On *ps-ro* semiopen fuzzy set and *ps-ro* fuzzy semicontinuous, semiopen functions

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Abstract

The aim of this paper is to introduce and characterize *ps-ro* semiopen (semiclosed) fuzzy sets, which are totally independent of the existing notion of fuzzy open (closed) sets and semiopen (semiclosed) fuzzy sets. Also, in term of these fuzzy sets and operators *ps-scl* and *ps-sint*, a class of functions named as *ps-ro* fuzzy semicontinuous and *ps-ro* fuzzy semiopen (closed) functions are defined and their various properties are studied. *ps-ro* fuzzy semicontinuity is indeed totally different from both the existing concepts of fuzzy continuity and fuzzy semicontinuity. Similarly, *ps-ro* fuzzy open (closed) and well known concept of fuzzy semiopen (closed) functions do not imply each other. These concepts are used as new tools to study different characterizations of the given fuzzy topological space, giving a new dimension in the study of fuzzy topological spaces.

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1 Introduction and Preliminaries

Ever since the introduction of fuzzy logic and fuzzy sets by L.A.Zadeh [13] and fuzzy topology and fuzzy continuity by C. L. Chang [2], there have been several generalizations and also various departures of different notions of general topological concepts in their fuzzy settings. K. K Azad [1] introduced and studied the concept of fuzzy semiopen(closed) sets, fuzzy semiopen(closed) functions and fuzzy semicontinuity. Some relevant development on these topics are found in [9], [6], etc.

In a fuzzy topological space (fts, for short) (X, τ), the family $i_\alpha(\tau) = \{\mu^\alpha : \mu \in \tau\}$ for all $\alpha \in I_1 = [0, 1)$ where $\mu^\alpha = \{x : \mu(x) \geq \alpha\}$ forms a topology called strong α-level topology on X. The study of strong α-level topology on X reveals various facts about the fts (X, τ) [8], [7]. In [3], it was observed that a strong α-level set μ^α need not be regular open on $(X, i_\alpha(\tau))$ for all $\alpha \in I_1$ if μ is fuzzy regular open on (X, τ) and also the regular openness of μ^α may fail to imply the fuzzy regularity of μ. This observation gave birth to a new family of fuzzy sets termed as pseudo regular open(closed) fuzzy sets on X. The family of all pseudo regular open fuzzy sets form a fuzzy topology on X which is coarser than τ and is termed as *ps-ro* fuzzy topology on X. Members of *ps-ro* fuzzy topology are called *ps-ro* open fuzzy sets and their complements are *ps-ro* closed fuzzy sets on (X, τ). In [4], interms of above *ps-ro* fuzzy sets, a fuzzy continuous type function called *ps-ro* fuzzy continuous function was introduced and different properties were studied.

In this paper, with the help of above mention *ps-ro* open fuzzy sets, a new class of fuzzy sets called *ps-ro* semiopen(semiclosed) fuzzy set is introduced and studied. It is seen that *ps-ro* semiopen (semiclosed) fuzzy sets are totally independent not only from the existing concepts of semiopen(closed) fuzzy sets but also from fuzzy open(closed) sets. Operator *ps-scl* is defined with

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the help of *ps*-semicluster point. Another similar operator *ps-int* is introduced and different characterizations and interrelations are studied. Further, interms of these fuzzy sets and operators, concept of *ps-ro* fuzzy semicontinuous, *ps-ro* fuzzy semiopen(semiclosed) functions are introduced and their various properties are studied. It is observed that every *ps-ro* fuzzy continuous function is *ps-ro* fuzzy semicontinuous but not conversely. Every *ps-ro* fuzzy semicontinuous functions are totally different from well known concepts of fuzzy continuous as well as fuzzy semicontinuous functions. Also, a characterization of such function is made interms of graph of the function in product topology. It was also seen that *ps-ro* fuzzy semiopen(closed) and fuzzy semiopen(closed)functions do not imply each other. It is worth mentioning here that the above concepts of *ps-ro* fuzzy sets and different functions are used as new tools to study different characterizations of the given fuzzy topological space, giving a new dimension in the study of fts.

To make this paper self content, we state a few known definitions and results here that we require subsequently.
Let X be a non-empty set and I be the closed interval $[0, 1]$. A fuzzy set μ on X is a function on X into I. If f is a fuction from X into a set Y and A, B are fuzzy sets on X and Y respectively, then $1-A$ (called complement of A), $f(A)$ and $f^{-1}(B)$ are fuzzy sets on X, Y and X respectively, defined by $(1-A)(x) = 1 - A(x) \forall x \in X$, $f(A)(y) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), when f^{-1}(y) \neq \varnothing \\ 0, \ otherwise \end{cases}$ and $f^{-1}(B)(x) = B(f(x))$ [13]. A collection $\tau \subseteq I^X$ is called a *fuzzy topology* on X if (i) $0, 1 \in \tau$ (ii) $\forall\ \mu_1, \mu_2, ..., \mu_n \in \tau \Rightarrow \wedge_{i=1}^{n} \mu_i \in \tau$ (iii) $\mu_\alpha \in \tau$, $\forall\ \alpha \in \Lambda$ (where Λ is an index set) $\Rightarrow \vee \mu_\alpha \in \tau$. Then, (X, τ) is called a fts [2]. A fuzzy set is called a fuzzy point, denoted by x_α, where $0 < \alpha \leq 1$, and defined as $x_\alpha(z) = \begin{cases} \alpha, & \text{for } z = x \\ 0, & \text{otherwise.} \end{cases}$. A fuzzy point x_α is said to q-coincident with a fuzzy set A, denoted by $x_\alpha q A$ if $\alpha + A(x) > 1$. If A and B are not q-coincident, we write $A \not{q} B$. A fuzzy set A is said to be a q-neighbourhood (or, *q-nbd.*) of a fuzzy point x_α if there is a fuzzy open set B such that $x_\alpha q B \leq A$ [10]. A set S in a topological space (X, τ) is called regular open if $S = int(clS)$ [11].
Let f be a function from a set X into a set Y. Then the following holds:
(i) $f^{-1}(1 - B) = 1 - f^{-1}(B)$, for any fuzzy set B on Y.
(ii) $A_1 \leq A_2 \Rightarrow f(A_1) \leq f(A_2)$, for any fuzzy sets A_1 and A_2 on X.
(iii) $B_1 \leq B_2 \Rightarrow f^{-1}(B_1) \leq f^{-1}(B_2)$, for any fuzzy sets B_1 and B_2 on Y.
(iv) $ff^{-1}(B) \leq B$, for any fuzzy set B on Y and the equality holds if f is onto. (v) $f^{-1}f(A) \geq A$, for any fuzzy set A on X, equality holds if f is one-to-one [2]. A fuzzy set μ on a fts (X, τ) is called fuzzy regular open if $\mu = int(cl\mu)$. A fuzzy set A on a fts (X, τ) is said to be semiopen fuzzy set if there exist a fuzzy open set U such that $U \leq A \leq cl(U)$. A is said to be semiclosed fuzzy set if there exist a fuzzy closed set V such that $int(V) \leq A \leq V$. For a function $f : X \to Y$, the graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$, for each $x \in X$, where X and Y are any sets. Let (X, τ_1) and (X, τ_2) be two fts. The fuzzy product space of X and Y is the cartesian product $X \times Y$ of sets together with the fuzzy topology $\tau_1 \times \tau_2$ generated by the family $\{(p_1^{-1}(A), p_2^{-1}(B)) : A \in \tau_1, B \in \tau_2\}$, where p_1 and p_2 are the projections of $X \times Y$ on X and Y respectively. The family $B = \{A \times B : A \in \tau_1, B \in \tau_2\}$ forms a base for the product topology $\tau_1 \times \tau_2$ on $X \times Y$ [1]. A function f from a fts (X, τ) to fts (Y, σ) is said to be
(i) fuzzy continuous [2], if $f^{-1}(\mu)$ is fuzzy open on X, for all fuzzy open set μ on Y.
(ii) fuzzy open (fuzzy closed) [12], if for each fuzzy open (fuzzy closed) set μ on X, $f(\mu)$ is fuzzy open (fuzzy closed) on Y.

(iii)fuzzy semicontinuous [1], if $f^{-1}(\mu)$ is semiopen fuzzy set on X, for all fuzzy open set μ on Y.
(iv) fuzzy semiopen (fuzzy semiclosed) [1], if for each fuzzy open (fuzzy closed) set μ on X, $f(\mu)$ is semiopen fuzzy set (semiclosed fuzzy) on Y.
For a fuzzy set μ in X, the set $\mu^\alpha = \{x \in X : \mu(x) > \alpha\}$ is called the strong α-level set of X. In a fts (X, τ), the family $i_\alpha(\tau) = \{\mu^\alpha : \mu \in \tau\}$ for all $\alpha \in I_1 = [0, 1)$ forms a topology on X called strong α-level topology on X [8], [7]. A fuzzy open(closed) set μ on a fts (X, τ) is said to be pseudo regular open(closed) fuzzy set if the strong α-level set μ^α is regular open(closed) in $(X, i_\alpha(\tau)), \forall \alpha \in I_1$. The family of all pseudo regular open fuzzy sets form a fuzzy topology on X called *ps-ro* fuzzy topology on X which is coarser than τ. Members of *ps-ro* fuzzy topology are called *ps-ro* open fuzzy sets and their complements as *ps-ro* closed fuzzy sets on (X, τ) [3]. A function f from fts (X, τ_1) to fts (Y, τ_2) is pseudo fuzzy *ro* continuous (in short, *ps-ro* fuzzy continuous) if $f^{-1}(U)$ is *ps-ro* open fuzzy set on X for each pseudo regular open fuzzy set U on Y [4]. Equivalently, f is *ps-ro* fuzzy continuous if $f^{-1}(A)$ is *ps-ro* open fuzzy set on X for each *ps-ro* open fuzzy set A on Y [5]. The union of all *ps-ro* open fuzzy sets, each contained in a fuzzy set A on a fts X is called fuzzy *ps*-interior of A and is denoted by *ps-int*(A). So, *ps-int*$(A) = \vee\{B : B \leq A,\ B$ is *ps-ro* open fuzzy set on $X\}$. Similarly, fuzzy *ps*-closure of A, *ps-cl*$(A) = \wedge\{B : A \leq B,\ B$ is *ps-ro* closed fuzzy set on $X\}$ [4], [5].

Theorem 1.1. *[4], [5] For any fuzzy sets A and B on a fts (X, τ), the following hold:*
(i) ps-cl(A) is the smallest ps-ro closed fuzzy set containing A and ps-int(A) is the largest ps-ro open fuzzy set contained in A.
(ii) ps-cl$(A) \leq$ ps-cl(B) and ps-int$(A) \leq$ ps-int(B) if $A \leq B$.
(iii) ps-cl$(A) = A$ iff A is ps-ro closed and ps-int$(A) = A$ iff A is ps-ro open.
(iv) ps-cl(ps-cl(A)) = ps-cl(A) and ps-int(ps-int(A)) = ps-int(A).
(v) ps-cl$(A \vee B)$ = ps-cl$(A) \vee$ ps-cl(B) and ps-int$(A \wedge B)$ = ps-int$(A) \wedge$ ps-int(B).
(vi)$1 -$ ps-int$A$ = ps-cl$(1 - A)$ and $1 -$ ps-cl$A$ = ps-int$(1 - A)$.

2 *ps-ro* semiopen(closed) fuzzy set and *ps* operators

Definition 2.1. A fuzzy set A on a fts (X, τ) is said to be *ps-ro* semiopen fuzzy set if there exist a *ps-ro* open fuzzy set U such that $U \leq A \leq$ *ps-cl*(U). A is said to be *ps-ro* semiclosed fuzzy set if there exist a *ps-ro* closed fuzzy set V such that *ps-int*$(V) \leq A \leq V$.

Remark 2.1. Clearly, on a fts (X, τ) every *ps-ro* open(closed) fuzzy set is fuzzy open(closed) set. Also, every *ps-ro* open(closed) fuzzy set is *ps-ro* semiopen(semiclosed). But the converse is not always true is shown by the following Example:

Example 2.1. *Let $X = \{a, b, c\}$ and A, B and C be three fuzzy sets on X defined by $A(x) = 0.1$ $\forall x \in X, B(a) = 0.2, B(b) = 0.2$ and $B(c) = 0.3$ and $C(x) = 0.4$ $\forall x \in X$. It is easy to verify that $\tau = \{0, 1, A, B\}$ is a fuzzy topology on X. B is not pseudo regular open fuzzy set as B^α is not regular open on the topological space $(X, i_\alpha(\tau))$ for $0.2 \leq \alpha < 0.3$, although B is fuzzy open on (X, τ). Here, the collection of all pseudo regular open fuzzy sets on X is $\{0, 1, A\}$. Hence, ps-ro fuzzy topology on X is $\{0, 1, A\}$ which is strictly coarser than τ. Here, B is fuzzy open but is not ps-ro open fuzzy set on the fts (X, τ). $1 - B$ is fuzzy closed but is not ps-ro closed fuzzy set on the fts (X, τ). Now, ps-cl$(A) = 1 - A$, where $(1 - A)(x) = 0.9$ $\forall x \in X$. So, $A \leq C \leq$ ps-cl(A). Hence, C is a ps-ro semiopen fuzzy set but it is neither fuzzy open nor ps-ro open fuzzy set on the fts (X, τ). Similarly, $1 - C$ is a ps-ro semiclosed fuzzy set but it is neither fuzzy closed nor ps-ro closed fuzzy set on the fts (X, τ).*

Example 2.2. *Let $Y = \{a, b, c\}$. Let A, B, C, D, E and F be fuzzy sets on Y defined by $A(y) = 0.3$ $\forall\ y \in Y$, $B(a) = 0.1, B(b) = 0.1$ and $B(c) = 0.2$, $C(y) = 0.2\ \forall\ y \in Y$, $D(y) = 0.4\ \forall\ y \in Y$, $E(a) = 0.5, E(b) = 0.6$ and $E(c) = 0.6$ and $F(y) = 0.5\ \forall\ y \in Y$. Then, $\tau_1 = \{0, 1, A, B, D, E\}$ is a fuzzy topology on Y. Also, B and E are not pseudo regular open fuzzy set for $0.1 \leq \alpha < 0.2$ and $0.5 \leq \alpha < 0.6$ respectively. Clearly, ps-ro topology on Y is $\{0, 1, A, D\}$. Here, $B \leq C \leq cl(B)$ and $cl(B) = 1 - E$. So, C is semiopen fuzzy set but it is not ps-ro semiopen fuzzy set on the fts (Y, τ_1) as there exist no ps-ro open fuzzy set U satisfying $U \leq C \leq$ ps-cl(U). Again, $A \leq F \leq$ ps-cl(A). So, F is ps-ro semiopen fuzzy set but not semiopen fuzzy set on the fts (Y, τ_1) as there do not exist any fuzzy open set V satisfying $V \leq F \leq cl(V)$. Similarly, $1 - C$ is a semiclosed fuzzy set but is not ps-ro semiclosed fuzzy set and $1 - F$ is ps-ro semiclosed fuzzy set but not semiclosed fuzzy set on the fts (Y, τ_1).*

Remark 2.2. Example (2.2) shows that *ps-ro* semiopen(*ps-ro* semiclosed) fuzzy set and semiopen (semiclosed) fuzzy set are totally independent of each other.

Remark 2.3. In Example (2.2), B is fuzzy open set but not *ps-ro* semiopen fuzzy set on the fts (Y, τ_1). In Example (2.1), C is *ps-ro* semiopen fuzzy set but is not fuzzy open on the fts (X, τ). This proves that *ps-ro* semiopen(*ps-ro* semiclosed) fuzzy set and fuzzy open(fuzzy closed) set are totally independent concept.

Theorem 2.1. A fuzzy set A on a fts (X, τ) is *ps-ro* semiopen fuzzy set iff $A \leq$ *ps-cl*(*ps-int*(A)).
Proof. Let A be ps-ro semiopen fuzzy set on X. Then, $\exists$ a ps-ro open fuzzy set U such that $U \leq A \leq$ ps-cl(U). As, ps-int$(U) \leq$ ps-int(A), $U \leq$ ps-int(A). So, ps-cl$(U) \leq$ ps-cl(ps-int(A)). This gives, $A \leq$ ps-cl(ps-int(A)). Conversely, let $A \leq$ ps-cl(ps-int(A)). Let ps-int$(A) = V$. So, $A \leq$ ps-cl(V). Again, ps-int$(A) \leq A$. So, $V \leq A \leq$ ps-cl(V). Hence, A is ps-ro semiopen fuzzy set on X.

Theorem 2.2. In a fts (X, τ), *ps-ro* semiopen and *ps-ro* semiclosed fuzzy sets are complement of each other.
Proof. Let A be a ps-ro semiopen fuzzy set on X, then $\exists$ a ps-ro open fuzzy set V on X such that $V \leq A \leq$ ps-cl(V). Hence, $1 - V \geq 1 - A \geq$ ps-int$(1 - V)$. $1 - V$ being ps-ro closed, $1 - A$ is ps-ro semiclosed fuzzy set on X.

Theorem 2.3. A fuzzy set A on a fts (X, τ) is *ps-ro* semiclosed fuzzy set iff *ps-int*(*ps-cl*(A)) $\leq A$.
Proof. Using Theorem (2.1) and Theorem (2.2) the result follows.

Theorem 2.4. Any union of *ps-ro* semiopen fuzzy sets is a *ps-ro* semiopen fuzzy set on a fts (X, τ).
Proof. Let $\{A_\alpha : \alpha \in \Lambda\}$, where Λ is an index set, be a collection of ps-ro semiopen fuzzy sets on X. Then $\exists$ ps-ro open fuzzy set U_α such that $U_\alpha \leq A_\alpha \leq$ ps-cl(U_α) for each $\alpha \in \Lambda$. Thus, $\vee U_\alpha \leq \vee A_\alpha \leq \vee$ps-cl$(U_\alpha) \leq$ ps-cl$(\vee U_\alpha) \Rightarrow \vee U_\alpha \leq \vee A_\alpha \leq$ ps-cl$(\vee U_\alpha)$. Also, $\vee U_\alpha$ is ps-ro open fuzzy set. Hence, $\vee A_\alpha$ is ps-ro semiopen fuzzy set. This proves the theorem.

Theorem 2.5. Any intersection of *ps-ro* semiclosed fuzzy sets is *ps-ro* semiclosed fuzzy set on a fts (X, τ).
Proof. Let $\{A_\alpha : \alpha \in \Lambda\}$, where Λ is a index set, be a collection of ps-ro semiclosed fuzzy sets on X then $\exists$ ps-ro closed fuzzy set U_α such that ps-int$(U_\alpha) \leq A_\alpha \leq U_\alpha$ for each α. Thus, $\wedge$ps-int$(U_\alpha) \leq \wedge A_\alpha \leq \wedge U_\alpha$. Hence, ps-int$(\wedge U_\alpha) \leq \wedge A_\alpha \leq \wedge U_\alpha$. $\wedge U_\alpha$ being ps-ro closed fuzzy set, it follows that $\wedge A_\alpha$ is ps-ro semiclosed fuzzy set on X.

Definition 2.2. A fuzzy set A on a fts (X, τ) is said to be
(a) *ps-ro* semi-neighborhood (*nbd*) of a fuzzy point x_α, if there is a *ps-ro* semiopen fuzzy set B on X such that $x_\alpha \in B \leq A$. (b) *ps-ro semi-quasi-nbd* or *ps-ro sq-nbd* of a fuzzy point x_α, if there is *ps-ro* semiopen fuzzy set V on X such that $x_\alpha q V \leq A$.

Definition 2.3. A fuzzy point x_α on a fts (X, τ) is called *ps*-semicluster point of a fuzzy set A on X if every *ps-ro sq-nbd* of x_α is q coincident with A. The set of all *ps*-semicluster points of A is called as *ps*-semi-closure of A denoted by *ps-scl*(A).

Theorem 2.6. For fuzzy set A on a fts (X, τ), *ps-scl*(A) is the intersection of all *ps-ro* semiclosed fuzzy sets each containing A.
Proof. Let B denote the intersection of all ps-ro semiclosed fuzzy sets each containing A. Suppose $x_\alpha \in B$ and if possible let $\exists$ a ps-ro sq-nbd N of x_α such that $N \not{q} A$. Then, $\exists$ a ps-ro semiopen fuzzy set V on X such that $x_\alpha q V \leq N$. As, w$V \not{q} A$, then $\forall\ x \in X$ we have $V(x) + A(x) \leq 1$. So, $A \leq 1 - V$. Since, $1 - V$ is ps-ro semiclosed fuzzy set on X, $B \leq 1 - V$. Since $x_\alpha \notin 1 - V$, $x_\alpha \notin B$, which is a contradiction. Therefore, for $x_\alpha \in B$ and any ps-ro sq-nbd N of x_α we have $N q A$. So, x_α is a ps-semicluster point of A. i.e. $x_\alpha \in$ ps-scl(A). Hence, $B \leq$ ps-scl(A). Now, we will show ps-scl$(A) \leq B$, which is same as to show $x_\alpha \in B$ if $x_\alpha \in$ ps-scl(A) or equivalently $x_\alpha \notin B \Rightarrow x_\alpha \notin$ ps-scl(A). Let $x_\alpha \notin B$, then $\exists$ ps-ro semiclosed fuzzy set $U \geq A$ such that $x_\alpha \notin U$ then, $x_\alpha q(1-U)$ and $A \not{q}(1-U)$. Thus, $x_\alpha \notin$ ps-scl(A). So, ps-scl$(A) \leq B$. Hence, $B =$ ps-scl(A). This completes the proof.

Theorem 2.7. A fuzzy set A on a fts (X, τ) is *ps-ro* semiclosed fuzzy set iff $A =$ *ps-scl*(A).
Proof. Let A be a ps-ro semiclosed fuzzy set on a fts (X, τ) then by Theorem (2.6), $A =$ ps-scl(A). Conversely, let $A =$ ps-scl(A), then by Theorem (2.5), A is ps-ro semiclosed fuzzy set.

Definition 2.4. In a fts (X, τ), ps-semi interior(*ps-sint*, in short) of a fuzzy subset A is defined as the union of all *ps-ro* semiopen fuzzy set on X contained in A. i.e. *ps-sint*$(A) = \vee\{B : B \leq A, B$ is *ps-ro* semiopen fuzzy set on $X\}$.

Theorem 2.8. For any fuzzy sets A and B on a fts (X, τ), the following hold:
(a) *ps-scl*(A) is the smallest *ps-ro* semiclosed fuzzy set containing A.
(b) *ps-scl*$(0) = 0$, *ps-scl*$(1) = 1$.
(c) $A \leq$ *ps-scl*(A).
(d) *ps-scl*(*ps-scl*$(A)) =$ *ps-scl*(A).
(e) *ps-scl*$(A) \leq$ *ps-scl*$(B), if A \leq B$.
(f) *ps-scl*$(A \vee B) =$ *ps-scl*$(A) \vee$ *ps-scl*(B).
(g) *ps-scl*$(A \wedge B) \leq$ *ps-scl*$(A) \wedge$ *ps-scl*(B).
(h) *ps-scl*$(A) \leq$ *ps-cl*(A).
Proof. Straightforward and hence omitted.

Theorem 2.9. For any fuzzy sets A and B on a fts (X, τ), the following hold:
(a) *ps-sint*(A) is the largest *ps-ro* semiopen fuzzy set contained in A.
(b) *ps-sint*$(0) = 0$, *ps-sint*$(1) = 1$.
(c) *ps-sint*$(A) \leq A$.
(d) A is *ps-ro* semiopen fuzzy set iff $A =$ *ps-sint*(A).
(e) *ps-sint*(*ps-sint*$(A)) =$ *ps-sint*(A).

(f) $ps\text{-}sint(A) \le ps\text{-}sint(B), if A \le B$.
(g) $ps\text{-}sint(A \wedge B) = ps\text{-}sint(A) \wedge ps\text{-}sint(B)$.
(h) $ps\text{-}sint(A \vee B) \ge ps\text{-}sint(A) \vee ps\text{-}sint(B)$.
(i) $ps\text{-}int(A) \le ps\text{-}sint(A)$.
(j) $1 - ps\text{-}sint(A) = ps\text{-}scl(1 - A)$.
(k) $1 - ps\text{-}scl(A) = ps\text{-}sint(1 - A)$.
Proof. Straightforward and hence omitted.

3 *ps-ro* fuzzy semicontinuous, semiopen (closed) functions

Definition 3.1. A function f from a fts (X, τ_1) to another fts (Y, τ_2) is called *ps-ro* fuzzy semicontinuous function if $f^{-1}(A)$ is *ps-ro* semiopen fuzzy set on X, for each *ps-ro* open fuzzy set A on Y.

Remark 3.1. Clearly, every *ps-ro* fuzzy continuous function is *ps-ro* fuzzy semicontinuous but the converse is not always true is shown by the following Example:

Example 3.1. *Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by $A(a) = 0.1, A(b) = 0.2$ and $A(c) = 0.2$ and $B(x) = 0.3, \forall x \in X$. Let C, D, E and F be fuzzy sets on Y defined by $C(t) = 0.3, \forall t \in Y$, $D(x) = 0.3, D(y) = 0.3$ and $D(z) = 0.4$, $E(t) = 0.4, \forall t \in Y$ and $F(x) = 0.1, F(y) = 0.1$ and $F(z) = 0.2$. $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D, E, F\}$ are fuzzy topologies on X and Y respectively. Clearly, A is not pseudo regular open fuzzy set for $0.1 \le \alpha < 0.2$ on X. So, the ps-ro topology on X is $\{0, 1, B\}$. Again, D and F are not pseudo regular open fuzzy set for $0.3 \le \alpha < 0.4$ and $0.1 \le \alpha < 0.2$, respectively on Y . So, the ps-ro topology on Y is $\{0, 1, C, E, \}$. Now, ps-cl$(B) = 1 - B$, where $(1 - B)(x) = 0.7, \forall x \in X$. Define a function f from the fts (X, τ_1) to fts (Y, τ_2) by $f(a) = x, f(b) = y$ and $f(c) = z$. E is ps-ro open fuzzy set on Y and $f^{-1}(E)(x) = 0.4, \forall x \in X$. So, $B \le f^{-1}(E) \le$ ps-cl(B). Thus, $f^{-1}(E)$ is ps-ro semiopen fuzzy set on X. Similarly, it can be verified that $f^{-1}(U)$ is ps-ro semiopen fuzzy set on X for every ps-ro fuzzy set U on Y. Hence, f is ps-ro fuzzy semicontinuous but is not ps-ro fuzzy continuous as $f^{-1}(E)$ is not ps-ro open fuzzy set on X.*

Remark 3.2. In Example (3.1), $F \in \tau_2$ but $f^{-1}(F) \notin \tau_1$, proving that f is *ps-ro* fuzzy semicontinuous but is not fuzzy continuous.

Remark 3.3. Let f be a fuzzy continuous function from a fts (X, τ_1) to fts (Y, τ_2) and A be a *ps-ro* open fuzzy set on Y. As, every *ps-ro* open fuzzy sets are fuzzy open, $f^{-1}(A)$ is fuzzy open on X which by Remark (2.3) is not in general fuzzy *ps-ro* semiopen fuzzy set on X. Hence, a fuzzy continuous function may not be *ps-ro* fuzzy semicontinuous. In view of this and Remark(3.2), *ps-ro* fuzzy semicontinuity and fuzzy continuity do not imply each other.

Example 3.2. *Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by $A(a) = 0.1, A(b) = 0.1$ and $A(c) = 0.2$ and $B(x) = 0.3, \forall x \in X$. Let C, D, E and F be fuzzy sets on Y defined by $C(t) = 0.3, \forall t \in Y$, $D(x) = 0.3, D(y) = 0.3$ and $D(z) = 0.4$, $E(t) = 0.4, \forall t \in Y$ and $F(t) = 0.2, \forall t \in Y$. $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D, E, F\}$ are fuzzy topologies on X and Y respectively. Clearly, A is not pseudo regular open fuzzy set for $0.1 \le \alpha < 0.2$ on X. So, the ps-ro topology on X is $\{0, 1, B\}$. Again, D is not pseudo regular open fuzzy set for $0.3 \le \alpha < 0.4$ on Y . So, the ps-ro topology on Y is $\{0, 1, C, E, F\}$. Define a function f from the fts (X, τ_1) to fts (Y, τ_2) by $f(a) = x, f(b) = y$ and $f(c) = z$. Here, f is fuzzy semicontinuous as $f^{-1}(U)$ is* *semiopen fuzzy set on X for every fuzzy open set U on Y but f is not ps-ro fuzzy semicontinuous since $f^{-1}(F)$ is not ps-ro semiopen fuzzy set on X.*

Remark 3.4. In Example(3.1), $f^{-1}(F)$ is not semiopen fuzzy set on X, showing that f is not fuzzy semicontinuous but is *ps-ro* fuzzy semicontinuous. Combining this result with Example(3.2), we see that the concept of *ps-ro* fuzzy semicontinuous and fuzzy semicontinuous functions are totally independent of each other.

Theorem 3.1. A function f from a fts (X, τ_1) to another fts (Y, τ_2) is *ps-ro* fuzzy semicontinuous function iff $f^{-1}(A)$ is *ps-ro* semiclosed fuzzy set on X, for each *ps-ro* closed fuzzy set A on Y.
Proof. Let f be ps-ro fuzzy semicontinuous and A be ps-ro closed fuzzy set on Y, then $f^{-1}(1-A) = 1 - f^{-1}(A)$ is ps-ro semiopen fuzzy set on X. Thus, $f^{-1}(A)$ is ps-ro semiclosed fuzzy set on X. Converse part can be proved similarly.

Theorem 3.2. Let f be a function from a fts (X, τ_1) to fts (Y, τ_2). f is *ps-ro* fuzzy semicontinuous iff for any fuzzy point x_α on X and any *ps-ro* open fuzzy set V on Y with $f(x_\alpha) \in V$, there exist *ps-ro* semiopen fuzzy set U on X such that $x_\alpha \in U$ and $f(U) \leq V$.
Proof. Let f be ps-ro fuzzy semicontinuous. Let x_α be any fuzzy point on X and V be any ps-ro fuzzy open set on Y containing $f(x_\alpha)$. $f^{-1}(V)$ is ps-ro semiopen fuzzy set which contains x_α. Taking $U = f^{-1}(V)$, the result follows. Conversely, let the given condition hold and V be any ps-ro open fuzzy set on Y. If $f^{-1}(V) = 0$, then the result is true. If $f^{-1}(V) \neq 0$, then there exist fuzzy point x_α in $f^{-1}(V)$. i.e. $f(x_\alpha) \in V$ and V is ps-ro open fuzzy set on Y. So, by the given hypothesis $\exists$ ps-ro semiopen fuzzy set U_{x_α} on X which contains x_α such that $x_\alpha \in U_{x_\alpha} \leq f^{-1}(V)$. Since x_α is arbitrary, taking union of all such relations, we get $f^{-1}(V) = \vee\{x_\alpha : x_\alpha \in f^{-1}(V)\} \leq \vee\{U_{x_\alpha} : x_\alpha \in f^{-1}(V)\} \leq f^{-1}(V)$. So, $\vee\{U_{x_\alpha} : x_\alpha \in f^{-1}(V)\} = f^{-1}(V)$. This shows $f^{-1}(V)$ is ps-ro semiopen fuzzy set. Hence, f is ps-ro fuzzy semicontinuous.

Theorem 3.3. A function f from a fts (X, τ_1) to fts (Y, τ_2) is *ps-ro* fuzzy semicontinuous iff for every fuzzy set A on X, $f(ps\text{-}scl(A)) \leq ps\text{-}cl(f(A))$.
Proof. Let f be ps-ro fuzzy semicontinuous and A be any fuzzy set on X. Then, $f^{-1}(ps\text{-}cl(f(A)))$ is ps-ro semiclosed fuzzy set on X.
Now, $A \leq f^{-1}(f(A)) \leq f^{-1}(ps\text{-}cl(f(A)))$
$\Rightarrow ps\text{-}scl(A) \leq ps\text{-}scl(f^{-1}(ps\text{-}cl(f(A)))) = f^{-1}(ps\text{-}cl(f(A)))$
$\Rightarrow f(ps\text{-}scl(A)) \leq f(f^{-1}(ps\text{-}cl(f(A)))) \leq ps\text{-}cl(f(A))$
$\Rightarrow f(ps\text{-}scl(A)) \leq ps\text{-}cl(f(A))$.
Conversely, let B be any ps-ro closed fuzzy set on Y, $ps\text{-}scl(f^{-1}(B))$ is ps-ro semiclosed fuzzy set on X. By the given condition we have, $f(ps\text{-}scl(f^{-1}(B))) \leq ps\text{-}cl(f(f^{-1}(B))) \leq ps\text{-}cl(B)$
$\Rightarrow f^{-1}f(ps\text{-}scl(f^{-1}(B))) \leq f^{-1}(ps\text{-}cl(B))$
$\Rightarrow ps\text{-}scl(f^{-1}(B)) \leq f^{-1}f(ps\text{-}scl(f^{-1}(B))) \leq f^{-1}(ps\text{-}cl(B))$
$\Rightarrow ps\text{-}scl(f^{-1}(B)) \leq f^{-1}(ps\text{-}cl(B)) = f^{-1}(B)$. *We have, $ps\text{-}scl(f^{-1}(B)) \geq f^{-1}(B)$. Hence we get, $ps\text{-}scl(f^{-1}(B)) = f^{-1}(B)$. Therefore, $f^{-1}(B)$ is ps-ro semiclosed fuzzy set on X. Hence, f is ps-ro fuzzy semicontinuous.*

Theorem 3.4. A function f from a fts (X, τ_1) to fts (Y, τ_2) is *ps-ro* fuzzy semicontinuous iff for every fuzzy set B on Y, $ps\text{-}scl(f^{-1}(B)) \leq f^{-1}(ps\text{-}cl(B))$.
Proof. Suppose f is ps-ro fuzzy semicontinuous and B be any fuzzy set on Y. Then, $f^{-1}(ps\text{-}cl(B))$ is ps-ro semiclosed fuzzy set on X. Now, $B \leq ps\text{-}cl(B) \Rightarrow f^{-1}(B) \leq f^{-1}(ps\text{-}cl(B))$

$\Rightarrow$ *ps-scl*$(f^{-1}(B)) \leq$ *ps-scl*$((f^{-1}($*ps-cl*$(B)))) = f^{-1}($*ps-cl*$(B))$
$\Rightarrow$ *ps-scl*$(f^{-1}(B))) \leq f^{-1}($*ps-cl*$(B))$. *Conversely, let* B *be any ps-ro closed fuzzy set on* Y*. Given,* *ps-scl*$(f^{-1}(B)) \leq f^{-1}($*ps-cl*$(B)) = f^{-1}(B)$. *So,* *ps-scl*$(f^{-1}(B)) \leq f^{-1}(B)$ *and* $f^{-1}(B) \leq$ *ps-scl*$(f^{-1}(B))$. *Therefore,* *ps-scl*$(f^{-1}(B)) = f^{-1}(B)$ *i.e.* $f^{-1}(B)$ *is ps-ro semiclosed fuzzy set on* X*. Hence,* f *is ps-ro fuzzy semicontinuous.*

Theorem 3.5. For a function f from a fts (X, τ_1) to fts (Y, τ_2), the following statements are equivalent:
(a) f is *ps-ro* fuzzy semicontinuous.
(b) *ps-int*(*ps-cl*$(f^{-1}(B))) \leq f^{-1}($*ps-cl*$(B))$, for each fuzzy set B on Y.
(c) $f($*ps-int*(*ps-cl*$(A))) \leq$ *ps-cl*$(f(A))$, for each fuzzy set A on X.
Proof.(a) $\Rightarrow$ *(b) Suppose* f *is ps-ro fuzzy semicontinuous and* B *be any fuzzy set on* Y*. So,* $f^{-1}($*ps-cl*$(B))$ *is ps-ro semiclosed fuzzy set on* X*. Hence,* *ps-int*(*ps-cl*$(f^{-1}($*ps-cl*$(B)))) \leq f^{-1}($*ps-cl*$(B))$*.* *Thus,* *ps-int*(*ps-cl*$(f^{-1}(B)) \leq f^{-1}($*ps-cl*$(B))$
(b) $\Rightarrow$ *(c) Let* A *be any fuzzy set on* X*. Let* $f(A) = B$*, then* $A \leq f^{-1}(B)$*, Therefore,* *ps-int*(*ps-cl*$(f^{-1}(B))) \leq f^{-1}($*ps-cl*$(B))$*. So,* *ps-int*(*ps-cl*$(A)) \leq f^{-1}($*ps-cl*$(B))$
$\Rightarrow f($*ps-int*(*ps-cl*$(A))) \leq f(f^{-1}($*ps-cl*$(B))) \leq$ *ps-cl*$(B) =$ *ps-cl*$(f(A))$*. Hence,* $f($*ps-int*(*ps-cl*$(A))) \leq$ *ps-cl*$(f(A))$*.*
(c) $\Rightarrow$ *(a) Suppose* U *be any ps-ro closed fuzzy set on* Y*. Let* $A = f^{-1}(U)$*, then* $f(A) \leq U$ *and* $f($*ps-int*(*ps-cl*$(A))) \leq$ *ps-cl*$(f(A)) \leq$ *ps-cl*$(U) = U$
$\Rightarrow f($*ps-int*(*ps-cl*$(A))) \leq U$
$\Rightarrow f^{-1}f($*ps-int*(*ps-cl*$(A))) \leq f^{-1}(U)$
$\Rightarrow$ *ps-int*(*ps-cl*$(A)) \leq f^{-1}f($*ps-int*(*ps-cl*$(A))) \leq f^{-1}(U)$
$\Rightarrow$ *ps-int*(*ps-cl*$(f^{-1}(U))) \leq ($*ps-cl*$(A)) \leq f^{-1}(U)$
So, $f^{-1}(U)$ *is ps-ro semiclosed fuzzy set on* X*. Hence,* f *is ps-ro fuzzy semicontinuous.*

Theorem 3.6. A function f from a fts (X, τ_1) to fts (Y, τ_2) is *ps-ro* fuzzy semicontinuous iff for every fuzzy set B on Y, $f^{-1}($*ps-int*$(B)) \leq$ *ps-sint*$(f^{-1}(B))$.
Proof. Let f *be ps-ro fuzzy semicontinuous and* B *be any fuzzy set on* Y*. Then,* $f^{-1}($*ps-int*$(B))$ *is ps-ro semiopen fuzzy set on* X*. Now,* *ps-int*$(B) \leq B$*. So, we get* $f^{-1}($*ps-int*$(B)) =$ *ps-sint*$(f^{-1}($*ps-int*$(B))) \leq$ *ps-sint*$(f^{-1}(B))$*. Conversely, let* B *be any ps-ro open fuzzy set on* Y*. Then,* $f^{-1}($*ps-int*$(B)) = f^{-1}(B) \leq$ *ps-sint*$(f^{-1}(B))$ *and* $f^{-1}(B) \geq$ *ps-sint*$(f^{-1}(B))$*, therefore,* $f^{-1}(B) =$ *ps-sint*$(f^{-1}(B))$*. Hence,* f *is ps-ro fuzzy semicontinuous.*

Theorem 3.7. Let a function f from a fts (X, τ_1) to fts (Y, τ_1) be one-to-one and onto. f is *ps-ro* fuzzy semicontinuous iff for every fuzzy set A on X, *ps-int*$f(A)) \leq f($*ps-sint*$(A))$.
Proof. Let f *be ps-ro fuzzy semicontinuous and* A *be any fuzzy set on* X*. Then,* $f^{-1}($*ps-int*$(f(A))$ *is ps-ro semiopen fuzzy set on* X*. Since,* f *is one-to-one,* $f^{-1}($*ps-int*$(f(A)) \leq$ *ps-sint*$(f^{-1}(f(A))) =$ *ps-sint*(A)*, i.e.* $ff^{-1}($*ps-int*$(f(A)) \leq f($*ps-sint*$(A))$*. Since f is onto,* *ps-int*$(f(A)) \leq f($*ps-sint*$(A))$*.* *Conversely, let* B *be any ps-ro open fuzzy set on* Y*. Since* f *is onto,* $B =$ *ps-int*$(B) =$ *ps-int*$(f(f^{-1}(B))) \leq f($*ps-sint*$(f^{-1}(B)))$*, i.e.* $f^{-1}(B) \leq f^{-1}(f($*ps-sint*$(f^{-1}(B))))$*. Again,* f *is one-to-one so,* $f^{-1}(B) \leq$ *ps-sint*$(f^{-1}(B))$ *and* $f^{-1}(B) \geq$ *ps-sint*$(f^{-1}(B))$*. Therefore,* $f^{-1}(B) =$ *ps-sint*$(f^{-1}(B))$*. Hence* f *is ps-ro fuzzy semicontinuous.*

Lemma 3.1. *[1]* Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. If A and B are fuzzy sets on X and Y respectively, then $g^{-1}(A \times B) = A \wedge f^{-1}(B)$.

Theorem 3.8. Let f be a function from a fts (X, τ_1) to fts (Y, τ_2). If the graph $g : X \to X \times Y$ of f is *ps-ro* fuzzy semicontinuous then f is also *ps-ro* fuzzy semicontinuous.

Proof. Let B be any ps-ro open fuzzy set on Y. Using Lemma (3.1), we get, $f^{-1}(B) = 1 \wedge f^{-1}(B) = g^{-1}(1 \times B)$. $1 \times B$ is ps-ro open fuzzy set on $X \times Y$ and since g is ps-ro fuzzy semicontinuous function, $g^{-1}(\times B)$ is ps-ro semiopen fuzzy set on X. Thus, $f^{-1}(B)$ is ps-ro semiopen fuzzy set on X. Hence f is ps-ro fuzzy semicontinuous.

Definition 3.2. A function f from a fts (X, τ_1) to another fts (Y, τ_2) is called
(a) *ps-ro* fuzzy semiopen function if $f(A)$ is *ps-ro* semiopen fuzzy set on Y for each *ps-ro* open fuzzy set A on X.
(b) *ps-ro* fuzzy semiclosed function if $f(A)$ is *ps-ro* semiclosed fuzzy set on Y for each *ps-ro* closed fuzzy set A on X.

Example 3.3. *Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by $A(a) = 0.1, A(b) = 0.2$ and $A(c) = 0.2$ and $B(x) = 0.3, \forall x \in X$. Let C, D, E and F be fuzzy sets on Y defined by $C(t) = 0.3, \forall t \in Y$, $D(x) = 0.3, D(y) = 0.3$ and $D(z) = 0.4$, $E(t) = 0.4, \forall t \in Y$ and $F(t) = 0.2, \forall t \in Y$. $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D, E, F\}$ are fuzzy topologies on X and Y respectively. Clearly, A is not pseudo regular open fuzzy set for $0.1 \leq \alpha < 0.2$ on X. So, the ps-ro topology on X is $\{0, 1, B\}$. Again, D is not pseudo regular open fuzzy set for $0.3 \leq \alpha < 0.4$ on Y . So, the ps-ro topology on Y is $\{0, 1, C, E, F\}$. Define a function f from the fts (X, τ_1) to fts (Y, τ_2) by $f(a) = x, f(b) = y$ and $f(c) = z$. Here, f is ps-ro fuzzy semiopen but f is neither fuzzy semiopen nor fuzzy open since $f(A)$ is neither semiopen nor open fuzzy set on Y. Here, f is ps-ro fuzzy semiclosed but f is neither fuzzy semiclosed nor fuzzy closed since $f(1 - A)$ is neither semiclosed nor closed fuzzy set on Y.*

Example 3.4. *Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by $A(a) = 0.3, A(b) = 0.3$ and $A(c) = 0.4$ and $B(x) = 0.3, \forall x \in X$. Let C and D be two fuzzy sets on Y defined by $C(t) = 0.4, \forall t \in Y$ and $D(x) = 0.2, D(y) = 0.3$ and $D(z) = 0.3$. $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D\}$ are fuzzy topologies on X and Y respectively. Clearly, A is not pseudo regular open fuzzy set for $0.3 \leq \alpha < 0.4$ on X. So, the ps-ro topology on X is $\{0, 1, B\}$. Again, D is not pseudo regular open fuzzy set for $0.2 \leq \alpha < 0.3$ on Y . So, the ps-ro topology on Y is $\{0, 1, C\}$. Define a function f from the fts (X, τ_1) to fts (Y, τ_2) by $f(a) = x, f(b) = y$ and $f(c) = z$. Here, f is fuzzy semiopen but f is not ps-ro fuzzy semiopen since $f(B)$ is not ps-ro semiopen fuzzy set on Y. Again, f is fuzzy semiclosed but f is not ps-ro fuzzy semiclosed since $f(1 - B)$ is not ps-ro semiclosed fuzzy set on Y.*

Remark 3.5. From Remark(3.3) and Remark(3.4), *ps-ro* fuzzy semiopen(closed) and fuzzy semiopen(closed)functions do not imply each other.

Theorem 3.9. Let f be a function from a fts (X, τ_1) to fts (Y, τ_2). f is *ps-ro* fuzzy semiopen iff $f(ps\text{-}int(A)) \leq ps\text{-}cl(ps\text{-}int(f(A)))$, for any fuzzy set A on X.

Proof. Let f be ps-ro fuzzy semiopen and A be a fuzzy set on X. $ps\text{-}int(A)$ being ps-ro open fuzzy set on X, $f(ps\text{-}int(A))$ is ps-ro semiopen fuzzy set on Y. By Theorem (2.1), we have, $f(ps\text{-}int(A)) \leq ps\text{-}cl(ps\text{-}int(f(ps\text{-}intA))) \leq ps\text{-}cl(ps\text{-}int(f(A)))$. Thus, $f(ps\text{-}int(A)) \leq ps\text{-}cl(ps\text{-}int(f(A)))$. Conversely, let A be any ps-ro open fuzzy set on X. $f(A) = f(ps\text{-}int(A)) \leq ps\text{-}cl(ps\text{-}int(f(A)))$ .i.e. $f(A) \leq ps\text{-}cl(ps\text{-}int(f(A)))$. By Theorem (2.1), $f(A)$ is ps-ro semiopen fuzzy set on Y. Hence f is ps-ro fuzzy semiopen.

Theorem 3.10. For a function f from a fts (X, τ_1) to fts (Y, τ_2), the following statement are equivalent:
(a) f is *ps-ro* fuzzy semiopen.
(b) $f(ps\text{-}int(A)) \leq ps\text{-}sint(f(A))$, for each fuzzy set A on X.
(c) $ps\text{-}int(f^{-1}(B)) \leq f^{-1}(ps\text{-}sint(B))$, for each fuzzy set B on Y.
Proof.(a) $\Rightarrow$ (b) *Let A be any fuzzy set on X, then $f(ps\text{-}int(A))$ is ps-ro semioprn fuzzy set. Now, $ps\text{-}int(A) \leq A$, so we get $f(ps\text{-}int(A)) = ps\text{-}sint(f(ps\text{-}int(A))) \leq ps\text{-}sint(f(A))$.*
(b) $\Rightarrow$ (c) *Let B be any fuzzy set on Y. Then, $f(ps\text{-}int(f^{-1}(B))) \leq ps\text{-}sint(f(f^{-1}(B))) \leq ps\text{-}sint(B)$. Thus, $ps\text{-}int(f^{-1}(B))) \leq f^{-1}f(ps\text{-}int(f^{-1}(B))) \leq f^{-1}(ps\text{-}sint(B))$. Therefore, $ps\text{-}int(f^{-1}(B))) \leq f^{-1}(ps\text{-}sint(B))$.*
(c) $\Rightarrow$ (a) *let A be any ps-ro open fuzzy set on X. As, $A \leq f^{-1}f(A)$ we have, $A = ps\text{-}int(A) \leq ps\text{-}int(f^{-1}f(A)) \leq f^{-1}(ps\text{-}sint(f(A)))$. So, $f(A) \leq ps\text{-}sint(f(A))$ and $f(A) \leq ps\text{-}sint(f(A)$. Hence, $f(A) = ps\text{-}sint(f(A)$ .i.e. f is ps-ro fuzzy semiopen.*

Theorem 3.11. For a function f from a fts (X, τ_1) to fts (Y, τ_2) the following statements are equivalent:
(a) f is *ps-ro* fuzzy semiclosed.
(b) $f(ps\text{-}cl(A)) \geq ps\text{-}int(ps\text{-}cl(f(A)))$, for any fuzzy set A on X.
(c) $ps\text{-}scl(f(A)) \leq f(ps\text{-}cl(A))$, for any fuzzy set A on X.
Proof.(a) $\Rightarrow$ (b) *Let A be any fuzzy set on X, $f(ps\text{-}cl(A))$ is ps-ro semiclosed fuzzy set on Y. Now by Theorem (2.3), $ps\text{-}int(ps\text{-}cl(f(ps\text{-}cl(A)))) \leq f(ps\text{-}cl(A))$. Hence, $ps\text{-}int(ps\text{-}cl(f(A))) \leq ps\text{-}int(ps\text{-}cl(f(ps\text{-}cl(A)))) \leq f(ps\text{-}cl(A))$.*
(b) $\Rightarrow$ (a) *Let A be any ps-ro closed fuzzy set on X. $ps\text{-}int(ps\text{-}cl(f(A))) \leq f(ps\text{-}cl(A)) = f(A)$ .ie. $f(A)$ is ps-ro semiclosed fuzzy set on Y. Hence, f is ps-ro fuzzy semiclosed.*
(a) $\Rightarrow$ (c) *Let A be any fuzzy set on X. $f(ps\text{-}cl(A))$ is ps-ro semiclosed fuzzy set on Y. $f(A) \leq f(ps\text{-}cl(A))$. So, $ps\text{-}scl(f(A)) \leq ps\text{-}scl(f(ps\text{-}cl(A))) = f(ps\text{-}cl(A))$. Hence, $ps\text{-}scl(f(A)) \leq f(ps\text{-}cl(A))$.*
(c) $\Rightarrow$ (a) *Let A be any ps-ro closed fuzzy set on X. Now, $ps\text{-}scl(f(A)) \leq f(ps\text{-}cl(A)) = f(A)$. We have $f(A) \leq ps\text{-}scl(f(A))$. Hence, $f(A) = ps\text{-}scl(f(A))$. i.e. $f(A)$ is ps-ro semiclosed fuzzy set on Y. Therefore, f is ps-ro fuzzy semiclosed.*

Theorem 3.12. Let a function f from a fts (X, τ_1) to fts (Y, τ_1) be one-to-one and onto. f is *ps-ro* fuzzy semiclosed iff for every fuzzy set B on Y, $f^{-1}(ps\text{-}scl(B)) \leq ps\text{-}cl(f^{-1}(B))$.
Proof. Let f be ps-ro fuzzy semiclosed and B be any fuzzy set on Y. As f is onto, $ps\text{-}scl(B) = ps\text{-}scl(f(f^{-1}(B))) \leq f(ps\text{-}cl(f^{-1}(B))$, i.e $f^{-1}(ps\text{-}scl(B)) \leq f^{-1}(f(ps\text{-}cl(f^{-1}(B)))$. Since, f is one-to-one we get $f^{-1}(ps\text{-}scl(B)) \leq ps\text{-}cl(f^{-1}(B))$. Conversely, let A be any ps-ro closed fuzzy set on X. Since, f is one-to-one, $f^{-1}(ps\text{-}scl(f(A)) \leq ps\text{-}cl(f^{-1}(f(A))) = ps\text{-}cl(A) = A$, i.e. $f(f^{-1}(ps\text{-}scl(f(A))) \leq f(A)$. Since, f is onto $ps\text{-}scl(f(A)) \leq f(A)$ and $ps\text{-}scl(f(A)) \geq f(A)$. Therefore, $ps\text{-}scl(f(A)) = f(A)$. Hence, f is ps-ro fuzzy semiclosed.

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