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HYERS–ULAM–RASSIAS STABILITY OF HOMOMORPHISMS IN QUASI-BANACH ALGEBRAS

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This paper is dedicated to Professor Themistocles M. Rassias.

Submitted by P. K. Sahoo

Abstract. Let q be a positive rational number and n be a nonnegative integer. We prove the Hyers–Ulam–Rassias stability of homomorphisms in quasi-Banach algebras and of generalized derivations on quasi-Banach algebras for the following functional equation:

$$\sum_{i=1}^{n} f\left(\sum_{j=1}^{n} q(x_i - x_j)\right) + nf\left(\sum_{i=1}^{n} qx_i\right) = nq\sum_{i=1}^{n} f(x_i).$$

This is applied to investigate isomorphisms between quasi-Banach algebras. The concept of Hyers–Ulam–Rassias stability originated from the Th.M. Rassias' stability theorem that appeared in his paper: On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc. **72** (1978), 297–300.

1. Introduction and preliminaries

Ulam [30] gave a talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of unsolved problems. Among these was the following question concerning the stability of homomorphisms.

We are given a group G and a metric group G' with metric $\rho(\cdot,\cdot)$. Given $\epsilon > 0$, does there exist a $\delta > 0$ such that if $f : G \to G'$ satisfies

$$\rho(f(xy), f(x)f(y)) < \delta$$

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for all $x, y \in G$, then a homomorphism $h : G \to G'$ exists with $\rho(f(x), h(x)) < \epsilon$ for all $x \in G$?

By now an affirmative answer has been given in several cases, and some interesting variations of the problem have also been investigated. We shall call such an $f : G \to G'$ an *approximate homomorphism.*

Hyers [11] considered the case of approximately additive mappings $f : E \to E'$, where E and E' are Banach spaces and f satisfies *Hyers inequality*

$$\|f(x + y) - f(x) - f(y)\| \le \epsilon$$

for all $x, y \in E$. It was shown that the limit

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

exists for all $x \in E$ and that $L : E \to E'$ is the unique additive mapping satisfying

$$\|f(x) - L(x)\| \le \epsilon.$$

No continuity conditions are required for this result, but if $f(tx)$ is continuous in the real variable t for each fixed $x \in E$, then L is linear, and if f is continuous at a single point of E then $L : E \to E'$ is also continuous.

Th.M. Rassias [24] provided a generalization of Hyers' Theorem which allows the *Cauchy difference to be unbounded.*

Theorem 1.1. (Th.M. Rassias). *Let $f : E \to E'$ be a mapping from a normed vector space E into a Banach space E' subject to the inequality*

$$\|f(x + y) - f(x) - f(y)\| \le \epsilon(\|x\|^p + \|y\|^p) \tag{1.1}$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then the limit

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

exists for all $x \in E$ and $L : E \to E'$ is the unique additive mapping which satisfies

$$\|f(x) - L(x)\| \le \frac{2\epsilon}{2 - 2^p}\|x\|^p \tag{1.2}$$

for all $x \in E$. If $p < 0$ then inequality (1.1) *holds for $x, y \neq 0$ and* (1.2) *for $x \neq 0$. Also, if for each $x \in E$ the mapping $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in E$, then L is linear.*

Th.M. Rassias [25] during the 27^{th} International Symposium on Functional Equations asked the question whether such a theorem can also be proved for $p \ge 1$. Gajda [9] following the same approach as in Th.M. Rassias [24], gave an affirmative solution to this question for $p > 1$. It was shown by Gajda [9], as well as by Th.M. Rassias and Šemrl [28] that one cannot prove a Th.M. Rassias' type Theorem when $p = 1$. The counterexamples of Gajda [9], as well as of Th.M. Rassias and Šemrl [28] have stimulated several mathematicians to invent new definitions of *approximately additive* or *approximately linear* mappings, cf. Găvruta [10], Czerwik [7], who among others studied the Hyers–Ulam stability of functional equations. The inequality (1.1) that was introduced for the first time by Th.M. Rassias [24] provided a lot of influence in the development of a generalization of the Hyers–Ulam stability concept. This concept is known as *Hyers–Ulam–Rassias stability* of functional equations (cf. the books of D.H. Hyers, G. Isac and Th.M. Rassias [12], S. Jung [16], P. Czerwik [8]; the papers of C. Baak and M.S. Moslehian [4], K. Jun, S. Jung and Y. Lee [13], Y. Lee and K. Jun [17], C. Park [19], C. Park and J. Hou [22], C. Park and Th.M. Rassias [23], Th.M. Rassias [26, 27]).

Beginning around the year 1980, the topic of approximate homomorphisms and their stability theory in the field of functional equations and inequalities was taken up by several mathematicians (cf. [12] and the references therein).

Recently, Jun and Kim [14] solved the stability problem of Ulam for a quadratic functional equation. Jun and Kim [15] introduced and investigated the following quadratic functional equation

$$\begin{aligned}\sum_{i=1}^{n} r_i Q\left(\sum_{j=1}^{n} r_j(x_i - x_j)\right) &+ \left(\sum_{i=1}^{n} r_i\right) Q\left(\sum_{i=1}^{n} r_i x_i\right) \\ &= \left(\sum_{i=1}^{n} r_i\right)^2 \sum_{i=1}^{n} r_i Q(x_i).\end{aligned}$$

In this paper we introduce the following functional equation

$$\sum_{i=1}^{n} L\left(\sum_{j=1}^{n} q(x_i - x_j)\right) + nL\left(\sum_{i=1}^{n} q x_i\right) = nq\sum_{i=1}^{n} L(x_i). \tag{1.3}$$

The purpose of the present paper is to study the Hyers–Ulam–Rassias stability of the functional equation (1.3).

We recall some basic facts concerning quasi-Banach spaces and some preliminary results.

Definition 1.2. ([6, 29]) Let X be a real linear space. A *quasi-norm* is a real-valued function on X satisfying the following:

(1) $\|x\| \geq 0$ for all $x \in X$ and $\|x\| = 0$ if and only if $x = 0$.

(2) $\|\lambda x\| = |\lambda| \cdot \|x\|$ for all $\lambda \in \mathbb{R}$ and all $x \in X$.

(3) There is a constant $K \geq 1$ such that $\|x + y\| \leq K(\|x\| + \|y\|)$ for all $x, y \in X$.

The pair $(X, \|\cdot\|)$ is called a *quasi-normed space* if $\|\cdot\|$ is a quasi-norm on X. The smallest possible K is called the *modulus of concavity* of $\|\cdot\|$. Obviously the balls with respect to $\|\cdot\|$ define a linear topology on X. By a *quasi-Banach space* we mean a complete quasi-normed space, i.e. a quasi-normed space in which every $\|\cdot\|$-Cauchy sequence in X converges. This class includes Banach spaces and the most significant class of quasi-Banach spaces which are not Banach spaces are the L_p spaces for $0 < p < 1$ with the quasi-norm $\|\cdot\|_p$.

A quasi-norm $\|\cdot\|$ is called a *p-norm* $(0 < p \leq 1)$ if

$$\|x + y\|^p \leq \|x\|^p + \|y\|^p$$

for all $x, y \in X$. In this case, a quasi-Banach space is called a *p-Banach space*.

Given a p-norm, the formula $d(x, y) := \|x-y\|^p$ gives us a translation invariant metric on X. By the Aoki–Rolewicz theorem [29] (see also [6]), each quasi-norm is equivalent to some p-norm. Since it is much easier to work with p-norms than quasi-norms, henceforth we restrict our attention mainly to p-norms.

Definition 1.3. ([2]) Let $(A, \|\cdot\|)$ be a quasi-normed space. The quasi-normed space $(A, \|\cdot\|)$ is called a *quasi-normed algebra* if A is an algebra and there is a constant $K > 0$ such that $\|xy\| \le K\|x\| \cdot \|y\|$ for all $x, y \in A$.

A *quasi-Banach algebra* is a complete quasi-normed algebra.

If the quasi-norm $\|\cdot\|$ is a *p-norm* then the quasi-Banach algebra is called a *p-Banach algebra*.

In this paper, assume that A is a quasi-normed algebra with quasi-norm $\|\cdot\|_A$ and that B is a p-Banach algebra with p-norm $\|\cdot\|_B$. Let K be the modulus of concavity of $\|\cdot\|_B$.

This paper is organized as follows: In Section 2, we prove the Hyers–Ulam–Rassias stability of homomorphisms in quasi-Banach algebras.

In Section 3, we investigate isomorphisms between quasi-Banach algebras.

In Section 4, we prove the Hyers–Ulam–Rassias stability of generalized derivations on quasi-Banach algebras.

2. Stability of homomorphisms in quasi-Banach algebras

Let q be a positive rational number. For a given mapping $f : A \to B$, we define $Df : A^n \to B$ by

$$\begin{aligned} Df(x_1, \cdots, x_n) : &= \sum_{i=1}^{n} f\left(\sum_{j=1}^{n} q(x_i - x_j)\right) \\ &+ nf\left(\sum_{i=1}^{n} qx_i\right) - nq\sum_{i=1}^{n} f(x_i) \end{aligned}$$

for all $x_1, \cdots, x_n \in X$.

We prove the Hyers–Ulam–Rassias stability of homomorphisms in quasi-Banach algebras.

Theorem 2.1. *Assume that $r > 2$ if $nq > 1$ and that $0 < r < 1$ if $nq < 1$. Let θ be a positive real number, and let $f : A \to B$ be an odd mapping such that*

$$\|Df(x_1, \cdots, x_n)\|_B \le \theta \sum_{j=1}^{n} \|x_j\|_A^r, \tag{2.1}$$

$$\|f(xy) - f(x)f(y)\|_B \le \theta(\|x\|_A^r + \|y\|_A^r) \tag{2.2}$$

for all $x, y, x_1, \cdots, x_n \in A$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\|_B \le \frac{\theta}{((nq)^{pr} - (nq)^p)^{\frac{1}{p}}} \|x\|_A^r \tag{2.3}$$

for all $x \in A$.

Proof. Letting $x_1 = \cdots = x_n = x$ in (2.1), we get

$$\|nf(nqx) - n^2qf(x)\|_B \le n\theta\|x\|_A^r$$

for all $x \in A$. So

$$\|f(x) - nqf(\frac{x}{nq})\|_B \le \frac{\theta}{(nq)^r}\|x\|_A^r$$

for all $x \in A$. Since B is a p-Banach algebra,

$$\begin{aligned}\|(nq)^l f(\frac{x}{(nq)^l}) \quad - & \quad (nq)^m f(\frac{x}{(nq)^m})\|_B^p \\ \le & \sum_{j=l}^{m-1} \|(nq)^j f(\frac{x}{(nq)^j}) - (nq)^{j+1} f(\frac{x}{(nq)^{j+1}})\|_B^p \qquad (2.4) \\ \le & \frac{\theta^p}{(nq)^{pr}} \sum_{j=l}^{m-1} \frac{(nq)^{pj}}{(nq)^{prj}} \|x\|_A^{pr}\end{aligned}$$

for all nonnegative integers m and l with $m > l$ and all $x \in A$. It follows from (2.4) that the sequence $\{(nq)^d f(\frac{x}{(nq)^d})\}$ is Cauchy for all $x \in A$. Since B is complete, the sequence $\{(nq)^d f(\frac{x}{(nq)^d})\}$ converges. So one can define the mapping $H : A \to B$ by

$$H(x) := \lim_{d\to\infty} (nq)^d f(\frac{x}{(nq)^d})$$

for all $x \in A$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.4), we get (2.3).

It follows from (2.1) that

$$\begin{aligned}\|DH(x_1, \cdots, x_n)\|_B &= \lim_{d\to\infty} (nq)^d \|Df(\frac{x_1}{(nq)^d}, \cdots, \frac{x_n}{(nq)^d})\|_B \\ &\le \lim_{d\to\infty} \frac{(nq)^d\theta}{(nq)^{dr}} \sum_{j=1}^{n} \|x_j\|_A^r = 0\end{aligned}$$

for all $x_1, \cdots, x_n \in A$. Thus

$$DH(x_1, \cdots, x_n) = 0$$

for all $x_1, \cdots, x_n \in A$. By Lemma 2.1 of [21], the mapping $H : A \to B$ is Cauchy additive.

By the same reasoning as in the proof of Theorem of [24], the mapping $H : A \to B$ is $\mathbb{R}$-linear.

It follows from (2.2) that

$$\begin{aligned}\|H(xy) \quad - & \quad H(x)H(y)\|_B \\ = & \lim_{d\to\infty} (nq)^{2d} \|f(\frac{xy}{(nq)^d(nq)^d}) - f(\frac{x}{(nq)^d})f(\frac{y}{(nq)^d})\|_B \\ \le & \lim_{d\to\infty} \frac{(nq)^{2d}\theta}{(nq)^{dr}} (\|x\|_A^r + \|y\|_A^r) = 0\end{aligned}$$

for all $x, y \in A$. So

$$H(xy) = H(x)H(y)$$

for all $x, y \in A$.

Now, let $T : A \to B$ be another mapping satisfying (2.3). Then we have

$$\begin{aligned}\|H(x) - T(x)\|_B &= (nq)^d \|H(\frac{x}{(nq)^d}) - T(\frac{x}{(nq)^d})\|_B \\ &\le (nq)^d K(\|H(\frac{x}{(nq)^d}) - f(\frac{x}{(nq)^d})\|_B + \|T(\frac{x}{(nq)^d}) - f(\frac{x}{(nq)^d})\|_B) \\ &\le \frac{2 \cdot (nq)^d K\theta}{((nq)^{pr} - (nq)^p)^{\frac{1}{p}} (nq)^{dr}} \|x\|_A^r,\end{aligned}$$

which tends to zero as $n \to \infty$ for all $x \in A$. So we can conclude that $H(x) = T(x)$ for all $x \in A$. This proves the uniqueness of H. Thus the mapping $H : A \to B$ is a unique homomorphism satisfying (2.3). □

Theorem 2.2. *Assume that $0 < r < 1$ if $nq > 1$ and that $r > 2$ if $nq < 1$. Let θ be a positive real number, and let $f : A \to B$ be an odd mapping satisfying* (2.1) *and* (2.2). *If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $H : A \to B$ such that*

$$\|f(x) - H(x)\|_B \le \frac{\theta}{((nq)^p - (nq)^{pr})^{\frac{1}{p}}} \|x\|_A^r \tag{2.5}$$

for all $x \in A$.

Proof. It follows from (2.1) that

$$\|f(x) - \frac{1}{nq} f(nqx)\|_B \le \frac{\theta}{nq} \|x\|_A^r$$

for all $x \in A$. Since B is a p-Banach algebra,

$$\begin{aligned}\|\frac{1}{(nq)^l} f((nq)^l x) \quad &- \quad \frac{1}{(nq)^m} f((nq)^m x)\|_B^p \\ &\le \quad \sum_{j=l}^{m-1} \|\frac{1}{(nq)^j} f((nq)^j x) - \frac{1}{(nq)^{j+1}} f((nq)^{j+1} x)\|_B^p \qquad (2.6) \\ &\le \quad \frac{\theta^p}{(nq)^p} \sum_{j=l}^{m-1} \frac{(nq)^{prj}}{(nq)^{pj}} \|x\|_A^{pr}\end{aligned}$$

for all nonnegative integers m and l with $m > l$ and all $x \in A$. It follows from (2.6) that the sequence $\{\frac{1}{(nq)^d} f((nq)^d x)\}$ is Cauchy for all $x \in A$. Since B is complete, the sequence $\{\frac{1}{(nq)^d} f((nq)^d x)\}$ converges. So one can define the mapping $H : A \to B$ by

$$H(x) := \lim_{d \to \infty} \frac{1}{(nq)^d} f((nq)^d x)$$

for all $x \in A$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.6), we get (2.5).

The rest of the proof is similar to the proof of Theorem 2.1. □

3. Isomorphisms between quasi-Banach algebras

Throughout this section, assume that A is a quasi-Banach algebra with quasi-norm $\|\cdot\|_A$ and unit e and that B is a p-Banach algebra with p-norm $\|\cdot\|_B$ and unit e'. Let K be the modulus of concavity of $\|\cdot\|_B$.

We investigate isomorphisms between quasi-Banach algebras.

Theorem 3.1. *Assume that $r > 2$ if $nq > 1$ and that $0 < r < 1$ if $nq < 1$. Let θ be a positive real number, and let $f : A \to B$ be an odd bijective mapping satisfying* (2.1) *such that*

$$f(xy) = f(x)f(y) \tag{3.1}$$

for all $x, y \in A$. If $\lim_{d\to\infty}(nq)^d f(\frac{e}{(nq)^d}) = e'$ and $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then the mapping $f : A \to B$ is an isomorphism.

Proof. The condition (3.1) implies that $f : A \to B$ satisfies (2.2). By the same reasoning as in the proof of Theorem 2.1, there exists a unique homomorphism $H : A \to B$, which is defined by

$$H(x) := \lim_{d\to\infty}(nq)^d f(\frac{x}{(nq)^d})$$

for all $x \in A$. Thus

$$\begin{aligned} H(x) &= H(ex) = \lim_{d\to\infty}(nq)^d f(\frac{ex}{(nq)^d}) = \lim_{d\to\infty}(nq)^d f(\frac{e}{(nq)^d}\cdot x) \\ &= \lim_{d\to\infty}(nq)^d f(\frac{e}{(nq)^d})f(x) = e'f(x) = f(x) \end{aligned}$$

for all $x \in A$. So the bijective mapping $f : A \to B$ is an isomorphism, as desired. □

Theorem 3.2. *Assume that $0 < r < 1$ if $nq > 1$ and that $r > 2$ if $nq < 1$. Let θ be a positive real number, and let $f : A \to B$ be an odd bijective mapping satisfying* (2.1) *and* (3.1)*. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$ and $\lim_{d\to\infty}\frac{1}{(nq)^d}f((nq)^d e) = e'$, then the mapping $f : A \to B$ is an isomorphism.*

Proof. The proof is similar to the proofs of Theorems 2.1, 2.2 and 3.1. □

4. Stability of generalized derivations on quasi-Banach algebras

Recently, several extended notions of derivations have been treated in the Banach algebra theory (see [18] and references therein). In addition, the stability of these derivations is extensively studied by many mathematicians; see [1, 5, 20].

Throughout this section, assume that A is a p-Banach algebra with p-norm $\|\cdot\|_A$. Let K be the modulus of concavity of $\|\cdot\|_A$.

Definition 4.1. [3] A *generalized derivation* $\delta : A \to A$ is $\mathbb{R}$-linear and fulfills the generalized Leibniz rule

$$\delta(xyz) = \delta(xy)z - x\delta(y)z + x\delta(yz)$$

for all $x, y, z \in A$.

We prove the Hyers–Ulam–Rassias stability of generalized derivations on quasi-Banach algebras.

Theorem 4.2. *Assume that $r > 3$ if $nq > 1$ and that $0 < r < 1$ if $nq < 1$. Let θ be a positive real number, and let $f : A \to A$ be an odd mapping satisfying (2.1) such that*

$$\begin{aligned} \|f(xyz) - f(xy)z &+ xf(y)z - xf(yz)\|_A \\ &\leq \theta(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r) \end{aligned} \tag{4.1}$$

for all $x, y, z \in A$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique generalized derivation $\delta : A \to A$ such that

$$\|f(x) - \delta(x)\|_A \leq \frac{\theta}{((nq)^{pr} - (nq)^p)^{\frac{1}{p}}}\|x\|_A^r \tag{4.2}$$

for all $x \in A$.

Proof. By the same reasoning as in the proof of Theorem 2.1, there exists a unique $\mathbb{R}$-linear mapping $\delta : A \to A$ satisfying (4.2). The mapping $\delta : A \to A$ is defined by

$$\delta(x) := \lim_{d\to\infty}(nq)^d f(\frac{x}{(nq)^d})$$

for all $x \in A$.

It follows from (4.1) that

$$\begin{aligned} &\|\delta(xyz) - \delta(xy)z + x\delta(y)z - x\delta(yz)\|_A \\ &\quad = \lim_{d\to\infty}(nq)^{3d}\|f(\frac{xyz}{(nq)^{3d}}) - f(\frac{xy}{(nq)^{2d}})\frac{z}{(nq)^d} \\ &\qquad + \frac{x}{(nq)^d}f(\frac{y}{(nq)^d})\frac{y}{(nq)^d} - \frac{x}{(nq)^d}f(\frac{yz}{(nq)^{2d}})\|_A \\ &\quad \leq \lim_{d\to\infty}\frac{(nq)^{3d}\theta}{(nq)^{dr}}(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r) = 0 \end{aligned}$$

for all $x, y, z \in A$. So

$$\delta(xyz) = \delta(xy)z - x\delta(y)z + x\delta(yz)$$

for all $x, y, z \in A$. Thus the mapping $\delta : A \to A$ is a unique generalized derivation satisfying (4.2). □

Theorem 4.3. *Assume that $0 < r < 1$ if $nq > 1$ and that $r > 3$ if $nq < 1$. Let θ be a positive real number, and let $f : A \to A$ be an odd mapping satisfying (2.1) and (4.1). If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique generalized derivation $\delta : A \to A$ such that*

$$\|f(x) - \delta(x)\|_A \leq \frac{\theta}{((nq)^p - (nq)^{pr})^{\frac{1}{p}}}\|x\|_A^r \tag{4.3}$$

for all $x \in A$.

Proof. By the same reasoning as in the proof of Theorem 2.2, there exists a unique $\mathbb{R}$-linear mapping $\delta : A \to A$ satisfying (4.3). The mapping $\delta : A \to A$ is defined by

$$\delta(x) := \lim_{d\to\infty} \frac{1}{(nq)^d} f((nq)^d x)$$

for all $x \in A$.

The rest of the proof is similar to the proof of Theorem 4.2. □

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