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ON STABLE COMPLETE HYPERSURFACES WITH VANISHING r-MEAN CURVATURE

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Abstract. A form of Bernstein theorem states that a complete stable minimal surface in euclidean space is a plane. A generalization of this statement is that there exists no complete stable hypersurface of an n-euclidean space with vanishing $(n-1)$-mean curvature and nowhere zero Gauss-Kronecker curvature. We show that this is the case, provided the immersion is proper and the total curvature is finite.

1. Introduction. Let $x\colon M^n \to \boldsymbol{R}^{n+1}$ be a hypersurface of the $(n+1)$-euclidean space $\boldsymbol{R}^{n+1}$. We assume that $M = M^n$ is orientable and fix an orientation for M. Let $g\colon M \to S_1^n \subset \boldsymbol{R}^{n+1}$ be the Gauss map in the given orientation, where S_1^n is the unit n-sphere in $\boldsymbol{R}^{n+1}$. Recall that the linear operator $A\colon T_pM \to T_pM$, $p \in M$, associated to the second fundamental form of x is given by

$$\langle A(X), Y\rangle = -\langle \bar{\nabla}_X N, Y\rangle\,, \qquad X, Y \in T_pM\,,$$

where $\bar{\nabla}$ is the covariant derivative of the ambient space and N is the unit normal vector of x in the given orientation. The map $A = -dg$ is self-adjoint and its eigenvalues are the principal curvatures $k_1, k_2, \dots, k_n$ of x.

Assume now that the immersion is complete. We will say that the total curvature of the immersion is finite if $\int_M |A|^n\, dM < \infty$, where $|A| = (\sum_i k_i^2)^{1/2}$.

Consider now the elementary symmetric functions S_r, $r = 0, 1, \dots, n$, of the principal curvatures $k_1, \dots, k_n$ of x:

$$S_0 = 1\,, \quad S_r = \sum_{i_1<\dots<i_r} k_{i_1}\dots k_{i_r}\,, \quad i_1, \dots, i_r = 1, \dots, n\,,$$

and their associated r-mean curvatures H_r given by

$$H_r = \binom{n}{r}^{-1} S_r\,.$$

Hypersurfaces in euclidean spaces with $H_r = 0$ generalize minimal hypersurfaces ($H_1 = 0$). The relation is even deeper, since minimal hypersurfaces are critical points of the functional $A_0 = \int_M H_0\, dM$ for compactly supported variations of M, whereas hypersurfaces with $H_{r+1} = 0$ are critical points of the functional $A_r = \int_M H_r\, dM$ also for compactly supported variations [11]. A breakthrough in the study of such hypersurfaces was made when Hounie

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and Leite [8] proved that the equation $H_{r+1} = 0$, $r \neq 0, n-1$, is elliptic provided that rank $A > r$. In the case $r = 0$, no such condition is necessary, since the equation of a minimal hypersurface is automatically elliptic.

In [1], a definition of stability was given for hypersurfaces of the euclidean space with $H_{r+1} = 0$ (see Section 2 for details) and the following theorems were proved for the special case where $r + 1 = n - 1$ (in this case, it is not difficult to see that the condition rank $A > r$ is equivalent to $H_n \neq 0$ everywhere).

THEOREM A (Theorem 1.2 of [1]). *Let $x: M^n \to \boldsymbol{R}^{n+1}$ be an orientable hypersurface with $H_{n-1} = 0$ and $H_n \neq 0$ everywhere. Let $D \subset M$ be a bounded domain with piecewise smooth boundary. Assume that*

$$\text{Area of } g(D) < \text{Area of a hemisphere of } S_1^n .$$

Then D is stable and the estimate is sharp.

THEOREM B (Corollary 1.7 of [1]). *Let $x: M^n \to \boldsymbol{R}^{n+1}$ and $D \subset M$ be as in Theorem* A. *Assume that the Gauss map g restricted to $\bar{D}$ is a covering map onto $g(\bar{D})$, and that the first eigenvalue $\lambda_1(g(D))$ of $g(D)$ for the spherical Laplacian satisfies $\lambda_1(g(D)) < n$. Then D is unstable.*

Theorem A generalizes a theorem of Barbosa and do Carmo (Theorem 1.3 of [2]), which gives a condition for stability of bounded domains of orientable minimal surfaces in $\boldsymbol{R}^3$, and Theorem B generalizes a theorem of A. Schwarz (see [2], Theorem 2.7) for instability of similar domains.

The question naturally arises of what can be said about hypersurfaces $x: M^n \to \boldsymbol{R}^{n+1}$ with $H_{n-1} = 0$ and $H_n \neq 0$ everywhere that are orientable, *complete* and stable in the sense that every bounded domain in M is stable. This is a very strong condition, and, in the minimal case, it has been proved that the only orientable, complete stable minimal surface in $\boldsymbol{R}^3$ is the plane (see [5] and [7]).

Based on the above considerations, in [1] the following conjecture was proposed. *There exists no complete, orientable, stable hypersurface $x: M^n \to \boldsymbol{R}^{n+1}$ with $H_{n-1} = 0$ and $H_n \neq 0$ everywhere.* Here we show that with some additional conditions the conjecture is true. Namely, we prove

THEOREM 1.1. *There exists no complete orientable, proper, stable hypersurface $x: M^n \to \boldsymbol{R}^{n+1}$, $n \geq 3$, with $H_{n-1} = 0$ and $H_n \neq 0$ everywhere and of finite total curvature.*

2. Proof of Theorem 1.1. Before going into the proof of Theorem 1.1, we need to fix some notation and to recall relevant facts on stability. Further details can be found in [11], [12], [3] and [1].

Let $x: M^n \to \boldsymbol{R}^{n+1}$ be an orientable hypersurface with $H_{r+1} = 0$. A *regular domain* $D \subset M$ is a domain with compact closure and piecewise smooth boundary. We say that D is *stable* if either $A_r''(0) > 0$ for all variations with compact support in D or $A_r''(0) < 0$ for all such variations. A justification for this definition can be found in the Introduction of [1]. If for some variation with compact support in D we have $A_r''(0) > 0$, while for some other such variation, we have $A_r''(0) < 0$, we say that D is *unstable*.

Following [11], we define a linear map P_r of T_pM by

$$P_0 = I, \quad P_r = S_r I - AP_{r-1},$$

where I is the identity matrix and A is the linear map defined in the Introduction. Next, we define a second order linear operator L_r by

$$L_r f = \operatorname{div}(P_r \nabla f), \tag{1}$$

where ∇f is the gradient of f. We then write the Jacobi equation of the variational problem that defines the hypersurfaces with $H_{r+1} = 0$:

$$T_r f \overset{\text{def}}{=} L_r f - (r+2) S_{r+2} f = 0. \tag{2}$$

The Jacobi equation (2) is the linearization of the equation $H_{r+1} = 0$. As we mentioned in the Introduction, $H_n \neq 0$ everywhere is a sufficient condition for (2) to be elliptic. By (1), this is equivalent to the fact that P_r has all its eigenvalues of the same sign. We denote by $\theta_i(r)$ the eigenvalues of $\sqrt{P_r}A$ when P_r is positive definite, and the eigenvalues of $\sqrt{-P_r}A$ when P_r is negative definite. We will assume for convenience that P_r is positive definite, leaving the details of the other case to the reader.

With this notation, we can rewrite the Jacobi operator T_r as ([1], Section 2)

$$T_r = L_r + \|\sqrt{P_r}A\|^2,$$

where $\|\sqrt{P_r}A\|^2 = \sum_i \theta_i^2(r)$. Finally, we define the Morse index form I_r of our variational problem as

$$I_r(f, g) = -\int_M f T_r(g)\, dM.$$

In the case $r + 1 = n - 1$, i.e., $H_{n-1} = 0$, it can be shown that ([1], Lemma 2.4)

$$(\theta_i(n-2))^2 = -S_n. \tag{3}$$

In the proof of our theorem, we are going to use Theorems A and B of the Introduction. Concerning Theorem A, it should be noticed that the fact that the area of $g(D)$ is smaller than the area of a hemisphere of S_1^n implies, by symmetrization, that $\lambda_1(g(D)) > n$, and the latter is what is used in the proof of Theorem 1.2 of [1]. Thus, Theorem A$'$ below holds:

THEOREM A$'$. *Let $x\colon M^n \to \boldsymbol{R}^{n+1}$ and $D \subset M$ be as in Theorem* A *of the Introduction. Assume that $\lambda_1(g(D)) > n$. Then D is stable.*

We also need a lemma which is proved in [1] (Lemma 2.7) and that will be quoted here as Lemma A. We use $C_0^\infty(D)$ to denote the space of differentiable functions that vanish on the boundary ∂D of a regular domain D, and $C_c^\infty(D)$ to denote those differentiable functions that have a (compact) support in D.

LEMMA A ([1], see also [14]). *The following statements are equivalent:*

(i) *There exists* $f \in C_c^\infty(D)$ *such that* $I_r(f, f) \leq 0$.
(ii) *There exists* $f \in C_c^\infty(D)$ *such that* $I_r(f, f) < 0$.
(iii) *There exists* $f \in C_0^\infty(D)$ *such that* $I_r(f, f) < 0$.

We still need a definition. We say that the boundary ∂D of a regular domain D is *a first conjugate boundary* if there exists a Jacobi field that vanishes on ∂D and there exists no Jabobi field that vanishes in (the open set) D. A *Jacobi field* fN is a normal vector field such that f satisfies the Jacobi equation (2).

Let D be a domain such that ∂D is a first conjugate boundary. We observe that every domain properly contained in D is stable and every domain that contains D properly is unstable. In fact, if $D' \subsetneq D$ is not stable, there exists $f \in C_c^\infty(D')$ such that $I_r(f, f) \leq 0$. By Lemma A, there exists $f \in C_0^\infty(D')$ such that $I_r(f, f) < 0$. By the Morse Index Theorem, there exists $D'' \subset D'$ and a Jacobi field vanishing on $\partial D''$. This is a contradiction and proves the first part of the statement.

To prove the second part of the statement, let $D'' \supsetneq D$. Since ∂D is a conjugate boundary, by the Morse Index Theorem there exists $f \in C_0^\infty(D'')$ with $I_r(f, f) < 0$. By Lemma A, there exists $f \in C_c^\infty(D'')$ with $I_r(f, f) < 0$, hence D'' is unstable.

REMARK 2.1. Although we have no need of it, it is not hard to show that the two-part statement that we just proved is an equivalent definition of a first conjugate boundary.

The proof of Theorem 1.1 will depend on Lemmas 2.2, 2.3 and 2.5 below.

LEMMA 2.2. *Let* $x: M^n \to \boldsymbol{R}^{n+1}$ *be an orientable hypersurface with* $H_{n-1} = 0$ *and* $H_n \neq 0$ *everywhere. Assume that its Gauss map* $g: M^n \to S^n$ *is injective. Let* $D \subset M$ *be a regular domain such that* ∂D *is a first conjugate boundary. Then the following hold.*

(a) *The first eigenvalue* $\lambda_1(g(D))$ *for the spherical Laplacian satisfies* $\lambda_1(g(D)) = n$.

(b) *Let* $f: g(D) \to \boldsymbol{R}$ *be the first eigenfunction of* $g(D)$. *Then* $u = f \circ g$ *satisfies the Jacobi equation,* $u > 0$ *in* D *and* $u = 0$ *on* ∂D.

PROOF. We will prove (a). Indeed, $\lambda_1(g(D))$ is not smaller than n. Otherwise, we could find a domain $D' \subset D$ such that $\lambda_1(g(D')) < n$. Thus D' is unstable by Theorem B and this contradicts the fact that every domain contained in D is stable (since ∂D is a first conjugate boundary). Also, it cannot occur that $\lambda_1(g(D)) > n$. Otherwise, we could find a domain $D'' \supset D$ such that $\lambda_1(g(D'')) > n$. By Theorem A$'$, D'' is stable, and this is a contradiction. Thus $\lambda_1(g(D)) = n$ and this proves (a).

We now prove (b). Since the Gauss map is injective, $\partial(g(D)) = g(\partial D)$, and then

$$\tilde{\Delta} u + nu = 0, \quad u > 0 \ \text{in} \ D\,, \quad u = 0 \ \text{on} \ \partial D\,,$$

where $\tilde{\Delta}$ is the Laplacian of the pullback metric $\langle\langle\ ,\ \rangle\rangle$ on M by g (we recall that $H_n \neq 0$). By Stokes theorem,

$$0 = \int_D (\|\tilde{\nabla} u\|^2 - nu^2) dS = \int_D (\|\tilde{\nabla} u\|^2 - nu^2)|S_n| dM\,, \tag{4}$$

where dM and $dS = |S_n|dM$ are the volume elements of the induced metric and the pullback metric, respectively.

For notational simplicity, we write $(\theta_i(n-2))^2 = \theta_i^2$. Since, by (3), $\theta_i^2 = -S_n$, we have, assuming that P_r is positive definite,

$$\frac{1}{n}\|\sqrt{P_r}A\|^2 = \frac{1}{n}\sum_j \theta_j^2 = |S_n|\,.$$

Also, denoting by λ_i the eigenvalues of

$$\frac{\sqrt{P_r}A}{\|\sqrt{P_r}A\|}\,,$$

we obtain that

$$\lambda_i^2 = \frac{\theta_i^2}{\sum_j \theta_j^2} = \frac{1}{n}\,.$$

Since, for any $X \in T_pM$,

$$\|X\|^2 = \left\|\left(\frac{\sqrt{P_r}A}{\|\sqrt{P_r}A\|}\right)^{-1}\right\|^2 \left\|\frac{\sqrt{P_r}A}{\|\sqrt{P_r}A\|}X\right\|^2,$$

we can write

$$\|\tilde{\nabla} u\|^2 = n\frac{\|\sqrt{P_r}A\tilde{\nabla} u\|^2}{\|\sqrt{P_r}A\|^2}\,.$$

Therefore, we have from (4),

$$\begin{aligned} 0 &= \int_D \left(\frac{n\|\sqrt{P_r}A\tilde{\nabla} u\|^2}{\|\sqrt{P_r}A\|^2} - nu^2\right)\frac{1}{n}\|\sqrt{P_r}A\|^2 dM \\ &= \int_D (\|\sqrt{P_r}A\tilde{\nabla} u\|^2 - \|\sqrt{P_r}A\|^2u^2)dM\,. \end{aligned}$$

By using that $\tilde{\nabla} = A^{-2}\nabla$ ([1], Lemma 2.9), that P_r commutes with A, and that $\langle\langle A^{-1}X, A^{-1}X\rangle\rangle = \langle X, X\rangle$, we have, by Stokes Theorem,

$$0 = \int_D (|\sqrt{P_r}\nabla u|^2 - \|\sqrt{P_r}A\|^2u^2)\, dM = I_r(u,u)\,.$$

Now we use that ∂D is a first conjugate boundary. Thus for every $\varphi \in C_0^\infty(D)$, we have that $I_r(\varphi,\varphi) \geq 0$. Otherwise, there exists $g \in C_0^\infty(D)$ with $I_r(g,g) < 0$; by the Morse Index Theorem, there exists a Jacobi field in $D' \subsetneq D$ vanishing in $\partial D'$, and this is a contradiction. Then, for any $v \in C_0^\infty(D)$, we obtain for all $t \in \boldsymbol{R}$,

$$0 \leq I_r(u+tv, u+tv) = 2tI_r(u,v) + t^2I_r(v,v)\,.$$

Hence $I_r(u,v) = 0$, and thus u satisfies the Jacobi equation. This proves (b) and completes the proof of Lemma 2.2. □

LEMMA 2.3. *Let $S_1^n \subset \boldsymbol{R}^{n+1}$ be the unit sphere of $\boldsymbol{R}^{n+1}$ and $p = (0, \dots, 0, 1) \in S_1^n$. Then there exist a domain D, symmetric relative to the equator of S_1^n, and a function $f: S_1^n \to \boldsymbol{R}$ such that $\lambda_1(D) = n$ and that f is the first eigenfunction of D. Furthermore, $\lim_{q\to\pm p} f(q) = -\infty$, $q \in S_1^n$, where $-p$ is the antipodal point to p.*

PROOF. This is an application of Lemma 2.2 to rotation hypersurfaces. Let $\boldsymbol{R}^{n+1}$ have coordinates $x_1, \dots, x_{n+1} = y$. Following [10], we let Ox_1 be the axis of rotation and let $y = h(x_1)$ be the equation of the generating curve C of the rotation hypersurface $x: M^n \to \boldsymbol{R}^{n+1}$ with $H_{n-1} = 0$. It is easily checked that $H_n \neq 0$ everywhere for such hypersurfaces and that the curve C is symmetric.

Now consider the domain $W \subset M$ bounded by the rotation of the points of contact of the tangent lines to C issued from the origin 0 of $\boldsymbol{R}^{n+1}$. It is known ([1], §3.7) that the support function $\langle x, N\rangle$ satisfies the Jacobi equation, is positive in W and vanishes in ∂W. Thus ∂W is a first conjugate boundary and, since the Gauss map of such rotation hypersurfaces is injective ([10], §2), Lemma 2.2 implies that the symmetric domain $D = g(W) \subset S_1^n$ satisfies $\lambda_1(D) = n$. Furthermore, if f is the first eigenfunction of D, then, again by Lemma 2.2, $u = f \circ g$ satisfies the Jacobi equation, $u > 0$ in W and $u = 0$ in ∂W. It follows that $u = \langle x, N\rangle$.

Since M behaves asymptotically like a parabola ([10], §2), we have that the support function transfered to S^n, with a convenient choice of orientation, tends to $-\infty$ on both ends of M. Thus f satisfies $\lim_{q\to\pm p} f(q) = -\infty$. □

REMARK 2.4. If we know the explicit expresion of the generating curve C, we can write explictly the function f. For instance, in the case of a rotation hypersurface $x: M^3 \to \boldsymbol{R}^4$ with $H_2 = 0$, we know that the generating curve C is given by

$$y = 1 + \frac{x_1^2}{4}\,.$$

A simple computation shows that the support function transfered to S_1^n, i.e., $\langle x, N\rangle \circ g^{-1} = f$ is given by

$$f(z) = \frac{1 - 2z^2}{\sqrt{1 - z^2}}\,, \quad z = g \circ x_1 = \frac{x_1}{\sqrt{4 + (x_1)^2}}\,.$$

Since f is a radial function, one can easily check, by using the expression of the Laplacian for radial functions (see, for instance, Sakai [13], p. 263) that

$$\tilde{\Delta} f + 3f = 0\,,$$

as it should be.

Lemma 2.5 below follows an argument of do Carmo and Silveira [4].

LEMMA 2.5. *Given finitely many points $p_1, \dots, p_k \in S_1^n$, there exists a domain $W \subset S_1^n$ that omits neighborhoods $U_i \subset S_1^n$ of p_i, $i = 1, \dots, k$, and satisfies $\lambda_1(W) = n$.*

PROOF. For each p_i, make a rotation of S_1^n so that $p_i = (0, \dots, 0, 1)$. Let D_i and f_i be the domain and the function given by Lemma 2.3. Set $h = \sum_i f_i$ and define W as a connected component of the set $\{p \in S_1^n \ ; \ h \geq 0\}$.

We recall that a hemisphere H of S_1^n has eigenvalue n and that, of all domains in S_1^n with the same area, the spherical cap has the smallest eigenvalue. Since $D_1 \cap D_2 \neq \emptyset$, the set $\{p \in S_1^n \ ; \ f_1 + f_2 \geq 0\}$ is not empty. Thus a connected component D_{12} of $\{p \in S_1^n \ ; \ \ f_1 + f_2 \geq 0\}$ has eigenvalue n with eigenfunction $f_1 + f_2$. By the above minimization property,

$$A(D_{12}) > A(H \subset S_1^n)\,,$$

where $A(\quad)$ denotes the area of the enclosed domain. By the same token, $A(D_i) > A(H)$, $i = 1, \dots, k$. Thus $D_{12} \cap D_3 \neq \emptyset$, and an induction shows that $A(W) > A(H)$. This shows that W is not empty. Clearly, $\lambda_1(W) = n$, and h is the first eigenfunction of W. Since $\lim_{p \to p_i} f_i = -\infty$, W omits neighborhoods U_i of p_i, as we desired. $\square$

PROOF OF THEOREM 1.1. The proof uses some recent results of [6] on finite total curvature, complete hypersurfaces of n-dimensional euclidean spaces. We assume the existence of an immersion $x : M^n \to \boldsymbol{R}^{n+1}$ as in Theorem 1.1. Since x is proper, has finite total curvature, and $H_n \neq 0$ everywhere, Theorems 1.1 and 4.1 of [6] imply that there exist a compact manifold $\bar{M}$ and points $q_1, \dots, q_k \in \bar{M}$ such that M is diffeomorphic to $\bar{M} - \{q_1, \dots, q_k\}$ and the Gauss map extends to a homeomorphism $\bar{g} : \bar{M} \to S_1^n$. Set $p_i = \bar{g}(q_i)$, $i = 1, \dots, k$. Let $W \subset S_1^n$ be the domain, given by Lemma 2.5, that omits neighborhoods U_i of p_i and is such that $\lambda_1(W) = n$. Let $W' \supsetneq W$ be a domain in S_1^n that still omits neighborhoods of p_i, and set $D = g^{-1}(W')$. Since g is bijective and $\lambda_1(g(D)) < n$, we conclude, by Theorem B, that D is unstable. This contradicts the assumption and completes the proof. $\square$

EXAMPLE. The following example shows that the hypothesis of stability in Theorem 1.1 cannot be dropped. As mentioned in [1], the hypersurface M in $\boldsymbol{R}^4$ generated by the rotation of the parabola $h(z) = 1 + z^2/4$ around the z-axis is a nonstable complete hypersurface with $H_2 = 0$ and $H_3 \neq 0$ everywhere. By using the orthogonal parametrization $x : M \to \boldsymbol{R}^4$, it is represented as

$$x(z, \theta, \varphi) = (h \cos\theta \sin\varphi, h \sin\theta \sin\varphi, h \cos\theta, z)\,,$$

from which we can easily compute that $|A|^3 = (27/8) f^{-9/2}$, and that

$$\int_M |A|^3 \, dM = \frac{27}{2}\pi^2\,.$$

Thus M has finite total curvature, and this proves our claim.

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