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GENERALIZED CONVERGENCE THEOREMS FOR DENJOY-PERRON INTEGRALS

We give a proof of the controlled convergence theorem that is real-line independent.

Let E be an interval in the n-dimensional space, that is, it is the set of all points $x = (x_1, \dots x_n)$ with $a_j \le x_j \le b_j$ for $j = 1,2,\dots,n$. Assume that a norm has been defined on the n-dimensional space. An open sphere $S(x,r)$ with centre x and radius r is the set of all y such that $\|x-y\| < r$. A division D of E is a finite colleciton of interval-point pairs (I,x) with the intervals non-overlapping and their union E. It is δ-fine if $I \subset S(x,\delta(x))$ where x is a vertex of I. Then a real number H is the value of the generalized Riemann integral of f over E if given $\epsilon > 0$ there is a positive function $\delta(x)$ such that

$$|(D) \sum f(x)|I| - H| < \epsilon$$

for all δ-fine division D of E.

A function F defined on E is $AC^{**}(X)$ if for every $\epsilon > 0$ there are a $\delta(x) > 0$ and a $\eta > 0$ such that for any two δ-fine partial divisions of X, D_1 and D_2, satisfying

$$(D_1 \setminus D_2) \sum|I| < \eta \quad \text{we have} \quad (D_1 \setminus D_2) \sum|F(I)| < \epsilon.$$

Here $D_1 \setminus D_2$ denotes the collection of component intervals I in $D_1 \setminus D_2$. A function is ACG^{**} if E is the union of a sequence of closed sets X_i,

$i = 1,2,\ldots$, on each of which F is $AC^{**}(X_i)$. A sequence of functions $\{F_n\}$ is $UACG^{**}$ if F_n is ACG^{**} uniformly in n.

Then we can prove the following

THEOREM If the following conditions are satisfied:

(i) $f_n(x) \longrightarrow f(x)$ everywhere in E as $n \longrightarrow \infty$ where each f_n if generalized Riemann integrable on E;

(ii) the primitives F_n of f_n are $UACG^{**}$,

then f is generalized Riemann integrable on E and we have

$$\int_E f_n \longrightarrow \int_E f \quad \text{as} \quad n \longrightarrow \infty.$$

Some applications are also mentioned.