrected:

p. 128, line 28; Replace "injective" by "surjective".

Department of Mathematics
Tokyo Institute of Technology

Bibliography

[1] H. Cartan, Variétés analytiques reélles et variétés analytiques complexes, Bull. Soc. Math. France, **85** (1957), 77-99.

[2] K. Shiga, Some aspects of real-analytic manifolds and differentiable manifolds, J. Math. Soc. Japan, **16** (1964), 128-142.

[3] H. Whitney, Analytic extensions of differentiable functions defined in closed sets, Trans. Amer. Math, Soc., **36** (1934), 63-89.