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ERRATUM TO "ASYMPTOTIC DIMENSION AND BOUNDARY DIMENSION OF PROPER CAT(0) SPACES"

By

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Abstract. The review on [1] in Mathematical Reviews points out that the proof of its main result is incorrect. The aim of this paper is to correct the previous paper's argument and clarify the statement.

In [2] it is stated that the proof of [1, Theorem 1.1] is incorrect, i.e., the map f does not satisfy $(*)_\rho$, as claimed on line 4 of the first paragraph on [1, p. 188]. In fact, diam $f(B(\psi^{i_k}(x_0), 1)) = \operatorname{diam} a_1(B(x_0, 1)) \neq 0$ for each $k \in \mathbf{N}$. In this paper, we redefine the map $f = \bigcup_{k \in \mathbf{N}} f_k : (Y, \rho) \to (\mathbf{B}^{n+1}, \sigma)$, in particular $f_k : \psi^{i_k}(B(x_0, k)) \to \mathbf{B}^{n+1}$, where let $B(x_0, r) = \{y \in X : d(x_0, y) \leq r\}$ for $r > 0$.

Let (X, d) be a proper CAT(0) space and let $\psi : (X, d) \to (X, d)$ be an isometry satisfying that $\{\psi^i(x) : i \in \mathbf{Z}\}$ is unbounded (see [1, Theorem 1.1]). Fix a point x_0 of X. For every $x \in X$, let $\xi_x : [0, d(x_0, x)] \to X$ be the geodesic from x_0 to x in (X, d). Recall the projection map $p_1 : X \to B(x_0, 1)$ in [1, p. 187] defined by $p_1(x) = \xi_x(\min\{d(x_0, x), 1\})$ for each $x \in X$.

Since $\{\psi^i(x) : i \in \mathbf{Z}\}$ is unbounded, we have a sequence $i_1, i_2, \dots$ of $\mathbf{N}$ satisfying an additional condition: $d(\psi^{i_k}(B(x_0, k)), \psi^{i_{k'}}(B(x_0, k'))) > \max\{k, k'\}$ whenever $k \neq k'$ (see the second line from the bottom of [1, p. 187]). For every $k \in \mathbf{N}$, now we define a continuous map $q_k : B(x_0, k) \to B(x_0, 1)$ by $q_k(x) = \xi_x(d(x_0, x)/k)$ for each $x \in B(x_0, k)$. Here, we *redefine* the map f_k in the first line of [1, p. 188] by a map $a_1 \circ q_k \circ \psi^{-i_k} : \psi^{i_k}(B(x_0, k)) \to \mathbf{B}^{n+1}$. Let $Y = \bigcup_{k \in \mathbf{N}} \psi^{i_k}(B(x_0, k))$ and let $f = \bigcup_{k \in \mathbf{N}} f_k : Y \to \mathbf{B}^{n+1}$.

We see that $p_1|_{B(x_0, k)}$ is homotopic to q_k. Indeed, we have a homotopy $H : B(x_0, k) \times [0, 1] \to B(x_0, 1) : p_1|_{B(x_0, k)} \simeq q_k$ defined by

$$H(x, t) = \xi_x((d(x_0, x)/k - \min\{d(x_0, x), 1\})t + \min\{d(x_0, x), 1\})$$

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for each $(x,t) \in B(x_0,k) \times [0,1]$. Thus, $0 \neq [a_k] = [a_1 \circ p_1] = [a_1 \circ q_k] \in H^{n+1}(B(x_0,k), S(x_0,k))$, where $a_k : (B(x_0,k), S(x_0,k)) \to (\mathbf{B}^{n+1}, \mathbf{S}^n)$ is the map in the fifth line from the bottom of [1, p. 187]. Therefore, every f_k is essential.

We show that $f : (Y,\rho) \to (\mathbf{B}^{n+1}, \sigma)$ has property $(*)_\rho$: for every $r > 0$ and every $\varepsilon > 0$, there exists a compact set K of Y such that $\operatorname{diam} f(B(x,r) \cap Y) < \varepsilon$ for all $x \in Y \backslash K$. Here $\rho = d|_Y$ and σ is the usual metric of $\mathbf{B}^{n+1}$. Let $r > 0$ and let $\varepsilon > 0$. By the uniformly continuity of a_1, there exists $\delta > 0$ such that for every $E \subset B(x_0,1)$ with $\operatorname{diam} E < \delta$, $\operatorname{diam} a_1(E) < \varepsilon$. We see that for every $r > 0$ there exists $k_0 \in \mathbf{N}$ with $k_0 > \max\{r, 4r/\delta\}$ such that for every $k \geq k_0$ and every $D \subset B(x_0,k)$ with $\operatorname{diam} D \leq 2r$, $\operatorname{diam} q_k(D) < \delta$. Let $k \geq k_0$ and let $x, y \in B(x_0,k)$ with $d(x,y) \leq 2r$. We note that $q_k(x) = \xi_x(d(x_0,x)/k)$ and $q_k(y) = \xi_y(d(x_0,y)/k)$. By the comparison triangle for $\triangle(x_0,x,y)$, we have $d(q_k(x), q_k(y)) \leq d(x,y)/k \leq 2r/k < \delta/2$. Thus, every $D \subset B(x_0,k)$ with $\operatorname{diam} D \leq 2r$ satisfies that $\operatorname{diam} q_k(D) < \delta$. Let $K = \bigcup_{k=1}^{k_0-1} \psi^{i_k}(B(x_0,k))$, let $x \in Y \backslash K$ and let $k \in \mathbf{N}$ with $k \geq k_0$ such that $x \in \psi^{i_k}(B(x_0,k))$. Since $B(x,r) \cap Y \subset \psi^{i_k}(B(x_0,k))$, $\psi^{-i_k}(B(x,r) \cap Y) = B(\psi^{-i_k}(x), r) \cap B(x_0,k) \subset B(x_0,k)$. Since $\operatorname{diam} \psi^{-i_k}(B(x,r) \cap Y) \leq 2r$, by the above we see that $\operatorname{diam} q_k \circ \psi^{-i_k}(B(x,r) \cap Y) < \delta$. Hence $\operatorname{diam} f(B(x,r) \cap Y) = \operatorname{diam} f_k(B(x,r) \cap Y) = \operatorname{diam} a_1(q_k \circ \psi^{-i_k}(B(x,r) \cap Y)) < \varepsilon$. Therefore, the map f has property $(*)_\rho$.

Thus, there exists an extension $\bar{f} : \overline{Y}^\rho \to \mathbf{B}^{n+1}$ of f. By the same manner on the second paragraph of [1, p. 188], we can show that $g = \bar{f}|_{\nu_\rho Y} : \nu_\rho Y \to \mathbf{B}^{n+1}$ is essential because every f_k is essential. Therefore, $\dim \nu_d X \geq \dim \nu_\rho Y \geq n+1$.

References

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