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Modeling of nonstationarity and long memory with RS-ARFIMA-GARCH model

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Abstract. We consider in this study the problem of confusion between the nonstationarity and the long memory. Many authors have pointed out, in empirical case, the existence of long memory in financial and economics time series, through processes supposed short memory stationary (See Mikosch and Stáricá (2004) and Lobato and Savin (1998)). This existence has been proved as being the consequence of nonstationarity, which is the non constancy of the unconditional variance or the changes in the mean of the series. The objective of this article is to find a model likely to take into account nonstationarity and long memory.

Key words: ARFIMA, GARCH, Regime Switching, homogeneity intervals, MCMC.
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Résumé. On considère dans cette étude le problème de confusion entre la non stationnarité et la longue mémoire. Plusieurs auteurs ont signalé, de façon empirique, l'existence d'un comportement de longue mémoire dans des séries économiques et financières, à travers des processus supposés stationnaires avec courte mémoire (voir Mikosch and Stáricá (2004) and Lobato and Savin (1998)). Son existence a été démontrée comme étant la conséquence de la non stationnarité, c'est à dire la non constance de la variance inconditionnelle ou les changements dans la moyenne de ces séries. L'objectif de cet article est de trouver un modèle capable de prendre en compte à la fois la nonstationnarité et la longue mémoire.

1. Introduction

The phenomena of nonstationarity and long memory observed on the financial series, of the exchange rate for example, constitute a subject which draws the attention of several researchers, Lobato and Savin (1998), Granger and Hyung (2000), Breidt and Hsu (2002), Bisaglia and Gerolimetto (2009) etc. In the statistical modeling of the financial series, we often have difficulty in distinguishing what is a matter of the nonstationarity and what is a matter of the long memory.

These two behaviors can be confidentially connected. Studies showed the existence of dependence long memory on the series submitted to structural changes. By consulting Lobato and Savin (1998) several possible sources of presence of long memory, in financial series presenting structural changes, were quoted: it is about the nonstationarity, the aggregation of series, the seasonal component of long memory, the distortion in size, the non-existence of higher order moments. Among articles that highlight such sources, we can particularly cited that of Breidt and Hsu (2002) with the distortion in size, that of Lamoureux and Lastrapes (1990) with the nonstationarity engendered by the changes of regime at the level of unconditional volatility,that of, Mikosch and Stáricá (2004) which, by proceeding to a concatenation of samples, outcomes of various stationaries models, showed that the resultant series, besides nonstationary, presents a long memory if we consider squares or absolute values of this values. So the questions that we ask ourselves are: what type of long memory have we got? Have we got a long memory resulting from a non stationarity: change in the unconditional variance or in the average? Or is it about the long memory which supposes that the events going back up of a distant past have an effect on the dynamics of the series? In the sequel we shall suppose only the first case. Indeed, with regard to this case Ding *and al.* (1993) have concluded that the nonstationarity can be a plausible explanation of the presence of long memory in series assumed stationaries.

In this work we are interested in a joint modeling of the nonstationarity and the long memory observed frequently in a empirical way on financial series (or their transformation adequate) supposed stationaries with short memory with structural changes. The object of this article is to look for the most adequate model to describe the nonstationarity and the long memory at the same time.

Because of path dependence of the conditional variances (the conditional variance depends on the whole past history of the state variables), maximum likelihood estimation is infeasible. By enlarging the parameter space to include the state variables, Markov chain Monte Carlo (MCMC) is feasible.

The article is organized as follows: the Section 2 introduce RS-ARFIMA-GARCH model. In this section, we recall the notion of long memory and the definitions of ARFIMA model and of regime switching Markov model. In Section 3, we explain how the model can be estimated in the MCMC framework. Section 4 is devoted to the simulation study. In Section 5, we apply our approach to a long time series of returns of exchange rate of the US Dollar (USD) towards the Euro (EURO).

2. The model

As it is henceforth widely allowed, the short memory stationary processes with structural changes can possess a long memory property. That is why, to work with the unconditional volatility while considering the long memory as of false nature, it is natural to use a regime switching or jump model. But, if we work with the structural changes in the variance and the long memory, we can consider an approach which takes into account both phenomena, in the sense where we introduce the long memory into type regime switching model to take into account the slow decay of the sample autocorrelation function as for example in the case of the absolute returns of the $S\&P500$: the model *RS-ARFIMA-GARCH* for example.

2.1. Long memory and ARFIMA model

The processes with long memory, appeared in the years 1895 from the observations of the astronomer Newcomb then of the chemist Student (1927), were initially reserved for very specific domains (hydrology, turbulence). The applications of such models multiplied in the years 1990, under the influence of several pioneers works showing the presence of the phenomena to long memory in the economic and financial series, for example on series of exchange rate (Cheung (1993b), Ferrara and Guégan (2000a)), on the asset prices quoted in stock exchange (Willinger *and al.* (1999)), on the electricity spot price (Diongue *and al.* (2003)), on London Stock Exchange index (FTSE) (Yanlin and Kin-Yip (2014)). Let us consider a price process $(X_t)_{t\in\mathbb{Z}}$ with autocorrelation function, noted ρ_X, define, for all $k \in \mathbb{Z}$, by

$$\rho(k) = \frac{\gamma_X(k)}{\gamma_X(0)} \tag{1}$$

where $\gamma_X(.)$ is the autocovariance function associated to the process X_t. In the literature several definitions of long memory exist, we can cite, in particular, the long memory in the covariance sense, the long memory in the sense of the spectral density, the long memory in the Allan variance sense and the long memory which based on the concept of mixture which allows to make the data asymptotically independent, see Rosenblatt (1956). The last two definitions are however respectively little useful and very difficult to use in the practice. But the first two definitions resting on the covariance and on the spectral density are most used in the practice. So from these two tools, we have the following definitions:

Definition 1.
In the temporal domain, a stationary process $(X_t)_{t\in\mathbb{Z}}$ is said long memory process if its autocorrelation function ρ is not absolutely summable, that is

$$\sum_{j=-\infty}^{\infty} |\rho(k)| = \infty$$

An example of such a process is given by the fractional processes. The fractional process $(X_t)_{t\in\mathbb{Z}}$ the simplest is the one under the form

$$(1-B)^d X_t = \varepsilon_t$$

where ε_t is white noise with mean 0 and variance σ^2, written $\varepsilon_t \sim WN(0,\sigma^2)$ and $(1-B)^d$ the difference operator.
For $0 < d < \frac{1}{2}$ and a constant, $c > 0$, we show that

$$\rho(k) \sim ck^{2d-1} \quad \text{when } k \to \infty \tag{2}$$

This equation with long memory are characterized by an autocorrelation function decreasing in an hyperbolic way towards zero.

Definition 2.
In the frequency domain, a process $(X_t)_{t\in\mathbb{Z}}$ possesses the long memory property if its spectral density f increases without limit when the frequency aims towards zero, i.e.

$$f(\lambda) = \infty, \quad \text{when } \lambda \to 0^+$$

Particularly, $(X_t)_{t\in\mathbb{Z}}$ fractional process is said to have long memory if $0 < d < \frac{1}{2}$ and verifies

$$f(\lambda) \sim c'|\lambda|^{-2d} \tag{3}$$

where $f(\lambda)$ is the spectral density of the process $(X_t)_{t\in\mathbb{Z}}$ at the frequency λ and c' a positive constant.

To take into account the presence of a long memory, we use the Autoregressive Fractionally Integrated Moving Average (*ARFIMA*) process which with the fractional gaussian noise (Mandelbrot and Van Ness (1968)), are the examples the most often evoked by long memory process. The *ARFIMA*(p,d,q) model is define by:

$$\Phi(B)(I-B)^d(X_t-\mu) = \Theta(B)\varepsilon_t, \tag{4}$$

where d is a fractional number, μ is the mean of the process $(X_t)_{t\in\mathbb{Z}}$, the polynomials $\Phi(B) = 1-\phi_1 B - \phi_2 B^2 - .... - \phi_p B^p$ and $\Theta(B) = 1-\theta_1 B - \theta_2 B^2 - .... - \theta_q B^q$ have no common zeroes and have their zeroes outside the unit circle. Here, the process $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a white noise with mean 0 and variance σ_ε^2 and B a backward shift operator $BX_t = X_{t-1}$. This model which was proposed by Granger and Joyeux (1980) and Hosking (1981) to model a behavior of persistent long memory, is a generalization of the models *ARIMA* of Box and Jenkins (1976). It allows to take into account in a modeling at the same time the short-term behavior through the autoregressive and the moving average parameters and the long term behavior by means of the parameter of fractional integration.

2.2. Regime switching Markov model

This model was introduced into the literature by Hamilton in 1989 to take into account the existence of structural changes not visible to the naked eye in the studied financial and economic series.
Let us consider a financial asset A; let X_t be its price model expressing as follows:

$$X_t = m_t + \varepsilon_t \tag{5}$$

where m_t is the mean of the process and ε_t a strong white noise.

Let us suppose that the variable m_t follows several behavior over the period analyzed $[0, T]$, we thus obtain a change of state on the level of the asset price.
Let us suppose that there are two regimes governed by an economic variable, for example a regime of high volatility and a regime of low volatility. Thus m_t depends on the regime on which the process is. Let $s_t \in \{1, 2\}$ the economic variable, representing the regimes at time t. In that case the average of the price is m_{s_t}. The equation (5) becomes,

$$X_t = m_{s_t} + \varepsilon_t, \tag{6}$$

and

$$m_{s_t} = (1 - s_t)m_0 + s_t m_1,$$

where probably $m_0 \leq m_1$, and where ε_t is a gaussian white noise of zero mean and finite variance σ^2 ($\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2)$), and where s_t and ε_t are independent for all t.

It often happens that the variable s_t is not observable in practice. Consequently, the model (6) cannot be estimated because the current regime s_t depends on the previous regime s_{t-1}. To remedy this situation, we use the probability of passage from a regime to the other one (it is about a joint probability). The problem is completely defined if the transition probabilities between the various regimes are known. Let p be a probability associated to the changes of state,

$$P(s_t|s_{t-1}) = \begin{cases} p_{00} & \text{if } s_t = s_{t-1} = 0 \\ 1 - p_{00} & \text{if } s_t = 1, s_{t-1} = 0 \\ p_{11} & \text{if } s_t = s_{t-1} = 1 \\ 1 - p_{11} & \text{if } s_t = 0, s_{t-1} = 1, \end{cases} \tag{7}$$

where $P(s_t|s_{t-1})$ represent the probability to be in the regime s_t at time t conditionally to the previous regime. Let M be the transition matrix of the chain s_t which characterizes the variable m_{s_t}, M is defined by:

$$\mathbf{M} = \begin{pmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{pmatrix}.$$

So, we find Hamilton regime switching model (1989) extended afterward to the Markov regime switching model. In the sequel we suppose that $p_{00} = p_{11}$, that is to say the regimes have the same probability to occur. The autocorrelation function of the Markov regime switching process (6) decreases exponentially fast toward zero, identical to that of a GARCH (Timmermann (2000)). This autocorrelation function was studied by Guegan and Rioublanc (2005) and is given by :

$$\Gamma(h) = \frac{(m_0 - m_1)^2(1 - p_{00})(1 - p_{11})\rho^h}{(2 - p_{00} - p_{11})^2[\pi_1 m_0^2 + \pi_2 m_1^2 + 1 - (\pi_1 m_0 + \pi_2 m_1)^2]}, \ \forall\, h > 0 \tag{8}$$

where

$$\rho = -1 + p_{00} + p_{11},$$

$$\pi_1 = P(s_t = 0) = \frac{1 - p_{11}}{2 - p_{00} - p_{11}} \quad et \quad \pi_2 = P(s_t = 1) = \frac{1 - p_{00}}{2 - p_{00} - p_{11}},$$

are the non conditional probabilities.
The autocorrelation function $\Gamma(h)$, can be rewritten as :

$$\Gamma(h) = C_{\mu_i, p_{ii}} \rho^h \ , \quad i = 1, 2,$$

with

$$C_{\mu_i, p_{ii}} = \frac{(m_0 - m_1)^2 (1 - p_{00})(1 - p_{11})}{(2 - p_{00} - p_{11})^2 [\pi_1 m_0^2 + \pi_2 m_1^2 + 1 - (\pi_1 m_0 + \pi_2 m_1)^2]} \ , \quad i = 1, 2,$$

2.3. Regime switching ARFIMA-GARCH model

Let ε_t be a white noise of mean 0 and conditional variance σ_t^2 define as following:

$$\sigma_t^2 = \alpha_{0s_t} + \sum_{i=1}^{p_2} \alpha_{is_t} \varepsilon_{t-i}^2 + \sum_{j=1}^{q_2} \beta_{js_t} \sigma_{t-j}^2$$

Let $\Phi_{s_t}(B)$ and $\Theta_{s_t}(B)$ the p_1^{th} and q_1^{th} degree autoregressive and moving average polynomials respectively with real coefficients dependant to the chaine s_t.
A process $(X_t)_{t \in \mathbb{Z}}$ is called regime switching fractional integrated Garch process, noted
RS-ARFIMA(p_1,d,q_1)-GARCH(p_2, q_2), if the following equation is satisfied

$$\Phi_{s_t}(B)(1 - B)^d (X_t - m_{s_t}) = \Theta_{s_t}(B) \varepsilon_t \tag{9}$$

where

$$m_{s_t} = (1 - s_t) m_0 + s_t m_1,$$

$$\Phi_{s_t}(B) = 1 - \sum_{i=1}^{p} \phi_{i,s_t} B^i, \quad \Theta_{s_t}(B) = 1 - \sum_{j=1}^{q} \theta_{j,s_t} B^j$$

$$\varepsilon_t \overset{iid}{\sim} WN(0, \sigma_t)$$

and where

- the long memory parameters d is fractional, B is the backward operator
- $m_0 \leq m_1$,
- s_t and ε_t are independents for all t.
- the unconditional variance of every subperiod correspond in every homogeneity intervals of the series is given by $\frac{\alpha_{0j}}{1 - \sum_{i=1}^{p_2} \alpha_{ij} - \sum_{i=1}^{q_2} \beta_{ij}}$, $j = 1, 2..., N$, N : number of homogeneity intervals.
- α_{ij} et β_{ij} are coefficients of the process σ_t^2.

We remind that the variance σ_t^2 is conditional with regard to all the past of the noise ε_τ and the trajectory of the regime $\tilde{s}_{\tau-1} = (s_{\tau-1}, s_{\tau-2}, ...)$ which is not observed, for $\tau < t$, that is to say the conditional variance is defined as follows : $\sigma_t^2 = V(\varepsilon_t | F_{t-1})$,

where
$F_{t-1} = \sigma(\varepsilon_{\tau_1}, \tilde{s}_{\tau_2}, \tau_1 < t, \tau_2 \leq t)$.

Notice that this model is a direct generalization of the *ARFIMA* model of Granger and Joyeux (1980), it contains several particular cases :

- The model of constant mean and unconditional variance, which can be formalized as following :

$$X_t = m + \varepsilon_t \tag{10}$$

with

$$\varepsilon_t \overset{iid}{\sim} WN(0, \sigma^2)$$

when $d = 0$, $p_1 = q_1 = 0$ et $p_2 = q_2 = 0$.

- The regime switching Markov which is formalized in the following way :

$$X_t = m_{s_t} + \varepsilon_t \tag{11}$$

with

$$m_{s_t} = (1 - s_t)m_0 + s_t m_1, \quad \varepsilon_t \overset{iid}{\sim} WN(0, \sigma^2)$$

when $d = 0$, $p_1 = q_1 = 0$ et $p_2 = q_2 = 0$.

- The Markov-Garch(1,1) model, which we can call also regime switching *GARCH (RS-GARCH(1,1))* :

$$X_t = m_{s_t} + \sigma_t \varepsilon_t \tag{12}$$

with

$$m_{s_t} = (1 - s_t)m_0 + s_t m_1,$$

$$\varepsilon_t \overset{iid}{\sim} WN(0, 1)$$

$$\sigma_t^2 = \alpha_{0s_t} + \alpha_{1s_t} X_{t-1}^2 + \beta_{1s_t} \sigma_{t-1}^2$$

and the unconditional variance of every subperiod of the series given by $\dfrac{\alpha_{0j}}{1 - \sum_{i=1}^{p_2} \alpha_{ij} - \sum_{i=1}^{q_2} \beta_{ij}}$, $j = 1, 2$, $p_1 = q_1 = 0$ and $p_2 = q_2 = 1$

- The regime switching-*ARIMA* model which is formalized as following:

$$\Phi_{s_t}(B)(1 - B)^d (X_t - m_{s_t}) = \Theta_{s_t}(B) \varepsilon_t \tag{13}$$

with

$$m_{s_t} = (1 - s_t)m_0 + s_t m_1 \ \text{ and } \ \varepsilon_t \overset{iid}{\sim} WN(0, \sigma^2)$$

σ^2 corresponding at unconditional variance of X_t, d an integer and $p_2 = q_2 = 0$.

- The regime switching *ARFIMA* model which is formalized as following:

$$\Phi_{s_t}(B)(1 - B)^d (X_t - m_{s_t}) = \Theta_{s_t}(B) \varepsilon_t \tag{14}$$

with

$$m_{s_t} = (1 - s_t)m_0 + s_t m_1 \ \text{ and } \ \varepsilon_t \overset{iid}{\sim} WN(0, \sigma^2)$$

σ^2 corresponding at unconditional variance of X_t, d a fractional number and $p_2 = q_2 = 0$.

In the present paper, we focus our attention on time series models with $p_2 = q_2 = 1$ given by

$$\Phi_{s_t}(B)(1-B)^d(X_t - m_{s_t}) = \Theta_{s_t}(B)\varepsilon_t \tag{15}$$

with

$$m_{s_t} = (1-s_t)m_0 + s_t m_1,$$

$$\varepsilon_t \overset{iid}{\sim} WN(0,\sigma_t)$$

$$\sigma_t^2 = \alpha_{0s_t} + \alpha_{1s_t}X_{t-1}^2 + \beta_{1s_t}\sigma_{t-1}^2. \tag{16}$$

The short memory parameters found in $\Phi_{s_t}(B)$ and $\Theta_{s_t}(B)$ could be modeled as functions of s_t. Since these parameters only affect the short-run dynamics of the process and our main interest is to study the estimation of the long memory parameters, thus we can set the short memory parameters to be zero, that's mean $p_1 = q_1 = 0$ and we obtain

$$(1-B)^d(X_t - m_{s_t}) = \varepsilon_t. \tag{17}$$

The polynomial $(I-B)^d$ admits the following development :

$$(I-B)^d = \sum_{k\geq 0} b_k(d)B^k$$

where

$$b_k(d) = \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)}.$$

with the function Γ define as following :

$$\begin{aligned}\Gamma(a) &= \int_0^\infty x^{a-1}e^{-x}dx, \quad \text{for all real } a > 0,\\ &= a^{-1}\Gamma(a+1), \quad \text{for } a < 0.\end{aligned}$$

According to Hosking (1981), one can easily show that

$$b_k(d) = \frac{k-1-d}{k}b_{k-1}(d), \qquad b_0(d) = 1 \tag{18}$$

Under the condition of stationarity : $d < \frac{1}{2}$, it exists then a unique stationary solution to the equation (15), and in the case of a fractionally integrated process ($\Phi_{s_t}(B) = \Theta_{s_t}(B) = 1$), the process $(X_t)_{t\in\mathbb{Z}}$ define by (17) can be written under its form moving average infinite form :

$$\begin{aligned}X_t &= m_{s_t} + (1-B)^{-d}\varepsilon_t\\ &= m_{s_t} + \sum_{k\geq 0} b_k(d)\varepsilon_{t-k}\end{aligned} \tag{19}$$

where $b_k(d)$ and $b_0(d)$ are defined in (18).

3. Estimation

3.1. Identification of intervals of homogeneity

The estimation of the parameters being made on subperiods, it is beforehand necessary to identify the intervals of homogeneity. For it we use the spectral test of Starica and Granger (2005). We consider the following linear process

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j \varepsilon_{t-j} \tag{20}$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ a centered white noise with variance σ^2, and

$$\psi(z) = \sum_{j=-\infty}^{\infty} \psi_j z^j$$

with $(\psi_j)_{j\in\mathbb{Z}}$ a sequence real number absolutely summable.

To detect the intervals of homogeneity, Stărică and Granger proposes the following test statistic :

$$T(n, X, \mathcal{M}_{\mu,\sigma^2,f_\psi}) = \sup_{\lambda\in[0,\pi]} \left| \int_{-\pi}^{\lambda} \left(\frac{I_{n,X}(y)}{f_\psi(y)} - \frac{\widehat{\sigma}^2}{\sigma^2} \right) dy \right| \tag{21}$$

where

$$I_{n,X}(\lambda) = \gamma_{n,X}(0) + 2\sum_{i=1}^{n-1} \gamma_{n,X}(h) cos(\lambda h) \tag{22}$$

represent the natural estimator of the spectral density f_X of the process $(X_t)_{t\in\mathbb{Z}}$, with $\gamma_{n,X}$ his autocovariance function, $f_\psi(.)$ defined by

$$f_\psi(\lambda) = \frac{\sigma^2}{2\pi} \left| \psi(e^{-i\lambda}) \right|^2 \quad \lambda \in [0, \pi], \tag{23}$$

the spectral density function of the linear process (20) and

$$\widehat{\sigma}^2 = \int_{-\pi}^{\pi} \frac{I_{n,X}(z)}{|\psi(e^{-iz})|^2} dz \tag{24}$$

estimator of σ^2 variance of the noise $(\varepsilon_t)_t$.

On the basis of the following Theorem, we notice that the asymptotic distribution of the test statistic (21) is connected to Brownien motion[1].

Theorem 1. ***(Kluppelberg* & *Mikosch, 1996).***
Assume that $E(\varepsilon_t) = 0$, $E(\varepsilon_t^4) < \infty$, and denote $Var(\varepsilon_t) = \sigma^2$.
Let X_t denote the linear processes (20) and $\widehat{\sigma}^2$ the estimate of σ^2 defined in equation (24). Then the following holds

$$\sqrt{n} \int_{-\pi}^{\lambda} \left(\frac{I_{n,X}(y)}{f_\psi(y)} - \frac{\widehat{\sigma}^2}{\sigma^2} \right) dy \xrightarrow{d} \pi B(\frac{\lambda}{\pi}) \quad in \ \mathbb{D}([0, \pi]) \tag{25}$$

where f_ψ defined in equation (23) represent the spectral density function of the linear process (20) and $B(.)$ is a Brownien bridge.

[1] A Brownian bridge on [0,1] is defined as $B(\lambda) = W(\lambda) - \lambda W(1)$ where W is a standard Brownian motion.

The following corollary yields the critical values for the hypothesis testing central to the methodology explained above.

Corollary 1.
Under the hypothesis and with the notation of Theorem 1, we have the following:
(a) If X_t is the linear processes (20), then

$$\sqrt{n} \sup_{\lambda\in[0,\pi]} \left| \int_{-\pi}^{\lambda} \left(\frac{I_{n,X}(y)}{f_\psi(y)} - \frac{\widehat{\sigma}^2}{\sigma^2} \right) dy \right| \xrightarrow{d} \pi \sup_{\lambda\in[0,\pi]} |B(\frac{\lambda}{\pi})| \tag{26}$$

(b) If X_t is a white noise, denote $\tilde{\sigma}^2 = \frac{1}{n}\sum_{t=1}^{n} X_t^2$, then

$$\sqrt{n} \int_{-\pi}^{\lambda} \left(\frac{I_{n,X}(y)}{\frac{\sigma^2}{2\pi}} - \frac{\tilde{\sigma}^2}{\sigma^2} \right) dy \xrightarrow{d} \pi B(\frac{\lambda}{\pi})$$

$$\sqrt{n} \sup_{\lambda\in[0,\pi]} \left| \int_{-\pi}^{\lambda} \left(\frac{I_{n,X}(y)}{\frac{\sigma^2}{2\pi}} - \frac{\tilde{\sigma}^2}{\sigma^2} \right) dy \right| \xrightarrow{d} \pi \sup_{\lambda\in[0,\pi]} |B(\frac{\lambda}{\pi})| \tag{27}$$

in $\mathbb{D}([0,\pi])$, *where* $B(.)$ *is a Brownien bridge defined on* $[0,1]$.

In the sequel, the linear process (20) with mean μ, noise variance σ^2, and spectral density f_ψ will be compactly denoted by $\mathcal{M}_{\mu,\sigma^2,f_\psi}$.

One assumes that the sample X_1,...,X_n, generated by X_t, presents subsamples $X_1^{(1)},...,X_{n_1}^{(1)},...,X_{n_r}^{(r)},...,X_n^{(r)}$ different which we suppose stationaries. The intervals of homogeneity on $X_1,...,X_n$ are constructed as follows. Let us consider the subsample $X_1^{(1)},...,X_{n_1}^{(1)}$, assume that he is described by $\mathcal{M}_{\mu,\sigma^2,f_\psi}$ a linear parametric model with mean μ, noise variance σ^2 and spectral density f_ψ. We want to decide if the interval of homogeneity containing the observations $X_1^{(1)}$ to $X_{n_1}^{(1)}$ can be extend with p observations, $X_{n_1+1},...,X_{n_1+p}$, that is if p observations, $X_{n_1+1},...,X_{n_1+p}$, also belong to the interval. To accomplish this, we use the statistical test $T(n,X,\mathcal{M}_{\mu,\sigma^2,f_\psi})$ which consists to test if the linear model $\mathcal{M}_{\mu,\sigma^2,f_\psi}$ fits well to the subsample X_{n_1+p-s}, X_{n_1+p} that contains p new points of data (s, a number constant, is the size of the subsample on which the test is conducted).

3.2. Estimation of parameters of the model

For the estimation, we use a Bayesian Markov chain Monte Carlo (MCMC) methods that circumvents the problem of path dependence by including the state variables in the parameter space. This method allows us to treat the latent state variables as parameters of the model and to construct the likelihood function assuming we know the states. This technique is called data augmentation, see Tanner and Wong (1987) and Dufays (2012). The properties of the estimator compare favorably with other approaches. It is straightforward to obtain smoothed estimates of volatility from MCMC output. Here, we present the Bayesian algorithm for a *RS-ARFIMA-GARCH* model with the case of two regimes and normality of the error term ε_t, and in Section 4, we illustrate that it recovers correctly the parameters of a simulated data generating process.

We denote by X_t the vector $(x_1, x_2, ..., x_t)$ and likewise $S_t = (s_1, s_2, ..., s_t)$ with transition probabilities $\{p_{ij} = P(s_t = i|s_{t-1} = j)\}$. The model parameters consist of $p = (p_{00}, p_{10}, p_{12}, p_{11})$, $\mu = (\mu_0, \mu_1)$, d, and $\theta = (\theta_0, \theta_1)$, where $\theta_k = (\alpha_{0j}, \alpha_{1j}, \beta_{1j})$ for $k = 0, 1$ and $j = 0, 1$. The conditional density of x_t is the Gaussian density

$$f(x_t|X_{t-1}, S_t, \mu, d, \theta) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(- \frac{(x_t - \mu_{s_t})^2}{2\sigma_t^2} \right) \tag{28}$$

The marginal density (or probability mass function) of s_t is specified by

$$f(s_t|X_{t-1}, p, S_{t-1}) = f(s_t|p, S_{t-1}) = p_{s_t s_{t-1}}. \tag{29}$$

Indeed, let us consider the realization $X = \{x_1, x_2, ..., x_n\}$, μ, θ and the vector of states $S = (s_1, s_2, ..., s_n)$, the conditional density function of X_t, generator of the observations and the regimes, is given by

$$\begin{aligned} f(X|S, \mu, d, \theta) &= f(x_n|x_1, ..., x_{n-1}, S, \mu, d, \theta)...f(x_2|x_1, S, \mu, \theta)f(x_1|S, \mu, d, \theta) \\ &= \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(- \frac{(x_t - \mu_{s_t})^2}{2\sigma_t^2} \right), \end{aligned} \tag{30}$$

where σ_t^2 is a function of θ through s_t, defined by equation (16). This would be the likelihood function to maximize if the sates were known. Notice that it does not depend on p. Given p and X the distribution of S is given by

$$f(S|X, p) = f(S|p) = \prod_{t=1}^{n} p_{s_t s_{t-1}} \tag{31}$$

which does not depend on μ, d and θ.
The joint density of X and S given the parameters is then obtained by taking the product of the densities in (28) and (29) over all observations :

$$f(X, S|\mu, d, \theta, p) = \prod_{i=1}^{n} \frac{1}{\sigma_t} \exp\left(- \frac{(x_t - \mu_{s_t})^2}{2\sigma_t^2} \right) p_{s_t s_{t-1}}. \tag{32}$$

To implement the MCMC algorithm, we implement a Gibbs sampling algorithm that allows us to sample from the full conditional posterior densities of blocks of parameters given by θ, μ, d, p and the elements of S. We explain what our prior densities are for θ, μ, d and p when we define the different blocks of the Gibbs sampler. We note by $S_{\neq t}$ the vector S without the element s_t. The steps in the MCMC algorithm are as follows :

1. sample $s_t|S_{\neq t}, \mu, \theta, p, d, Y$
2. sample $\theta|S, \mu, p, d, Y$
3. sample $\mu|S, \theta, p, d, Y$
4. sample $d|S, \mu, \theta, p, Y$
5. sample $p|S, \mu, \theta, d, Y$
6. goto 1

A pass through 1-5 provides a draw from the posterior. We repeat this several 5000 times and collect these draws after an initial burn-in period. For detailed steps of the algorithm see Bauwens *and al.* (2010, 2013).

4. Simulation study

In this section, we make some simulations, on the one hand, to illustrate the behavior of the model, and on the other hand, to make the estimation of the parameters of the model. Because there is no known technique for generating an exact *RS-ARFIMA-GARCH*, we will approximate the infinite moving average (19) by the truncation moving average :

$$X_t = m_{s_t} + \sum_{k=1}^{M} b_k(d)\varepsilon_{t-k}, \quad t = 1, ..., n \tag{33}$$

where the non-random constants b_k are defined by (14) and M is the truncation parameter. As in Gray *and al.* (1989), we fixe $M = 29000$.
`First`, we show that this *RS-ARFIMA-GARCH* framework can also distinguish between the *ARFIMA* and *RS-GARCH* DGP$_s$. To verify that, we simulate two DGP corresponding respectively to *ARFIMA(0,d,0)* with two cases according to the parameter d :

1. $d = 0.2$,
2. $d = 0.4$

and a *RS-GARCH(1,1)* in two cases according the following means and transition probabilities :

1. *RS-GARCH(1,1)* : with $m_0 = -0.5$, $m_1 = 0.5$ with transition probabilities $p_{00} = p_{11} = 0.99$,
2. *RS-GARCH(1,1)* : withe $m_0 = -2$, $m_1 = 2$ with transition probabilities $p_{00} = 0.98$ et $p_{11} = 0.99$,

with, for every case, the parameters : $\alpha_{00} = 0.015$, $\alpha_{10} = 0.12$, $\beta_{10} = 0.25$ and $\alpha_{01} = 0.065$, $\alpha_{11} = 0.35$, $\beta_{11} = 0.18$. And we fit both the simulated data into the *RS-ARFIMA(0,d,0)GARCH (1,1)* model.
`Secondly`, to see how *RS-ARFIMA-GARCH* framework performs when the DGP is composed of both long memory and regime switching with *GARCH* noise, we simulate a *RS-ARFIMA-GARCH* : $(1 - B)^d(X_t - m_{s_t}) = \varepsilon_t$ under the two following forms :

1. *RS-ARFIMA(0,d,0)GARCH(1,1)* : with $m_0 = -0.5$, $m_1 = 0.5$, $d = 0.2$, and with transition probabilities $p_{00} = p_{11} = 0.99$.
2. *RS-ARFIMA(0,d,0)GARCH(1,1)* : with $m_0 = -2$, $m_1 = 2$, $d = 0.4$, and with transition probabilities $p_{00} = 0.98$ and $p_{11} = 0.99$.

Each simulated data set is then fitted into the original *RS-ARFIMA(0,d,0)-GARCH(1,1)* model. We plotted in Figure 1 the trajectories of the process $(X_t)_{t\in\mathbb{Z}}$ *RS-ARFIMA-GARCH* when the *GARCH* coefficients are $\alpha_{00} = 0.015$, $\alpha_{10} = 0.12$, $\beta_{10} = 0.25$ and $\alpha_{01} = 0.065$, $\alpha_{11} = 0.35$, $\beta_{11} = 0.18$ for each of the two models. In these two graphs, we observe that the underlying processes seem to be locally stationary as soon as we stay inside a regime but seem to be globally stationary if the means in absolute value become small.

After the simulation of a series, X_t, coming from each of the models listed above,

Figure 1 : Trajectory of the simulated models, n=1000.

we make the estimations of the parameters by Markov Chain Monte Carlo (MCMC) methods. In Table 4.1 and Table 4.2, we report the estimations of the parameters for the models corresponding to the two DGP.

Estimation of parameters of simulated *ARFIMA(0,d,0)* and *RS-GARCH(1,1)* data fitted into *RS-ARFIMA(0,d,0)-GARCH(1,1)* model

Table 4.1

	DGP ARFIMA	RS-ARFIMA-GARCH		DGP RS-GARCH		RS-ARFIMA-GARCH	
s_t		0	1	0	1	0	1
p_{ii}	/	0.9272	0.8949	0.99	0.99	0.9893	0.9872
m_i	/	-0.0328	0.0094	-0.5	0.5	-0.5391	0.4927
α_{0i}	/	0.0148	0.0119	0.015	0.065	0.0117	0.0628
α_{1i}	/	-0.0095	0.0038	0.12	0.35	0.1196	0.2992
β_{1i}	/	0.0075	0.0016	0.25	0.18	0.2419	0.1793
d	0.2	0.2146		/		0.0328	
p_{ii}	/	0.8365	0.7948	0.98	0.99	0.9795	0.9779
m_i	/	-0.0753	0.0528	-2	2	-2.1091	2.1423
α_{0i}	/	0.0128	0.0025	0.015	0.065	0.0134	0.0719
α_{1i}	/	0.0018	-0.0032	0.12	0.35	0.1256	0.3291
β_{1i}	/	0.0025	-0.0018	0.25	0.18	0.2584	0.1916
d	0.4	0.4126		/		0.0124	

Estimation of parameters of simulated *RS-ARFIMA(0,d,0)-GARCH(1,1)* data fitted into *RS-ARFIMA(0,d,0)-GARCH(1,1)* models.

Table 4.2

	DGP_1 values		RS-ARFIMA-GARCH		DGP_2 values		RS-ARFIMA-GARCH	
S_t	0	1	0	1	0	1	0	1
p_{ii}	0.99	0.99	0.9885	0.9863	0.98	0.99	0.9937	0.9975
m_i	-0.5	0.5	-0.4835	0.5182	-2	2	-2.0096	1.9628
α_{0i}	0.015	0.065	0.0094	0.0587	0.015	0.065	0.0146	0.0628
α_{1i}	0.12	0.35	0.1268	0.3482	0.12	0.35	0.1194	0.2992
β_{1i}	0.25	0.18	0.2486	0.1863	0.25	0.18	0.2416	0.1793
d	0.2		0.2138		0.4		0.3948	

In Table 4.1, we find that the mean estimates of d are close to true values for *ARFIMA* DGP and close to 0 for *RS-GARCH* DGP. In addition, for *RS-GARCH*, mean estimates of all parameters, particularly of p_{11} and p_{22}, are also close to their corresponding true values. Thus, we further argue that *RS-ARFIMA-GARCH* model can consistently identify the states of *RS-GARCH*. These results show that, the *RS-ARFIMA-GARCH* model can distinguish between the pure *ARFIMA* and pure *RS-GARCH* DGP_s and provide consistent estimates of parameters.
In Table 4.2, it can be seen that all estimated values of parameters from *RS-ARFIMA-GARCH* model, for each regime, are quite close to the true values. As a result, it suggests that *RS-ARFIMA-GARCH* model is capable of identifying the states.
We also see that, *RS-ARFIMA-GARCH* framework can further provide consistent estimate of long memory parameter and can consistently identify the volatility states.

5. Application on real data

For evaluate the capacity of our *RS-ARFIMA-GARCH* framework to describe the data, we compare it with *RS-ARFIMA*, *RS-GARCH*, standard *ARFIMA* and standard Markov regime switching models.

5.1. Data

The data of exchange rate being very volatile, we choose them as our study. So, we are interested in the daily series of exchange rate of the US Dollar (USD) towards the Euro (EURO), from January 04th, 1999 to March 14th, 2014, daily frequency (5 days a week), that is a trajectory of length $T = 3965$. We note $(X_t)_{t=1,\ldots,T}$ this series which the graphic representation is given at Figure 2. We then considered the logarithmic transformation

$$\log X_t - \log X_{t-1}, \quad \forall\, t \geq 1$$

The studied series thus become the daily variations, expressed in percentage. These data come from the FRED database maintained by the Federal Reserve Bank of Saint-Louis, available on-line on the site of the bank [2] in the section: *Exchange rate, balance of payments and trade data*. The choice of this exchange rate recovers from its status as world currency and his role on foreign exchange market.

The presence of long memory in the series of exchange rate, supposed stationaries, was shown in a empirical way in Klaassen (2005). Intervals of homogeneities were identified on these series, what proves their nonstationary character.
We present in the below Table 5.1 the characteristics of the series.

Characteristics	USD/EURO
Mean	1.2205
Median	1.2719
Maximum	1.6010
Minimum	0.8270
Ecart type	0.1854
Skewness	-0.4663
Kurtosis	2.2769
Jarque Bera	230.081

Table 5.1-Descriptive statistics of exchange rate X_t

It emerges from this Table that the distribution of the exchange rate USD/EURO does not seem to be normally distributed, because the sign of Skewness statistic is negative for this series and we see well also that the Jarque-Bera statistic is widely superior to the critical value of Chi-deux (5.991) at the 5% level of significance, what brings us to believe that the distribution of our series is non normal, what is a general characteristic of the financial series.

Figure 2 presents on the left the evolution of the daily exchange rate, on the middle the trajectory of the daily returns of exchange rate and on the right the autocorrelation function in absolute value of returns of exchange rate. The left graph reveals the existence of an increase trend for the parity, from 2002 until

[2] http://www.stls.frb.org/

2008 year of financial and economic crisis, thus a priori a nonstationarity of the series. Consequently, we need a test to confirm or invalidate this behavior of nonstationarity. We use for that purpose, Augmented Dickey-Fuller (ADF) unit root.

Table 5.2 - Augmented Dickey-Fuller test on the logarithm of the parity[3].

Parity	lag order p	Statistic of the test	Prob	significance level 1%	5%
USD/EURO	6	-23.158	0.01	-3.9614	-3.4115

The logarithm of USD/EURO parity is not stationary in level and stationary in first differency, of this fact the study will carry on the returns of USD/EURO parity.

The empirical autocorrelation function of the series, right graph, decrease in a slow way towards positive values, what constitutes an indicator of the presence of long memory. This slow decay can be explained by the second term of the relation (2.4) in Mikosch and Stáricá (2004).

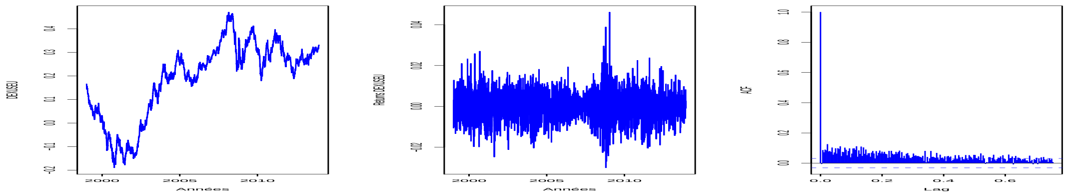

Figure 2 : Evolution (top left), Trajectory (top middle) and ACF (top right).

5.2. Regime change

Table 5.3 presents the results of the identification test of intervals of homogeneity (Starica and Granger (2005)). On the whole period considered, the studied series seems to be more stable than over the last two years of our sample. Indeed, we identify by means of the test several significant jumps over the period of study, decomposing so the whole period in several periods where the unconditional variance seems to be constant. We so identify the last interval of homogeneity from 10/08/2012 to 14/03/2014.

[3] The choice of p was made by means of the criteria of Schwartz and Akaike.

Table 5.3
Homogeneity intervals from 04/01/1999–14/03/2014

USD/EURO	
Statistic T	Intervals
0.829	04/01/1999 - 07/09/2001
0.515	09/02/2001 - 01/11/2002
0.469	05/04/2002 - 21/03/2003
0.514	23/08/2002 - 08/08/2003
0.441	10/01/2003 - 26/12/2003
0.515	30/05/2003 - 01/10/2004
0.297	05/03/2004 - 08/07/2005
0.304	10/12/2004 - 26/10/2007
0.623	30/03/2007 - 25/09/2009
0.323	27/02/2009 - 26/08/2011
0.291	10/09/2010 - 01/06/2012
0.301	17/06/2011 - 26/07/2013
0.421	10/08/2012 - 14/03/2014

By putting $p = 100$ new points of data, $s = 250$ the size of subsample on which the test is driven, $m = 700$ the size of the first block, the statistical homogeneity test practiced on the subsample $X_{m+p-s}, ..., X_{m+p}$ supplies a value of the identification test $T = 0.829$ widely upper to the critical value at the risk $\alpha = 5\%$, what makes that the block $X_1, ..., X_{700}$ cannot be extend to $X_1, ..., X_{800}$. So, from January, 1999, we found the first interval of homogeneous data which corresponds to a subsample of 700 data, what seems to be in agreement with the regular depreciation of the Euro, as soon as its introduction in 1999, compared with the Dollar until 2001. A structural change thus occurred according to our analysis between $m = 700$ and $m = 800$, what could correspond to the end of the abnormal depreciation of the Euro face to face of the Dollar between 2000 and 2001. This change was short because immediately we were up in the interval of homogeneity with the subsample $X_{550}, ..., X_{1000}$. But of $m = 1000$ data, what would correspond with the end of 2002 or the beginning of 2003, until $m = 1400$ which corresponds to roughly at the beginning of 2004, we detected many structural changes on the data. From 2004, aside some two changes which occurred between $m = 1500$ and $m = 1600$ and between $m = 1700$ and $m = 1800$, we found data which are homogeneous for the mostly; this situation of the data a little bit similar to that of before 2002, could correspond on the contrary to the appreciation of the Euro opposite of the Dollar. We notice a change between $m = 2300$ and $m = 2400$ what seems to correspond to the completion, in the second quarter 2008, of the bullish movement introduced to the second quarter of 2002 with extreme points situated at 0.8230 and at 1.6030 approximately. Then comes a break between $m = 2800$ and $m = 3000$ situation which can be explained by the fall of the Euro in the beginning summer 2010 reaching 1.1959 on 07/06/2010. Since then, we notice a certain form of homogeneity to between $m = 3400$ and $m = 3500$ which correspond respectively to the dates 12/01/2012 and 01/06/2012. This break between these two dates is apparently connected to the resumption of the downward trend of the series in May, 2012 reaching the minimal value 1.2062 on 24/07/2012. With the resumption of the upward trend, the series seems to be homogeneous since 07/09/2012. It thus seems that this series is really informative as regards the detection of structural changes.

5.3. Estimation

Studying the performance on simulated data, we verify the reliability of our model on the real data. In Table 5.4, we report the parameters from the estimation of different models using the estimation sample. The estimated models include the two regimes RS-ARFIMA-GARCH model defined by equations (15) and (17), the RS-ARFIMA, the RS-GARCH, the standard ARFIMA and the standard Markov regime switching. The estimated value for d in ARFIMA model is 0.1808 and significantly greater than 0, suggesting that the long memory is present. The presence of a long memory in the series of exchange rate is in agreement with the works of Booth *and al.* (1982) detecting such a phenomenon by means of the analysis R/S or still those of Cheung (1993b) using the procedure R/S and the method of estimation of the processes ARFIMA of Geweke and Porter-Hudak (1983). The estimated values for α_{11} et α_{12} in RS-GARCH are $\hat{\alpha}_{11} = 0.0236$ and $\hat{\alpha}_{12} = 0.0817$, indicate that the exhange rate has conditional volatility. Turning to the RS-Markov model, estimates of p_{11} and p_{22} are significant and greater than 0.98. This suggests a significant regime-switching process, with a small frequency to switch between states. Concerning the RS-ARFIMA-GARCH, estimated d is 0.1348. Since it is significantly greater than 0, the long memory is expected to exist for the exchange rate USD/EURO. Comapred with the other estimated values d from other models, we can see that the estimate of d is more small. As to GARCH parameters, estimates of RS-ARFIMA-GARCH model are fairly close to those of RS-GARCH model. In terms of p_{11} and p_{22}, RS-ARFIMA-GARCH model generates consistent estimates with those in RS-Markov model. In terms of model performance evaluations, logarithm of likelihood, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) all suggest that RS-ARFIMA-GARCH model outperform all other models.
In conclusion, using the empirical results of the exchange rate USD/EURO, we demonstrate that RS-ARFIMA-GARCH framework is capable of estimating the true long memory parameter and identifying the nonstationarity. Compared with ARFIMA model, it can control for the effects of regime switching and generate more reliable estimate of long memory. It overall outperforms all the models which we considered in this study and could be a widely useful tool for modeling financial time series submitted to structural changes.

Table 5.4
Parity US/EURO

	RS-ARFIMA-GARCH		RS-ARFIMA		RS-GARCH		ARFIMA	RS-Markov	
S_t	1	2	1	2	1	2	One regime	1	2
p_{ii}	0.9924	0.9946	0.9938	0.9842	0.9969	0.9858	/	0.9986	0.9968
m_i	0.02964	-0.02753	0.03156	-0.02573	0.02925	-0.02974	0.03267	0.02738	-0.01951
d	0.1348		0.1693		/		0.1826	/	
α_{0i}	0.0053	0.0241	/		0.0049	0.0256	/	/	
α_{1i}	0.0345	0.0325	/		0.0316	0.0297	/	/	
β_{1i}	0.8281	0.8516	/		0.8123	0.8537	/	/	
L_n	2429.75		2417.23		2419.31		2412.48	2425.30	
AIC	-4857.50		-4835.27		-4831.04		-4851.42	-4843.61	
BIC	-4552.93		-4547.32		-4529.61		-4431.57	-4446.03	

Conclusion

In this work we are interested in a joint modeling of the nonstationarity and the long memory observed on processes supposed short memory stationary. In this viewpoint, we have developed a regime-switching univariate RS-ARFIMA-GARCH model for take into account this two phenomena. The model cannot be estimated by the ML method because of the path dependence problem. To circumvent this problem, we use a Bayesian Markov chain Monte Carlo (MCMC) algorithm. For evaluate his capacity to describe the data, we have compare it with RS-ARFIMA, RS-GARCH, standard ARFIMA and RS-Markov models. We have found that the the RS-ARFIMA-GARCH framework for modeling financial time series submitted to structural changes outperforms all these models which we considered in this study.

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