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CORRECTION TO "ON CONJUGATE LOCI AND CUT LOCI OF COMPACT SYMMETRIC SPACES I"

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In our previous paper [3], Lemma 3.1 was incomplete. The correct statement is as follows.

LEMMA 3.1. *Let* $H\in\tilde{C}\cap\bar{S}$. *If* $H\notin\tilde{Q}$, *we have* $\bar{S}\cap\Gamma-\{0\}\neq\emptyset$ *and*

$$m_{\bar{S}\cap\Gamma}(H)=|H|.$$

PROOF. By Theorem 3.1 there exists $A\in\Gamma-\{0\}$ such that $|H|=|H-A|$ and $|H|\leqq|H-A'|$ for any $A'\in\Gamma$. Put $H'=H-A$. Then the argument in the proof of Theorem 2.7 in Sakai [2] shows

$$|2(\gamma, H')|\leqq 1 \qquad \text{for any } \gamma\in\Sigma .$$

On the other hand, $H\in\bar{S}-\tilde{Q}$ implies

$$0\leqq 2(\gamma, H)<1 \qquad \text{for any } \gamma\in\Sigma_{+} .$$

Thus, recalling that $2(\gamma, A)=2(\gamma, H)-2(\gamma, H')\in \boldsymbol{Z}$ for any $\gamma\in\Sigma$, we get $2(\gamma, A)=0$, 1 or -1 for any $\gamma\in\Sigma$, and hence we have

$$(*) \qquad \gamma\in\Sigma,\ (\gamma, A)>0 \Longrightarrow (\gamma, H)\geqq 0.$$

Now we define

$$\mathfrak{B}=\{\gamma\in\Sigma\,;\ (\gamma, A)>0 \quad \text{or} \quad (\gamma, A)=0,\ (\gamma, H)\geqq 0\}.$$

Then $\mathfrak{B}$ is a closed system of roots containing γ or $-\gamma$ for each $\gamma\in\Sigma$. Thus by a characterization of Borel-Hirzebruch ([1], Corollary 4.10) for systems of positive roots, which is valid also for a general (not necessarily reduced) root system, there exists an order $>'$ on $\mathfrak{a}'$ such that $\mathfrak{B}$ contains the set Σ'_{+} of all positive roots with respect to $>'$. By (*) we have then

$$0\leqq 2(\gamma, A)\leqq 1, \quad 0\leqq 2(\gamma, H)<1 \qquad \text{for any } \gamma\in\Sigma'_{+} .$$

Therefore

$$S'=\{h\in\mathfrak{a}\,;\ 0<2(\gamma, h)<1 \quad \text{for any } \gamma\in\Sigma'_{+}\}$$

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is a fundamental cell whose closure $\bar{S}'$ contains 0, A and H. Since $H\in\bar{S}\cap\bar{S}'$, we can find $s\in W$ such that $sS'=S$ and $sH=H$. Put $B=sA$. Then $B\in\bar{S}\cap\Gamma-\{0\}$ and

$$|H-B|=|s(H-A)|=|H-A|=|H|.$$

This completes the proof.

References

[1] Borel, A. and Hirzebruch, F., Characteristic classes and homogeneous spaces I, Amer. J. Math. **80** (1958), 459-538.

[2] Sakai, T., On cut loci of compact symmetric spaces, Hokkaido Math. J. **6** (1977), 136-161.

[3] Takeuchi, M., On conjugate loci and cut loci of compact symmetric spaces I, Tsukuba J. Math. **2** (1978), 35-68.

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