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Superposability of the equations of magneto-hydrodynamics.

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§0. The equations of magneto-hydrodynamics in an incompressible viscous electrically conducting fluid are given by

$$\frac{\epsilon\mu}{C^2}\frac{\partial^2\vec{H}}{\partial t^2}-\Delta_2\vec{H}+\frac{4\pi\sigma\mu}{C^2}\Big[\frac{\partial\vec{H}}{\partial t}+\mathrm{Curl}(\vec{H}\times\vec{q})\Big]=0\,, \tag{0.1}$$

$$\frac{\partial\vec{q}}{\partial t}-\vec{q}\times\mathrm{Curl}\,\vec{q}+\frac{\mu}{4\pi\rho}\vec{H}\times\mathrm{Curl}\,\vec{H}$$
$$+\mathrm{grad}\Big(\frac{1}{2}\vec{q}^2+\frac{p}{\rho}-U\Big)-\gamma\Delta_2\vec{q}=0\,, \tag{0.2}$$

$$\mathrm{div}\,\vec{q}=0,\qquad \mathrm{div}\,\vec{H}=0\,, \tag{0.3}$$

the notations being classical. As in the case of equations of motion in hydrodynamics, here also, the equations contain non-linear terms and the solutions cannot be superposed in general. We examine below the necessary and sufficient conditions under which two solutions of the system of equations (0.1), (0.2), (0.3) can be superposed on one another, i.e., we seek the conditions such that when $(\vec{q}_1, \vec{H}_1, p_1, U_1)$ and $(\vec{q}_2, \vec{H}_2, p_2, U_2)$ are solutions of the system, $(\vec{q}_1+\vec{q}_2, \vec{H}_1+\vec{H}_2, p_1+p_2+\pi, U_1+U_2)$ is also a solution of the system. Similar investigations for the hydrodynamical case have been made in some detail [1].

§1. The equations for the first two fields are

$$\frac{\epsilon\mu}{C^2}\frac{\partial^2\vec{H}_i}{\partial t^2}-\Delta_2\vec{H}_i+\frac{4\pi\sigma\mu}{C^2}\Big[\frac{\partial\vec{H}_i}{\partial t}+\mathrm{Curl}(\vec{H}_i\times\vec{q}_i)\Big]=0\,, \tag{1.1}$$

$$\frac{\partial q_i}{\partial t}-\vec{q}_i\times\mathrm{Curl}\,\vec{q}_i+\frac{\mu}{4\pi\rho}\vec{H}_i\times\mathrm{Curl}\,\vec{H}$$
$$+\mathrm{grad}\Big(\frac{1}{2}\vec{q}_i^{\,2}+\frac{p_i}{\rho}-U_i\Big)=\gamma\Delta_2\vec{q}_i^{\,2}\,, \tag{1.2}$$

$$\mathrm{div}\,\vec{q}_i=0,\qquad \mathrm{div}\,\vec{H}_i=0\qquad (i=1,2) \tag{1.3}$$

and the equations for the field resulting from the superposition are

$$\frac{\epsilon\mu}{C^2}\frac{\partial^2(\vec{H}_1+\vec{H}_2)}{\partial t^2}-\Delta^2(\vec{H}_1+\vec{H}_2)+\frac{4\pi\sigma\mu}{C^2}\Big[\frac{\partial(\vec{H}_1+\vec{H}_2)}{\partial t}$$

$$+\text{Curl}(\vec{H}_1+\vec{H}_2)\times(\vec{q}_1+\vec{q}_2)\Big]=0\,, \tag{1.4}$$

$$\frac{\partial(\vec{q}_1+\vec{q}_2)}{\partial t}-(\vec{q}_1+\vec{q}_2)\times\text{Curl}(\vec{q}_1+\vec{q}_2)+\frac{\mu}{4\pi\rho}[(\vec{H}_1+\vec{H}_2)$$

$$\times\text{Curl}(\vec{H}_1+\vec{H}_2)]+\text{grad}\Big[\frac{1}{2}(\vec{q}_1+\vec{q}_2)^2$$

$$+\frac{p_1+p_2+\pi}{\rho}-(U_1+U_2)\Big]=\gamma\Delta_2(\vec{q}_1+\vec{q}_2)\,. \tag{1.5}$$

From equations (1.1), (1.2). (1.4) and (1.5) we get

$$\text{Curl}[\vec{H}_1\times\vec{q}_2+\vec{H}_2\times\vec{q}_1]=0\,, \tag{1.6}$$

$$\vec{q}_1\times\text{Curl }\vec{q}_2+\vec{q}_2\times\text{Curl }\vec{q}_1-\frac{\mu}{4\pi\rho}(\vec{H}_1\times\text{Curl }\vec{H}_2+\vec{H}_2\times\text{Curl }\vec{H}_1)$$

$$-\text{grad}\Big(\vec{q}_1\cdot\vec{q}_2+\frac{\pi}{\rho}\Big)=0\,. \tag{1.7}$$

The field $(\vec{q}, \vec{H}, p, U)$ is self superposable when

$$\text{Curl}(H\times q)=0\,, \tag{1.8}$$

$$\text{Curl}\Big[\vec{q}\times\text{Curl }\vec{q}-\frac{\mu}{4\pi\rho}\vec{H}\times\text{Curl }\vec{H}\Big]=0\,. \tag{1.9}$$

We examine in some detail the case where the vectors $\vec{q}$, $\vec{H}$ are coplanar, when they are both two-dimensional and axially symmetric vectors.

§ 2. Two-dimensional motion

$$\vec{q}=q_x\vec{i}_x+q_y\vec{i}_y,\qquad \vec{H}=H_x\vec{i}_x+H_y\vec{i}_y\,,$$

$$q_x=-\frac{\partial\psi}{\partial y}\,,\qquad q_y=\frac{\partial\psi}{\partial x}\,, \tag{2.1}$$

$$H_x=-\frac{\partial\phi}{\partial y}\,,\qquad H_y=\frac{\partial\phi}{\partial x}\,, \tag{2.2}$$

where the stream lines and the lines of force are given by $\psi=$ constant and $\phi=$ constant respectively.

From (1.6) we see that there exists a scalar function F such that

$$\vec{H}_1\times\vec{q}_2+\vec{H}_2\times\vec{q}_1=\text{grad }F\,. \tag{2.2}$$

In terms of the functions ψ and ϕ this condition becomes

$$\frac{\delta(\phi_1,\psi_2)}{\delta(x,y)}+\frac{\delta(\phi_2,\psi_1)}{\delta(x,y)}=0\,. \tag{2.3}$$

Condition (1.7) now becomes

$$\frac{\delta(\psi_1, \Delta_2\psi_2)}{\delta(x, y)} + \frac{\delta(\psi_2, \Delta_2\psi_1)}{\delta(x, y)} = \frac{\mu}{4\pi\rho}\left[\frac{\delta(\phi_1, \Delta_2\phi_2)}{\delta(x, y)} + \frac{\delta(\phi_2, \Delta_2\phi_1)}{\delta(x, y)}\right]. \tag{2.4}$$

§ **3.** Conditions for superposability in the case of axial symmetry

$$\vec{q} = q_x\vec{i}_x + q_{\bar{\omega}}\vec{i}_{\bar{\omega}} + q_\omega\vec{i}_\omega ,$$

$$\vec{H} = H_x\vec{i}_x + H_{\bar{\omega}}\vec{i}_{\bar{\omega}} + H_\omega\vec{i}_\omega .$$

The conditions $\operatorname{div}\vec{q} = 0$ and $\operatorname{div}\vec{H} = 0$ are satisfied by taking

$$q_x = -\frac{1}{\bar{\omega}}\frac{\partial\psi}{\partial\bar{\omega}}, \quad q_{\bar{\omega}} = \frac{1}{\bar{\omega}}\frac{\partial\psi}{\partial x},$$

$$H_x = -\frac{1}{\bar{\omega}}\frac{\partial\phi}{\partial\bar{\omega}}, \quad H_{\bar{\omega}} = \frac{1}{\bar{\omega}}\frac{\partial\phi}{\partial x},$$

and we may write $q_\omega = f(x, \bar{\omega})$, and $H_\omega = g(x, \bar{\omega})$.

From (2.2) we now may write

$$\frac{1}{\bar{\omega}}\left[\frac{\partial\phi_1}{\partial x}f_2 - \frac{\partial\psi_2}{\partial x}g_1 + \frac{\partial\phi_2}{\partial x}f_1 - \frac{\partial\psi_1}{\partial x}g_2\right]dx$$

$$+\frac{1}{\bar{\omega}}\left[\frac{\partial\phi_1}{\partial\bar{\omega}}f_2 - \frac{\partial\psi_2}{\partial\bar{\omega}}g_1 + \frac{\partial\phi_2}{\partial\bar{\omega}}f_1 - \frac{\partial\psi_1}{\partial\bar{\omega}}g_2\right]d\bar{\omega} = dF - \frac{\partial F}{\partial t}\delta t \tag{3.1}$$

and

$$\frac{\delta(\phi_1, \psi_2)}{\delta(x, \bar{\omega})} + \frac{\delta(\phi_2, \psi_1)}{\delta(x, \bar{\omega})} = 0. \tag{3.2}$$

By means of cross differentiation we may write from (3.1) a single equation

$$\frac{\delta(\phi_1, f_2)}{\delta(x, \bar{\omega})} + \frac{\delta(\phi_2, f_1)}{\delta(x, \bar{\omega})} - \frac{1}{\bar{\omega}}\left[\frac{\partial\phi_1}{\partial x}f_2 + \frac{\partial\phi_2}{\partial x}f_1\right]$$

$$= \frac{\delta(\psi_1, g_2)}{\delta(x, \bar{\omega})} + \frac{\delta(\psi_2, g_1)}{\delta(x, \bar{\omega})} - \frac{1}{\bar{\omega}}\left[\frac{\partial\psi_1}{\partial x}g_2 + \frac{\partial\psi_2}{\partial x}g_1\right]. \tag{3.3}$$

From (1.7) we get

$$\frac{1}{\bar{\omega}^2}\left[\frac{\partial\psi_1}{\partial x}E^2\psi_2 + \frac{\partial\psi_2}{\partial x}E^2\psi_1 + \frac{\partial}{\partial x}(\bar{\omega}^2 f_1 f_2)\right.$$

$$\left.-\frac{\mu}{4\pi\rho}\left(\frac{\partial\phi_1}{\partial x}E^2\phi_2 + \frac{\partial\phi_2}{\partial x}E^2\phi_1 + \frac{\partial}{\partial x}(\bar{\omega}^2 g_1 g_2)\right)\right]\vec{i}_x$$

$$+\frac{1}{\bar{\omega}^2}\left[\left(\frac{\partial\psi_1}{\partial\bar{\omega}}E^2\psi_2 + \frac{\partial\psi_2}{\partial\bar{\omega}}E^2\psi_1 + \frac{\partial}{\partial\bar{\omega}}(\bar{\omega}^2 f_1 f_2)\right)\right.$$

$$\left.-\frac{\mu}{4\pi\rho}\left(\frac{\partial\phi_1}{\partial\bar{\omega}}E^2\phi_2 + \frac{\partial\phi_2}{\partial\bar{\omega}}E^2\phi_1 + \frac{\partial}{\partial\bar{\omega}}(\bar{\omega}^2 g_1 g_2)\right)\right]\vec{i}_{\bar{\omega}}$$

$$+\frac{1}{\varpi^2}\Big[\Big(\frac{\partial\psi_1}{\partial\varpi}\frac{\partial}{\partial x}(\varpi f_2)-\frac{\partial\psi_1}{\partial x}\frac{\partial}{\partial\varpi}(\varpi f_2)+\frac{\partial\psi_2}{\partial\varpi}\frac{\partial}{\partial x}(\varpi f_1)-\frac{\partial\psi_2}{\partial x}\frac{\partial}{\partial\varpi}(\varpi f_1)$$

$$-\frac{\mu}{4\pi\rho}\Big(\frac{\partial\phi_1}{\partial\varpi}\frac{\partial}{\partial x}(\varpi g_2)-\frac{\partial\phi_1}{\partial x}\frac{\partial}{\partial\varpi}(\varpi g_2)+\frac{\partial\phi_2}{\partial\varpi}\frac{\partial}{\partial x}(\varpi g_1)-\frac{\partial\phi_2}{\partial x}\frac{\partial}{\partial\varpi}(\varpi g_1)\Big)\Big]\vec{i}_\omega$$

$$=\operatorname{grad}\Big(\vec{q}_1\cdot\vec{q}_2+\frac{\pi}{\rho}\Big), \tag{3.4}$$

where E^2 is the differential operator

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial\varpi^2}-\frac{1}{\varpi}\frac{\partial}{\partial\varpi}.$$

Writing $\frac{\pi}{\rho}+\vec{q}_1\cdot\vec{q}_2=G$ we have the right hand of (3.4) in the form

$$\frac{\partial G}{\partial x}\vec{i}_x+\frac{\partial G}{\partial\varpi}\vec{i}_\varpi+\frac{1}{\varpi}\frac{\partial G}{\partial\omega}\vec{i}_\omega. \tag{3.5}$$

Since $\vec{q}_1$ and $\vec{q}_2$ are axially symmetric and π also is to be axially symmetric, we have $\frac{\partial G}{\partial\omega}=0$. In view of this we get from (3.4)

$$\frac{1}{\varpi^2}\Big[\frac{\partial\psi_1}{\partial x}E^2\psi_2+\frac{\partial\psi_2}{\partial x}E^2\psi_1+\frac{\partial}{\partial x}(\varpi^2 f_1f_2)$$

$$-\frac{\mu}{4\pi\rho}\Big\{\frac{\partial\phi_1}{\partial x}E^2\phi_2+\frac{\partial\phi_2}{\partial x}E^2\phi_1+\frac{\partial}{\partial x}(\varpi^2 g_1g_2)\Big\}\Big]dx$$

$$+\frac{1}{\varpi^2}\Big[\frac{\partial\psi_1}{\partial\varpi}E^2\psi_2+\frac{\partial\psi_2}{\partial\varpi}E^2\psi_1+\frac{\partial}{\partial\varpi}(\varpi^2 f_1f_2)$$

$$-\frac{\mu}{4\pi\rho}\Big\{\frac{\partial\phi_1}{\partial\varpi}E^2\phi_2+\frac{\partial\phi_2}{\partial\varpi}E^2\phi_1+\frac{\partial}{\partial\varpi}(\varpi^2 g_1g_2)\Big\}\Big]d\varpi$$

$$=dG-\frac{\partial G}{\partial t}\delta t \tag{3.6}$$

and

$$\frac{\delta(\psi_1,\varpi f_2)}{\delta(x,\varpi)}+\frac{\delta(\psi_2,\varpi f_1)}{\delta(x,\varpi)}=\frac{\mu}{4\pi\rho}\Big\{\frac{\delta(\phi_1,\varpi g_2)}{\delta(x,\varpi)}\ \frac{\delta(\phi_2,\varpi g_1)}{\delta(x,\varpi)}\Big\}. \tag{3.7}$$

From (3.6) we may derive by means of cross differentiation

$$\frac{\delta(\psi_1,E^2\psi_2)}{\delta(x,\varpi)}+\frac{\delta(\psi_2,E^2\psi_1)}{\delta(x,\varpi)}-\frac{2}{\varpi}\Big\{\frac{\partial\psi_1}{\partial x}E^2\psi_2+\frac{\partial\psi_2}{\partial x}E^2\psi_1+\frac{\partial}{\partial x}(\varpi^2 f_1f_2)\Big\}$$

$$=\frac{\mu}{4\pi\rho}\Big[\frac{\delta(\phi_1,E^2\phi_2)}{\delta(x,\varpi)}+\frac{\delta(\phi_2,E^2\phi_1)}{\delta(x,\varpi)}-\frac{2}{\varpi}\Big\{\frac{\partial\phi_1}{\partial x}E^2\phi_2+\frac{\partial\phi_2}{\partial x}E^2\phi_1+\frac{\partial}{\partial x}(\varpi^2 g_1g_2)\Big\}\Big]. \tag{3.8}$$

Equation (3.6) can also be written in the form

$$d\Big(\frac{\pi}{\rho}\Big)=\frac{1}{\varpi^2}\Big[d\psi_1E^2\psi_2+d\psi_2E^2\psi_1+d(\varpi^2 f_1f_2)-\frac{\mu}{4\pi\rho}\{(d\phi_1E^2\phi_2$$

$$+d\phi_2 E^2\phi_1 + d(\varpi^2 g_1 g_2)\}\Big] - d\Big\{\frac{1}{\omega^2}\Big(\frac{\partial\psi_1}{\partial x}\frac{\partial\psi_2}{\partial x} + \frac{\partial\psi_1}{\partial\omega}\frac{\partial\psi_2}{\partial\omega}\Big)\Big\} \tag{3.9}$$

and determines the adjusted pressure.

We now indicate two specific examples of flows that are superposable.

In the case where the fields $\vec{q}$ and $\vec{H}$ are both coplanar and independent of the space coordinate Z, we can assume them in the form given in (2.1), or using polar coordinates r, θ in the xoy-plane, we can also take them as

$$\vec{q}_i = \Big(\frac{1}{r}\frac{\partial\psi_i}{\partial\theta},\ -\frac{\partial\psi_i}{\partial r},\ o\Big), \tag{4.1}$$

$$\vec{H}_i = \Big(\frac{1}{r}\frac{\partial\phi_i}{\partial\theta},\ -\frac{\partial\phi_i}{\partial r},\ o\Big) \qquad (i=1,2). \tag{4.2}$$

The necessary and sufficient conditions (viz., 2.3, 2.4) for the addition of two can be written in the form

$$\Big[-\frac{\partial}{\partial\theta}\Big\{\frac{1}{r^2}\Big(\frac{\partial(\psi_1,\phi_2)}{\partial(r,\theta)} + \frac{\partial(\psi_2,\phi_1)}{\partial(r,\theta)}\Big)\Big\},$$
$$\frac{\partial}{\partial r}\Big\{\frac{1}{r}\Big(\frac{\partial(\psi_1,\phi_2)}{\partial(r,\theta)} + \frac{\partial(\psi_2,\phi_1)}{\partial(r,\theta)}\Big)\Big\},\ o\Big] = 0 \tag{4 3}$$

and

$$\frac{1}{r}\Big[o,\ o,\ +\frac{\partial(\psi_1,\Delta_2\psi_2)}{\partial(r,\theta)} + \frac{\partial(\psi_2,\Delta_2\psi_1)}{\partial(r,\theta)}\Big]$$
$$-\Big[o,\ o,\ \frac{\mu}{4\pi\rho}\frac{1}{r}\Big(\frac{\partial(\phi_1,\Delta_2\phi_2)}{\partial(r,\theta)} + \frac{\partial(\phi_2,\Delta_2\phi_1)}{\partial(r,\theta)}\Big)\Big] = 0. \tag{4.4}$$

Writing $\sqrt{\frac{\mu}{4\pi\rho}}\,\phi_i = \bar{\phi}_i$, $i=1,2$, these equations give

$$\frac{\partial}{\partial\theta}\Big[\frac{\partial(\psi_1,\bar{\phi}_2)}{\partial(r,\theta)} + \frac{\partial(\psi_2,\bar{\phi}_1)}{\partial(r,\theta)}\Big] = 0, \tag{4.5}$$

$$\frac{\partial}{\partial r}\Big[\frac{1}{r}\frac{\partial(\psi_1,\bar{\phi}_2)}{\partial(r,\theta)} + \frac{\partial(\psi_2,\bar{\phi}_1)}{\partial(r,\theta)}\Big] = 0, \tag{4.6}$$

$$\frac{\partial(\psi_1,\Delta_2\psi_2)}{\partial(r,\theta)} + \frac{\partial(\psi_2,\Delta_2\psi_1)}{\partial(r,\theta)} - \frac{\partial(\bar{\phi}_1,\Delta_2\bar{\phi}_2)}{\partial(r,\theta)} - \frac{\partial(\bar{\phi}_2,\Delta_2\bar{\phi}_1)}{\partial(r,\theta)} = 0. \tag{4.7}$$

Cataldo Agostinelli* [**2**] has shown that the fields (4.1) and (4.2) satisfy the fundamental equations (0.1) and (0.2) if

$$\phi_i = -\frac{1}{2}h_i r^2 + f_i(r,\theta,t), \tag{4.8}$$

$$\psi_i = -\frac{1}{2}\omega_i r^2 + \beta f_i(r,\theta,t) \qquad (i=1,2), \tag{4.9}$$

* The results of [**2**] referred to here were available to the author only from the Math. Rev., 18 (1957), p. 849.

where h_i, ω_i, β are constants and

$$\text{(i) } \Delta_2 f_i = 0 \quad \text{or} \quad \text{(ii) } \beta = \alpha = \sqrt{\frac{\mu}{4\pi\rho}} \text{ and}$$

$$\Delta_2\left(\frac{\partial f_i}{\partial t} - \gamma \Delta_2 f_i + (\omega_i - \alpha h_i)\frac{\partial f_i}{\partial \theta}\right) = 0 .$$

It is easily seen that fields given in (4.8) and (4.9) satisfy the condition (4.7) and from (4.5) and (4.6) we get

$$\frac{\partial(\psi_1, \overline{\phi}_2)}{\partial(r, \theta)} + \frac{\partial(\psi_2, \overline{\phi}_1)}{\partial(r, \theta)} = g(r) , \tag{4.10}$$

$$\frac{\partial}{\partial r}\left[\frac{g(r)}{r}\right] = 0 . \tag{4.11}$$

Whence we deduce that

$$f_K(r) = g_K(r) + a_K\theta + b_K \qquad (K = 1, 2) \tag{4.12}$$

with $g_K(r) =$ a harmonic function.

We may therefore write

$$\phi_i = -\frac{1}{2} h_i r^2 + (A_i + B_i \log r) + a_i\theta + b_i , \tag{4.13}$$

$$\psi_i = -\frac{1}{2}\omega_i r^2 + \beta(A_i + B_i \log r + a_i\theta + b_i) \qquad (i = 1, 2) \tag{4.14}$$

and the corresponding fields are

$$\vec{q}_i = \left(\frac{a_i}{r}\beta,\ \omega_i r - \beta\frac{B_i}{r}, o\right) , \tag{4.15}$$

$$\vec{H}_i = \left(\frac{a_i}{r},\ h_i r - \frac{B_i}{r}, o\right) . \tag{4.16}$$

Hence we have the result that

The fields defined in (4.15), (4.16) *are superposable.* When the functions $f_i(r, \theta)$ of (4.8), (4.9) satisfy the condition

$$\Delta_2\left(-\gamma \Delta_2 f_i + [\omega_i - \alpha h_i]\frac{\partial f_i}{\partial \theta}\right) = 0 \qquad (i = 1, 2) . \tag{4.17}$$

The conditions (4.5) and (4.6) and (4.7) are reduced to the form

$$\frac{\partial(\psi_1, \phi_2)}{\partial(r, \theta)} + \frac{\partial(\psi_2, \phi_1)}{\partial(r, \theta)} = g(r) , \qquad \frac{\partial}{\partial r}\left[\frac{g(r)}{r}\right] = 0 \tag{4.18}$$

and

$$(\omega_1 - \alpha h_1)\frac{\partial}{\partial \theta}(\Delta_2 f_2) + (\omega_2 - \alpha h_2)\frac{\partial}{\partial \theta}(\Delta_2 f_1) = 0 . \tag{4.19}$$

We may write (4.19) also in the form (compare (4.17))

$$\Delta_2(-r\Delta_2 f_2)+\Delta_2(-r\Delta_2 f_1)=0\,. \tag{4.20}$$

After some calculation we get from this that

$$f_K=g_K(r)+a_K\theta+b_K \qquad (K=1,2)\,,$$

where $g_K(r)$ is a biharmonic function, i.e.,

$$g_K(r)=C_K+d_K\log r+\frac{A_K}{4}r^2+\frac{B_K}{4}(\log r-1)r^2\,. \tag{4.21}$$

Hence we have

$$\psi_i=-\frac{1}{2}\omega_i r^2+\Big[C_i+D_i\log r+\frac{A_i}{4}r^2+\frac{B_i}{4}(\log r-1)r^2\Big]\beta$$

$$+\beta(a_K\theta+b_K)\,, \tag{4.22}$$

$$\phi_i=-\frac{1}{2}h_i r^2+\Big[C_i+D_i\log r+\frac{A_i}{4}r^2+\frac{B_i}{4}(\log r-1)r^2\Big]$$

$$+a_K\theta+b_K \tag{4.23}$$

and the corresponding fields are

$$\vec{q}_i=\Big[\frac{\beta a_i}{r}\,,\ \omega_i r-\Big(\frac{D_i}{r}+\frac{A_i}{2}r+\frac{B_i}{2}r(\log r-1)+\frac{B_i}{4}r\Big)\beta+o,o\Big]\,, \tag{4.24}$$

$$\vec{H}_i=\Big[\frac{a_i}{r}\,,\Big(h_i r-\frac{D_i}{r}-A_i r-B_i r(\log r-1)-\frac{B_i}{4}r,0\Big)\Big]\,. \tag{4.25}$$

We have therefore the result:

The motions given by (4.24) *and* (4.25) *are superposable.*

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