H. W. Pu, Department of Mathematics, Texas A&M University, College Station, Texas 77843 and H. H. Pu, Department of Mathematics, Soochow University, Taipei, ROC.

A Problem of Marcus

Zahorski proved in [8]:

If a continuous function f possesses a derivative f' (finite or infinite) everywhere on I_0, then the set

$$E(\alpha, \beta) = \{x \in I_0 : \alpha < f'(x) < \beta\}$$

is in the class M_3 for each pair of numbers α, β, $-\infty \leqq \alpha < \beta \leqq +\infty$.

Marcus posed the problem in [4]:

Is the above theorem still true if the ordinary derivative f' and $E(\alpha,\beta)$ are replaced by the approximate derivative f'_{ap} and

$$E_{ap}(\alpha,\beta) = \{x \in I_0 : \alpha < f'_{ap}(x) < \beta\} ?$$

Recently, the authors prove the following general theorem which yields an affirmative answer to this problem [6].

If f is of Baire type one, has the Darboux property, possesses an approximate derivative f'_{ap} (finite or infinite) everywhere on a fixed interval I_0 and f'_{ap} is of Baire type one, then $E_{ap}(\alpha,\beta) \in M_3$.

The proof for this theorem is based on Bruckner, Burkill and Neugebauer's work (see [1], [2], [5]) and parallels Clarkson and Weil's with several essential modifications (see [3], [7]).

References

1. A. M. Bruckner, An affirmative answer to a problem of Zahorski and some consequences, Mich. Math. J., 13(1966) pp. 15-26.

2. J. C. Burkill and U. S. Haslam-Jones, The derivates and approximate derivates of measurable functions, Proc. London Math. Soc. (2) 32(1931) pp. 346-355.

3. J. A. Clarkson, A property of derivatives, Bull. Amer. Math. Soc. 53(1947) pp. 124-125.

4. S Marcus, On a theorem of Denjoy and on approximate derivative, Monatsh Math. 66(1962) pp. 435-440.

5. C. J. Neugebauer, Darboux functions of Baire class one and derivative, Proc. Amer. Math. Soc. 13(1962) pp. 838-843.

6. H. W. Pu and H. H. Pu, An affirmative answer to a problem of Marcus, (to be submitted).

7. C. E. Weil, On property of derivatives, Trans. Amer.

Math. Soc. 114(1965) pp. 363-376.

8. Z. Zahorski, Sur la premiere derivee, Trans. Amer. Math. Soc. 69(1950) pp. 1-54.

Received February 16, 1976