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Paradoxical Decompositions Using Lipschitz Functions [1]

Let G_k denote the group of isometries of $\mathbb{R}^k$. Sets $A, B \subset \mathbb{R}^k$ are called *equidecomposable*, if there are partitions $A = A_1 \cup \ldots \cup A_n$, $B = B_1 \cup \ldots \cup B_n$ and isometries $f_1, \ldots, f_n \in G_k$ such that $f_i(A_i) = B_i \quad (i = 1, \ldots, n)$. By a well-known theorem of S. Banach and A. Tarski [2], any two bounded sets in $\mathbb{R}^k \quad (k \geq 3)$ with non-empty interior are equidecomposable. In $\mathbb{R}^2$ such a paradox does not exist [1]. Still, paradoxical sets do exist in $\mathbb{R}^2$. S. Mazurkiewicz and W. Sierpiński showed in [4] that there is a non-empty set $A \subset \mathbb{R}^2$ which can be decomposed into two disjoint subsets congruent to A. Sierpiński later showed that such a paradox does not exist in $\mathbb{R}$. Moreover, no set $A \subset \mathbb{R}$ can be partitioned into two subsets which are equidecomposable to A (see [5], p. 56). In spite of this fact that paradoxical sets do not exist in $\mathbb{R}$ if only isometries are used, there are paradoxical decompositions in $\mathbb{R}$ which use Lipschitz functions (in particular, contractions.) This talk presents some recent results and open problems concerning this type of decomposition.

References

[1] M. S. Banach, Sur le problème de la measure, *Fund. Math.* 4 (1923), 7-33.

[2] M. S. Banach and A. Tarski, Sur la decomposition des ensembles de point en parties respectivement congruents, *Fund. Math.* 6 (1924), 244-277.

[3] M. Laczkovich, Paradoxical decompositions using Lipschitz functions, (submitted.)

[4] S. Mazurkiewicz and W. Sierpiński, Sur un ensemble superposables avec chacune de ses deux parties, *C. R. Acad. Sci. Paris,* 158 (1914), 618-619.

[5] W. Sierpiński, *On the congruence of sets and their equivalence by finite decomposition,* Lucknow, 1954. Reprinted by Chelsea, 1967.

[1] A more complete summary of this talk is in the Inroads section of this issue.