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"A Lower Bound for the Packing Measure which is a Multiple of the Hausdorff Measure"

Claude Tricot, Jr. has solved S. James Taylor's conjecture that when the Hausdorff dimension of a set E is equal to the packing dimension of the set E, then the Hausdorff measure of E is equal to the packing measure of E only when the dimension is an integer. Tricot's solution affirmed Taylor's conjecture. So the question now changes to how much larger is the packing measure of E in relation to the Hausdorff measure of E when the dimension is not an integer. For symmetric sets on the real line, and under certain conditions, a lower bound for the packing measure that is a multiple of the Hausdorff measure is given.

Definition 1: A *symmetric set* is defined as follows:

Let $N = 1$. Take an interval B_1^1 of length b_1 from the center of $[0, 1]$ leaving intervals on the left, A_1^1, and on the right, A_1^2, of $[0, 1]$ each of length a_1. Take intervals $\{B_{k+1}^i\}_{i=1}^{2^k}$ each of length b_{k+1} from the center of each of the 2^k intervals $\{A_k^i\}_{i=1}^{2^k}$ of length a_k leaving 2^{k+1} intervals $\{A_{k+1}^i\}_{i=1}^{2^{k+1}}$ of length a_{k+1}. Then E, the symmetric set, is the intersection of the sets $E_n = \cup_{i=1}^{2^n} A_n^i$.

The lower bound for the packing measure that is a multiple of the Hausdorff measure is given in the following theorem. In this theorem, $\mu^\alpha(E)$ will denote the Hausdorff measure and $(\alpha - p)(E)$ will denote the packing measure, where α is the dimension.

<u>Theorem 1</u>: Let E be a symmetric set. If the Hausdorff dimension of E is equal to the packing dimension of E and if $\overline{\lim}_{n\to\infty} (a_{n+1}/a_n) < (1/2)$, then

$(\alpha - p)(E) \geq [1 + \beta]^\alpha \mu^\alpha(E)$ where $\beta = \underline{\lim}_{n\to\infty} (b_{n+1}/a_n)$.

An upper bound for the packing measure of a symmetric set E can also be found in relation to the Hausdorff measure of E.

Theorem 2: If $E = \cup_{n=1}^{\infty} E_n$ where $\{E_n\}_{n=1}^{\infty}$ are disjoint, if $\lim_{r \to o} \mu^{\alpha} [E_n \cap (x - r, x + r)]/(2r)^{\alpha} \geq d_n > o$, and if

$$\sum_{n=1}^{\alpha} d_n^{-1} \mu^{\alpha} (E_n) < \infty,$$

then $(\alpha - p)(E) \leq \sum_{n=1}^{\infty} d_n^{-1} \mu^{\alpha} (E_n)$.

Symmetric sets with Hausdorff dimension equal to packing dimension have the following property:

Theorem 3: Let E be a symmetric set on the real line. If the Hausdorff dimension of E is equal to the packing dimension of E, then E is porous.

However, their are a class of symmetric sets which are porous and have Hausdorff dimension strictly less then their packing dimension.