

QUERIES

This section has been reorganized into four parts. The first will consist of COMMENTS received on the queries. QUERIES FROM THE TEXT will list appropriate problems posed in the Survey and Inroads articles to draw further attention to them. The problems from Volume 1 No. 1 and Volume 2 No. 1 are included in this issue. ADDITIONAL QUERIES and SOLUTIONS will include the problems and solutions received within the past six months.

COMMENTS

Query 3. The original statement is incorrect. It should be:

3. If f maps a subset E of \mathbb{R}^n into \mathbb{R}^n , is continuous on E and $|x-f(x)| = 1$ for each x in E , can $\dim(E) < \dim(f(E))$, where $\dim()$ denotes Hausdorff dimension?

Query 4*. T. McLaughlin points out that in the solution to Query 4, parts a and b (Vol. 2 No.1 p. 69), CH is used only to show that the union of fewer than c sets of measure 0 is of measure 0. This also follows from Martin's Axiom, one form of which states that no compact Hausdorff space of countable cellularity can be covered by fewer than c nowhere dense sets. Restated,

4*. Can Query 4, parts b and c, be answered in ZFC?

Query 9. (Vol. 2 No. 1 p. 78) In the note showing that the set of points at which a function is increasing on the right need not be measurable, E should be defined to be an arbitrary dense subset of R . The note is due to Larry Eifler.

QUERIES FROM THE TEXT

Background information for these queries is contained in the article in which they occur.

12. (Bagemihl, Piranian, Young) Does there exist a continuous complex-valued function f in H having the 3-segment property at each point of a set of positive measure or of the second category on R ? (Vol. 1 No. 1 p. 16).
13. Characterize the space $BVS^m[a,b]$ for the case in which m is not continuous on $[a,b]$. (Vol. 1 No. 1 p. 41).
14. Are there functions which are modified A_nP -integrable but not A_nP -integrable? (Vol. 1 No. 1 p. 57). See also reference [8], (Vol. 1 No. 1 p. 61).
15. Can a suitable notion similar to that of an AC_nG^* function be defined so that a result for the A_nP -integral similar to Theorem (5,n,0) holds? (Vol. 1 No. 1 p. 57-58).

16. Characterize the class of functions whose product with each summable derivative is a summable derivative. (Vol. 2 No. 1 p. 16).

17. Let \mathcal{F} be a two parameter family of functions with the property that if x_1, x_2, y_1, y_2 are real numbers, $x_1 \neq x_2$, there exists a unique $f \in \mathcal{F}$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. What conditions on \mathcal{F} will guarantee that the intersections of a typical continuous function with members of \mathcal{F} will have properties analogous to those of the intersections of a typical continuous function with the family of lines $\{ax + b\}$? For example, is it sufficient for each $f \in \mathcal{F}$ to satisfy a Lipschitz condition?

(Vol. 2 No. 1 p. 37)

18. Let P_n denote the family of polynomials of degree at most n . What can one say about the intersections of a typical continuous f with the members of P_n ? For example, is it true that the intersection of a typical f with $p \in P_n$ contains at most $n + 1$ isolated points? And must there exist a $p \in P_n$ whose intersection with f does have $n + 1$ isolated points. In general, if Q is a subset of P_n obtained by fixing k specified coefficients and allowing the remaining coefficients to be arbitrary, then what intersection properties does a typical f have with the members of Q ?

(Vol. 2 No. 1 p. 37)

19. If $C [0,1]$ is replaced with some other complete metric space of functions, (e.g. the set of bounded Darboux Baire 1) functions furnished with the "sup metric", what can now be said about intersections of typical functions in the class with straight lines?
(Vol. 2 No. 1 p. 38)
20. If one replaces $C [0,1]$ with the family of continuous functions defined on a square, what are the intersections of typical functions with straight lines or with planes? Information here might yield results about differentiability properties of typical continuous functions of several variables.
(Vol. 2 No. 1 p. 38)
21. Does the gradient of a differentiable function have the Denjoy property?
(Vol. 2 No. 1 p. 57)
22. Can every ACG* function be written as $f \circ g$ where f is differentiable and g is monotone and absolutely continuous?
(Vol. 2 No. 2 p. 92)
23. Does there exist a differentiable function whose graph is not sparse?
(Vol. 2 No. 2 p. 93)

24. How can the class of functions of the form $f \circ g$, where f is a homeomorphism and g is ACG, be characterized?
(Vol. 2 No. 2 p. 98)
25. How can the class of functions of the form $f \circ g$, where f is a homeomorphism and g is BVG be characterized?
(Vol. 2 No. 2 p. 98)
26. How can the class of functions of the form $f \circ g$, where f is a homeomorphism and g satisfies Lusin's condition (N) be characterized?
(Vol. 2 No. 2 p. 98)
27. How can the class of functions of the form $f \circ g$, where f is a homeomorphism and g satisfies Banach's condition T_2 be characterized?
(Vol. 2 No. 2 p. 98)
28. How may the functions of the form $f \circ g$, where f is an absolutely continuous homeomorphism and g is differentiable, be characterized?
(Vol. 2 No. 2 p. 99)
29. How may the functions of the form $f \circ g$, where f is an absolutely continuous homeomorphism and g is ACG, be characterized?
(Vol. 2 No. 2 p. 99)

30. How may the functions of the form $f \circ g$, where f is an absolutely continuous homeomorphism and g is BVG, be characterized?
(Vol. 2 No. 2 p. 99)
31. If $f(x)$ has a sparse graph and g is a homeomorphism, must both $g \circ f$ and $f \circ g$ have sparse graphs?
(Vol. 2 No. 2 p. 100)
32. Can every continuous function be written as the sum of two functions which satisfy Lusin's condition (N)?
(Vol. 2 No. 2 p. 101)
33. Can any continuous function which is BVG be written as the sum of two continuous generalized monotone functions?
(Vol. 2 No. 2 p. 101)
34. Does there exist a continuous function which possesses a symmetric derivative everywhere on the real line, but fails to be differentiable on a set of positive Hausdorff dimension?
(Vol. 2 No. 2 p. 105)
35. Suppose that $\mathcal{R}_\ell f'(x)$ exists on $[0,1]$. Does the function $\mathcal{R}_\ell f'(x)$ belong to the family of the honorary functions of the second Baire class? Is it true for selective derivatives?
(Vol. 2 No. 2 p. 117)

ADDITIONAL QUERIES

No additional queries have been received by the Exchange within the past six months.

SOLUTIONS

No solutions have been received within the past six months.