involve
a journal of mathematics

The influence of education in reducing the HIV epidemic

Renee Margevicius and Hem Raj Joshi

msp

2013 vol. 6, no. 2

INVOLVE 6:2 (2013)
dx.doi.org/10.2140/involve.2013.6.127

The influence of education in reducing the HIV epidemic

Renee Margevicius and Hem Raj Joshi

(Communicated by Suzanne Lenhart)

We use an SIRE (susceptible, infected, removed, and education) model to study and evaluate the effectiveness of Uganda's education campaigns from the last 25 years in reducing the prevalence of AIDS and HIV infection. We divide the susceptible class into four subgroups with different infection rates due to their differing beliefs on sexual conduct. We use data from Uganda about the epidemic and educational influences to help estimate the infection rates, and then we simulate the model and compare our results to real data from 1996–2007.

1. Introduction

HIV is a slow working virus that often causes AIDS, in which the immune system begins to fail. The disease is transmitted by mother to child at birth, or by sharing needles, or through unsafe sex. At the time of this writing, an estimated 34 million people were living with HIV throughout the world [WHO 2010], about two-thirds of them in sub-Saharan Africa.

Uganda, a country located in that region, has had a major influence in HIV prevention. In 1987, the Ugandan government created a campaign called ABC, standing for *abstinence*, *being faithful*, and *use of condoms*, to promote ways of preventing the spread of the virus through safer sexual behavior [Green et al. 2002; 2006]. The first part of the campaign, *abstinence*, promotes no sex until marriage. The *being faithful* portion supports those couples that only practice sex with one partner. Lastly, the *use of condoms* promotes safe sex for those with multiple partners. This three-pronged approach mirrors the recommendations of international organizations created throughout the world to help educate people on HIV/AIDS and slow its spread. But Uganda has been more successful than most countries; throughout the nineties, the prevalence of HIV in Uganda fell, and many observers credit this to the ABC prevention campaign. Over the last ten years the incidence of HIV/AIDS in Uganda has largely stabilized [Uganda 2010], even as the

MSC2010: 34K60, 35K55.
Keywords: SIRE model, ABC strategy, mathematical model.

country shifted its prevention policy away from ABC and towards abstinence-only programs, which many experts believe may lead to a rise in risky behavior.

Our goal in this paper is to model the effects of education on the dynamics of the HIV epidemic. In [Joshi et al. 2008], we modified the basic *susceptible, infected, and removed* (SIR) model to include an *education* class (we call the new model an SIRE model). The education class represents the proportion of organizations that are involved in spreading the ABC campaign. We split the susceptible class into three subclasses: the general susceptibles S who do not change their behavior due to the campaign, a class S_{AB} of susceptibles who have been influenced by the *abstinence* and *being faithful* portions, and a class S_C of susceptibles who begin to use *condoms* due to the campaign. Here we extend that work by further dividing S_{AB} into two subclasses S_A and S_B, consisting of those susceptibles who have chosen to practice abstinence and those who have chosen to be faithful to one partner as a result of the campaign.

We collected information such as the history of the HIV epidemic, government statistics, and behavioral records. We used this information in our SIRE model, a system of ordinary differential equations, in order to explain the effects of the ABC campaign. After estimating the parameters using collected data, we simulated the model using MATLAB. The educational influences should cause infection and HIV-related death rates to slow down.

The outline of this paper is as follows. Section 2 will provide an overview of the standard SIR model, as well as the modified SIRE model with the educational influences. Section 3 discusses parameter estimates. In Section 4, we simulate the SIRE model and compare the results to collected data. In Section 5, we present future directions and conclusions.

2. Modified SIRE model

A basic SIR model [Edelstein-Keshet 1988], with susceptible, infected, and removed classes, takes the form

$$S' = -\beta SI + b(S+I) - dS, \quad I' = \beta SI - \gamma I, \quad R' = \gamma I, \tag{1}$$

where d is the natural death rate, b is the birth rate, γ is the death rate due to infection, and β is the infection rate.

We will augment the basic SIR model (1) by introducing educational influence. We will take into account changes in behavior of some susceptibles in the adult population only (ages 15–49), since the promotional campaigns are designed to influence adult behavior. As a result of educational influence, our susceptible class will exhibit different behaviors. Figure 1 is schematic diagram of our model and it shows the connectivity of the different classes.

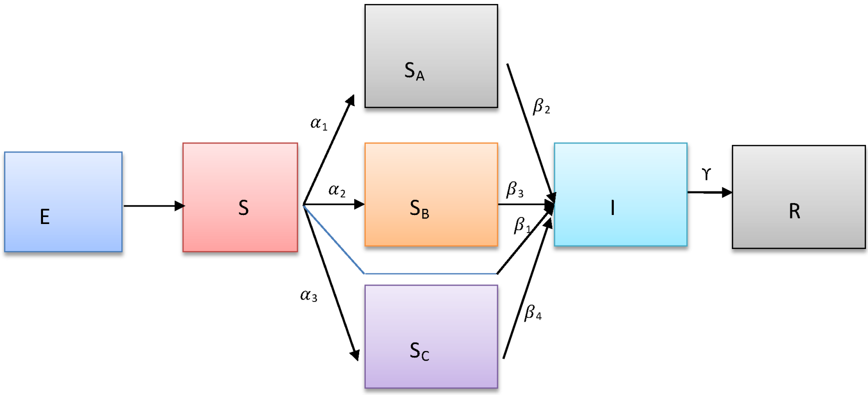

Figure 1. Schematics of the our SIRE model.

The influences of the educational information will break the susceptible population into four different types of behavior. The first will be the initial population in which there is no change in behavior due to education, denoted by S, the general susceptible population. The next part of the population counts those who have been influenced to choose abstinence, which will be denoted by S_A. Another type of change in behavior for the susceptible class will be choosing to be faithful, which will be indicated by S_B. The last type of behavior change is use of condoms, and those susceptibles will be denoted by S_C. In addition, an E class is needed to show the influence of the educational information given about the A, B, and C type behaviors. This E class causes some members of the susceptible class, S, to move into the A, B, and C categories. The size of the E class depends on the fraction of organizations providing the information on HIV. The proportions of the organizations providing the education on each of the three types of behaviors causes the split among the A, B, and C categories. Due to influence of E, people from class S will move into the S_A, S_B, and S_C classes at given rates. In addition, the entry rate for people into the general susceptible class, and death rates where people leave, must be taken into consideration.

The infection rate β for each class will vary due to the influence of education and change in behavior. Thus we will have four different infection rates for the four different susceptible classes. The infected class I will move to the removed class R at the rate γ, where the removed class is the number of people who have died from HIV/AIDS.

For the education class, E, we use a logistic model with a growth rate which will increase as the number of infective increase. We multiply the growth rate, r, by the ratio of the populations of the infected to the living.

The following model is an extension of the one in [Joshi et al. 2008]. The main modification is the use of two separate equations for those with changed behavior due to abstinence and being faithful. Having two separate susceptible equations for these changes in behavior will result in two different infection rates. These infection rates will be cause members of the respective susceptible subclasses to be added to the infected class and will change the education class. With these parameters and alterations included, our new SIRE model is

$$
\begin{aligned}
S' &= -\alpha_1 ES - \alpha_2 ES - \alpha_3 ES - \beta_1 SI + b(S + S_A + S_B + S_C + I) - dS,\\
S'_A &= \alpha_1 ES - \beta_2 S_A I - dS_A \quad (\alpha_1 = 0.02),\\
S'_B &= \alpha_2 ES - \beta_3 S_B I - dS_B \quad (\alpha_2 = 0.08),\\
S'_C &= \alpha_3 ES - \beta_4 S_C I - dS_C \quad (\alpha_3 = 0.8),\\
I' &= \beta_1 SI + \beta_2 S_A I + \beta_3 S_B I + \beta_4 S_C I - \gamma I,\\
R' &= \gamma I,\\
E' &= \frac{I}{I + S + S_A + S_B + S_C} rE(1 - E),
\end{aligned}
\tag{2}
$$

where α_1, α_2 and α_3 are the transfer rates from S to S_A, S_B, and S_C, respectively. The initial conditions for this system are $S(0)$, $S_A(0)$, $S_B(0)$, $S_C(0)$, $I(0)$, $R(0)$, and $E(0)$. The entering adult rate is b and the general death rate is d. For this case, new adults will enter the general susceptible class S only. Thus, there are four different susceptible classes for which four infection rates are needed: β_1, β_2, β_3, β_4, for S, S_A, S_B, S_C, respectively, as they relate to the infected class I. A proportion of the susceptibles leave the general susceptible class S into S_A, S_B, or S_C when the individuals in class S and the educational campaign class E interact. In addition, when the infected class interacts with the susceptible class, individuals leave according to their rates into the infected class. As a result of HIV, individuals from the infected class leave and are moved into the removed class R with death rate γ.

3. Parameter estimations

The data needed for this model contained information about population, death rates, percentage of adults ages 15–64, the growth of the adult class, adult prevalence rates, and the percent of adult population infected [UNICEF 2010]. In order to determine the organizational estimates for the educational influence rates, we consulted literature, essays, subject matter experts, and surveys. These types of data will influence the relationship between the E and S classes, in addition to the split amongst the A, B, and C behavior types.

The initial conditions for the set of differential equations depend on the data provided. Since the first educational data collected occurred in 1996, we will begin with that year for the model and use the data to determine the initial conditions for $S(0)$ and $I(0)$. Thus we assume that, prior to 1996, no one followed the A, B, and C type behaviors. In addition, the removed class, R, accumulates the deaths from HIV only. In 1996, the entire population (July 1996 est.) of Uganda was 20,158,176 with adult population comprising 48% [CIA 1997]. Therefore the initial susceptible and infected classes $(S(0)+I(0))$ will have a total of 9,675,924 people for that year. The HIV prevalence rate for adults was estimated to be 12.1%; thus 1,161,110 people are in the infected $(I(0))$ class. As a result, there will be 8,365,991 people in the susceptible $(S(0))$ class. Note that, for $E(0)$, there was an initial estimate of 30% of organizations involved in the ABC campaign. This estimate is an approximation, so the numerical runs will vary. Thus, these are the initial conditions:

$$\begin{gathered} S(0)=9.67, \quad S_A(0)=0, \quad S_B(0)=0, \quad S_C(0)=0, \\ I(0)=1.16, \quad R(0)=0.11, \quad E(0)=0.30. \end{gathered} \tag{3}$$

4. SIRE model simulation

The time span for this model is 12 years (1996–2007). All rates, b, d, r, β_1, β_2, β_3, β_4, γ, were assumed to be constant for all our model simulations. Using data from this time period, we were able to calculate the number of new adults as a percentage of all adults. The entering adult rates for the 12 years were averaged to obtain an entry rate for the susceptible class. The natural death rate was also averaged from UN data, over each five year period. The adults for the general susceptible population had an entering rate $b=0.055$ and death rate $d=0.0176$. For γ, we took an average of the death rates due to HIV for a few years and found $\gamma=0.14$.

As for the parameters, we had to make many assumptions and estimations. For the infection rate parameters, β_1, β_2, β_3, and β_4, we assumed that β_2, β_3, and β_4 were proportional to β_1, so we only needed to determine one infection parameter. We predicted that β_1 was larger than β_2, expecting that the A behavior led to a lower infection rate compared to the general susceptible class. For example, $\beta_2=0.01\beta_1$ $(\beta_2 \ll \beta_1)$. As for the infection rate for the B behavior compared to the general susceptible class, we took $\beta_3=0.03\beta_1$ $(\beta_2<\beta_3\ll\beta_1)$. Lastly, we took the infection rate for the C behavior as $\beta_4=0.4\beta_1$ $(\beta_2<\beta_3\ll\beta_4\lll\beta_1)$. We determined the range of values for β_1 that best fit observations, since this infection rate was the hardest to estimate. In addition, we varied r in increments to determine which value, together with β_1, gave a model best fitted for the data.

For the determination of β_1, we first fixed the bounds $0.0001\le\beta_1\le0.1$, and let it vary in increments of 0.001, giving 100 values for β_1. Similarly the growth rate r was assumed to lie in the range $0.2\le r\le 2$. We used increments for r of 0.01, giving 180 values. For each pair (β_1, r) we ran the set of differential equations with a MATLAB differential equation solver to give model estimates for the values for

Year	Susceptible	Infected	Removed	Education
1997	9.99	1.046	0.1195	600/1,200
1998	10.42	1.021	0.1205	
1999	10.72	0.975	0.121	
2000	10.96	0.921	0.1215	700/1,200
2001	11.27	0.879	0.120	717/1,200
2002	11.61	0.824	0.118	
2003	12.05	0.807	0.113	
2004	12.47	0.773	0.104	
2005	12.82	0.756	0.089	778/1,200
2006	13.25	0.755	0.084	
2007	14.22	0.754	0.079	

Table 1. Historical data table (population numbers in millions).

each class for each year. These model numbers were then compared to the found data from 1997–2007 [UNAIDS 2009; WHO 2010; Joshi et al. 2008; AVERT 2010; Uganda 2010; UNICEF 2010]. The data points used are shown in Table 1.

We next show our results after running the simulations against our data. Figure 2 represents the true total susceptible data versus the model prediction. The population for this graph is given in millions. The model for the S class has data points close, but a few data points are not as close as the others to the graph. For Figure 3, we graphed the model output alongside the infected data. This model shows similarities

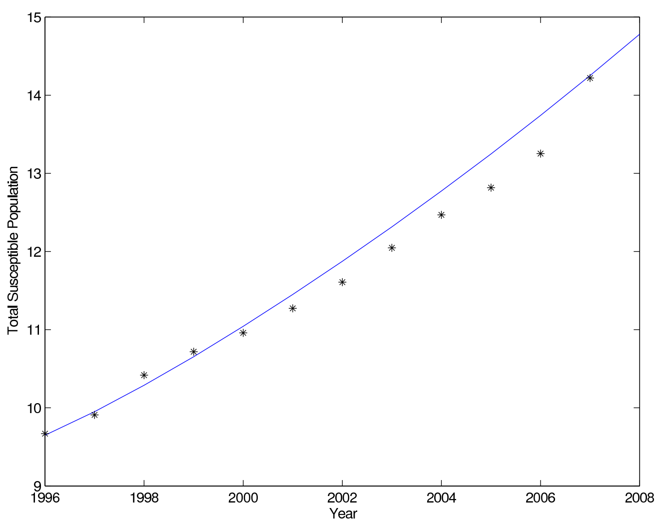

Figure 2. Susceptible population, in millions: model prediction (solid line) and data ($*$).

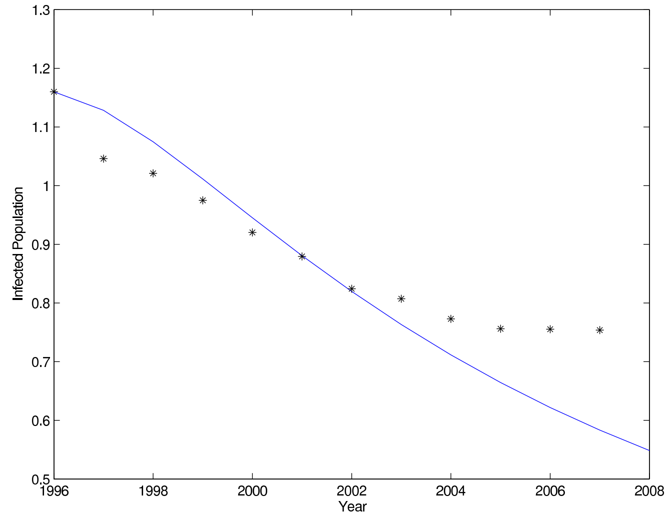

Figure 3. Infected population, in millions: model prediction (solid line) and data ($*$).

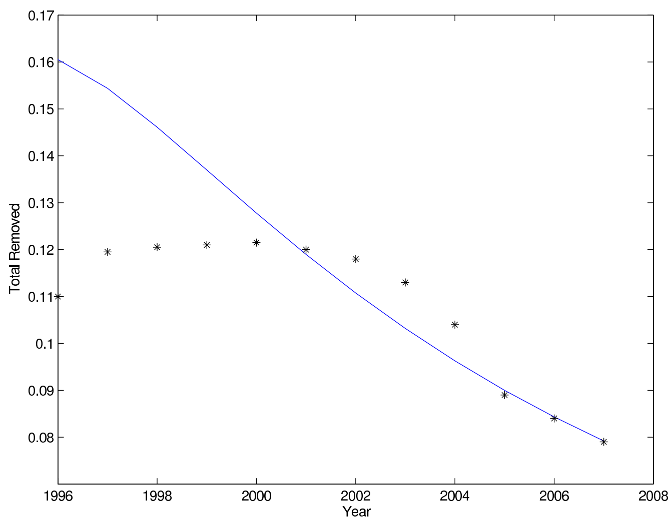

Figure 4. Number of HIV related deaths, in millions: model prediction (solid line) and data ($*$).

with the graph. Figure 4 represents the data of deaths per year compared to the model. This model shows a similar shape as the majority of the data points above the model. Figure 5 shows the model for the education class, illustrating fairly close data points to the equation.

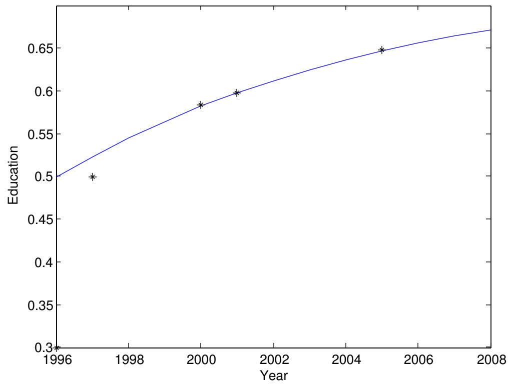

Figure 5. Education influence: model prediction (solid line) and data ($*$).

5. Conclusions and future research

This work illustrated altering the susceptible class based on behavior changes due to education. We found that, for the most part, the model's predictions were close to the data. Finding more data and including other features to make this model more realistic is important.

Future research includes refining the S and I classes based on age, gender, and stages of the disease. Past research has shown that females have been more responsive to these educational campaigns than males. Therefore, gender differentiation would be interesting to consider for future modeling efforts. In addition, the involvement of different types of organizations could be studied in variants of the model.

References

[AVERT 2010] AVERT, "HIV and AIDS in Uganda", web page, 2010, http://www.avert.org/aidsuganda.htm.

[CIA 1997] "The world factbook 1996", CIA, 1997, http://www.cia.gov/library/publications/the-world/factbook.

[Edelstein-Keshet 1988] L. Edelstein-Keshet, *Mathematical models in biology*, Random House, New York, 1988. MR 90i:92001 Zbl 0674.92001

[Green et al. 2002] E. Green, V. Nantulya, R. Stoneburner, and J. Stover, "What happened in Uganda?", case study, USAID, 2002, www.unicef.org/lifeskills/files/WhatHappenedInUganda.pdf.

[Green et al. 2006] E. C. Green, D. T. Halperin, V. Nantulya, and J. A. Hogle, "Uganda's HIV prevention success: The role of sexual behavior change and the national response", *AIDS and Behavior* **10**:4 (2006), 335–346.

[Joshi et al. 2008] H. Joshi, S. Lenhart, K. Albright, and K. Gipson, "Modeling the effect of information campaigns on the HIV epidemic in Uganda", *Math. Biosci. Eng.* **5**:4 (2008), 757–770. MR 2478986 Zbl 1154.92037

[Uganda 2010] "UNGASS country progress report: Uganda", progress report, Government of Uganda, 2010, http://tinyurl.com/962kucq.

[UNAIDS 2009] UNAIDS, "Epidemiological fact sheets on HIV/AIDS and sexually transmitted infections. 2009 update", technical report, UNAIDS, 2009.

[UNICEF 2010] "Uganda: Statistics", web page, 2010, http://www.unicef.org/infobycountry/uganda_statistics.html.

[WHO 2010] World Health Organization, "Global summary of the AIDS epidemic", web chart, 2010, http://www.who.int/hiv/data/2011_epi_core_en.png.

Received: 2011-05-12 Revised: 2012-05-29 Accepted: 2012-06-22

margeviciusr1@xavier.edu *Department of Mathematics and Computer Science, Xavier University, Cincinnati, OH 45207, United States*

joshi@xavier.edu *Department of Mathematics and Computer Science, Xavier University, Cincinnati, OH 45207, United States*

involve

msp.org/involve

See inside back cover or msp.org/involve for submission instructions. The subscription price for 2013 is US \$105/year for the electronic version, and \$145/year (+\$35, if shipping outside the US) for print and electronic. Subscriptions, requests for back issues from the last three years and changes of subscribers address should be sent to MSP.

Involve (ISSN 1944-4184 electronic, 1944-4176 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840, is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

Involve peer review and production are managed by EditFLOW® from Mathematical Sciences Publishers.

PUBLISHED BY

mathematical sciences publishers

nonprofit scientific publishing

http://msp.org/

involve

2013 vol. 6 no. 2