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CORRECTION: EXTREME STABILITY AND ALMOST PERIODICITY IN A DISCRETE LOGISTIC EQUATION

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The authors of the article [1] have noticed a few errors in [1] and the following corrections have to be made:

(I) The inequalities in (2.4) should read

$$x_{\max} = \frac{K^*}{r_*}\exp(r^* - 1)\,, \qquad x_{\min} = K_*\exp(r_* - \frac{r^*}{K_*}x_{\max})\,.$$

In view of this redefinition of $x_{\min}$, Lemma 2.1 needs a revised proof and the following is a complete new proof of Lemma 2.1 with the above modification of the inequalities in (2.4). Define $f_n(x),\ F(x),\ g(x), x_{\max}, x_{\min}$ as follows:

$$f_n(x) = x\ \ \exp\left[r(n)\left(1 - \frac{x}{K(n)}\right)\right], \qquad F(x) = x\ \ \exp\left(r^* - \frac{r_* x}{K^*}\right)$$

$$g(x) = K_*\exp\left(r_* - \frac{r^* x}{K_*}\right); \quad x_{\max} = F\left(\frac{K^*}{r_*}\right), \quad x_{\min} = g(x_{\max})\,.$$

Proof of Lemma 2.1 is divided into four steps for convenience:

Step 1. We have from

$$x(n+1) = f_n(x(n)) \le F(x(n)) \le \sup_{x>0} F(x) = x_{\max}$$

that

$$\limsup_{n\to\infty} x(n) \le x_{\max} \qquad \text{since} \quad x(n) \le F\left(\frac{K^*}{r_*}\right) \quad \text{for all} \quad n \ge 1\,.$$

Step 2. Suppose there exists an integer N such that $x(n+1) \ge x(n)$ for all $n \ge N$. Then one can show that

$$\liminf_{n\to\infty} x(n) \ge x_{\min}\,;$$

for instance we have from the boundedness of $\{x(n)\}$, that $\lim_{n\to\infty} x(n) = x^*$ exists and is finite. By letting $n \to \infty$ in the relation

$$x(n+1) = x(n)\exp\left[r(n)\left(1 - \frac{x(n)}{K(n)}\right)\right]\,,$$

we see that $x(n)/K(n) \to 1$ as $n \to \infty$, since $r(n) \geq r_* > 0$. Therefore for arbitrary $\varepsilon \in (0,1)$, there is a positive integer M such that $x(n) \geq (1-\varepsilon)K(n)$ for all $n \geq M$. Note that $x(n+1) \geq x(n)$ implies $1 - x(n)/K(n) \geq 0$ and so

$$\begin{aligned} x(n+1) &= x(n)\exp\left[r(n)\left(1-\frac{x(n)}{K(n)}\right)\right] \\ &\geq (1-\varepsilon)K(n)\exp\left[r_* - \frac{r^* x_{\max}}{K_*}\right] \\ &\geq (1-\varepsilon)K_*\exp\left[r_* - \frac{r^* x_{\max}}{K_*}\right], \end{aligned}$$

from which we have

$$\liminf_{n\to\infty} x(n) \geq (1-\varepsilon)K_*\exp\left[r_* - \frac{r^* x_{\max}}{K_*}\right] \geq (1-\varepsilon)x_{\min}$$

and the assertion follows since ε is arbitrary.

Step 3. If $x(m+1) \leq x(m)$ for some m, then $x(m+1) \geq x_{\min}$ and this can be verified as follows: note that $x(m+1) \leq x(m)$ implies that $K(m) \leq x(m)$ and hence

$$\begin{aligned} x(m+1) &= x(m)\exp\left[r(m)\left(1-\frac{x(m)}{K(m)}\right)\right] \\ &\geq K(m)\exp\left[r_* - \frac{r^* x_{\max}}{K_*}\right] \\ &\geq K_*\exp\left[r_* - \frac{r^* x_{\max}}{K_*}\right] \end{aligned}$$

which is what has been asserted in this step.

Step 4. We now show that

$$\liminf_{n\to\infty} x(n) \geq x_{min}\,.$$

If $x(n) \geq x_{\min}$ for all sufficiently large n, then the above assertion follows; hence let us suppose that there is a subsequence $\{x(n_j)\}_{j=1}^{\infty}$ such that $x(n_j) < x_{\min}$. First we shall verify that $n_{j+1} = n_j + 1$ for all $j \geq 1$. Since otherwise, $n_{j+1} - n_j \geq 2$ for some j. We can suppose that $x(n_{j+1}-1) \geq x_{\min}$. Then $x(n_{j+1}-1) \geq x_{\min} > x(n_{j+1})$, which by Step 3 implies $x(n_{j+1}) \geq x_{\min}$ and this is a contradiction. Therefore we conclude that $n_{j+1} = n_j + 1$ for all j or $n_j = n_1 + (j-1)$. Also it will follow from Step 3 that the sequence $\{x(n)\}_{n=n_1}^{\infty}$ is monotone increasing from which the assertion of Step 4 will follow by Step 2. This completes the proof of Lemma 2.1.

(II) The assumption in (3.1) of Lemma 3.1 should be modified to read

$$0 < r_* \qquad \text{and} \qquad \frac{r^* K^*}{r_* K_*}\exp(r^*-1) < 2\,.$$

Under this assumption, the inequality in (3.2) can be obtained from the following:

$$1 > 1 - r(n)\frac{x(n)}{K(n)} \geq 1 - \frac{r^* x_{\max}}{K_*} = 1 - \frac{r^* K^*}{r_* K_*}\exp(r^*-1) > 1 - 2 = -1\,.$$

(III) The inequality (3.3) and its proof are erroneous and can be omitted under the revised hypothesis in (II) above. The inequality (3.3) is used only in the proof of the linear stability of the almost periodic solution $\tilde{x}(n)$ on page 122 of [1]. The linear stability of $\tilde{x}(n)$ can be established in the following way under the new hypothesis on $r(n)$ given in (II) above.

We let

$$w(n) = \ln \frac{x(n)}{\tilde{x}(n)}$$

and obtain by using the persistence of the species

$$w(n+1) = w(n) \left\{ 1 - \frac{r(n)}{K(n)} \theta(n) \right\}$$

where $\theta(n)$ lies between $x(n)$ and $\tilde{x}(n)$. It follows from

$$1 > 1 - \frac{r(n)}{K(n)} \theta(n) \geq 1 - \frac{r^*}{K_*} x_{\max} = 1 - \frac{r^* K^*}{r_* K_*} \exp(r^* - 1) > 1 - 2 = -1$$

that $|w(n)|$ is nonincreasing and it can be shown that $x(n) \to \tilde{x}(n)$ as $n \to \infty$ which proves the global stability of the almost periodic solution $\tilde{x}(n)$.

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REFERENCES

[1] S. MOHAMAD AND K. GOPALSAMY, Extreme stability and almost periodicity in a discrete logistic equation, Tohoku Math. J. 52 (2000), 107–125.

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