Functional analysis and generalized integrals

by Ralph Henstock

Continuous linear functionals on spaces of absolutely and non-absolutely integrable functions, and suitable norms, were discussed. A conjecture concerning the geometric difference between division spaces that give rise to the two kinds of integrals, was shown false by Washek Pfeffer. I hope to give a direct approach to the Alexiewicz-Sargent-Thomson theory.

Finally a simple problem:- A well-behaved non-negative function f satisfies (1) $f(x)\to 0$ as $x\to-\infty$ and $\int_{-\infty}^{x}\{f(t)\}^n dt = \{\int_{-\infty}^{x} f(t)\,dt\}^n$, for some constant $n>0$. Find f.

The solution is that if $a>0$ satisfies $a^{n-1}=n$, if E is the set of x where $f(x)>0$, with characteristic function χ_E, if meas(E)>0, and if b is a point of metric density of E, then a.e.

$$f(x) = \begin{cases} \chi_E(x)\ A \exp\ [a.\text{meas}\{E\cap(b,x)\}] & (x>b) \\ \chi_E(x)\ A \exp\ [-a.\text{meas}\{E\cap(x,b)\}] & (x<b) \end{cases}$$

for some constant $A>0$, such that (1) is true in the set of measure zero where the last formula does not hold. $A=0$ when $f=0$ a.e. (Bulletin of the Institute of Mathematics and its applications, 22 (1986), numbers 3/4, pp. 60, 61.)

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