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Extreme points and the convexity theorem of A. A. Lyapunov

The aim of this paper is to give a short proof of theorem 4.7 on p.153 in [4], see also [5], pp.51, 52.

Theorem 1 *Let $(T, \mathcal{A}, m)$ be a complete measure space, $m : \mathcal{A} \to \mathbb{R}^p$ a non-atomic measure and $F : T \to \mathbb{R}^n$ an integrably bounded point-closed-convex measurable multifunction. Then*

$$\int_A F dm = \int_A (ext F) dm \text{ for every } A \in \mathcal{A}.$$

The rather short proof is based on (1) the fact that the set of integrable selectors of F is weakly compact in $L_1(T, \mathbb{R}^n)$; (2) the Eberlein-Šmulian theorem; (3) corollary 14, p.422 in [2]; (4) Lindenstrauss's proof of the convexity theorem of Lyapunov in [3]; (5) theorem 3.2, p. 175 in [1].

A short survey is also given of the role of extreme points in connection with the convexity theorem of Lyapunov, with specific reference to papers by Dvoretsky, Wald and Wolfowitz, Richter, Karlin, Uhl, Elton and Hill, Legut, and Akemann and Anderson.

References

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[3] J. Lindenstrauss, *J. Math. Mech.* **15** (1966), 971-972.

[4] P. Maritz, *Real Analysis Exchange* **11** (1985-86), 134-158.

[5] P. Maritz, *Real Analysis Exchange* **12** (1986-87), 43-54.