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Remarks on Xia's inequality and Chevet's inequality concerned with cylindrical measures

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§1. Introduction

In [5] D. Xia has established a certain inequality concerned with quasi-invariant measures, and thereafter his result was extended to the cylindrical measure case by W. Linde [2] and the author [4]. On the other hand, in [1] S. Chevet has established a similar inequality concerned with kernels of cylindrical measures.

The main purpose of the present paper is to give the generalizations of their results. Explicitly stating, we shall prove the following theorems.

THEOREM 1.1. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and suppose that F is barrelled. Let f be a function defined on E^* (but not necessarily everywhere finite) which satisfies the following two conditions;*

(1) $0 \leqq f(tx^*) \leqq tf(x^*) \leqq \infty$, *for every* $t>0$ *and every* $x^* \in E^*$,

(2) *for every* $y \in F$, *there is a* $\delta>0$ *such that the inequality* $f(x^*)<\delta$ *implies* $|\langle x^*, T(y)\rangle|<1$, *for every* $x^* \in E^*$.

Then there exists a neighborhood V of zero in F such that for every $x^* \in E^*$, the inequality

$$\sup_{y \in V} \left|\left\langle x^*, T(y)\right\rangle\right| \leqq f(x^*)$$

holds.

THEOREM 1.2. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and suppose that F is of the second category. Let $\{f_n\}$ be a sequence consisting of functions defined on E^* (but not necessarily everywhere finite) which satisfies the following two conditions;*

(1) $0 \leqq f_n(tx^*) \leqq tf_n(x^*) \leqq \infty$, *for every* $t>0$, *every natural number* n *and every* $x^* \in E^*$,

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(2) *for every* $y \in F$, *there are a* $\delta > 0$ *and a natural number* n *such that the inequality* $f_n(x^*) < \delta$ *implies* $|\langle x^*, T(y)\rangle| < 1$, *for every* $x^* \in E^*$.

Then there exist a natural number n *and a neighborhood* V *of zero in* F *such that for every* $x^* \in E^*$, *the inequality*

$$\sup_{y \in V} \left|\left\langle x^*, T(y)\right\rangle\right| \leqq f_n(x^*)$$

holds.

We now remark that Theorem 1.1 holds for a locally convex space F if and only if it is barrelled, and Theorem 1.2 holds for a locally convex space F if and only if it satisfies that for every sequence $\{B_n\}$ consisting of convex balanced closed subsets of F such that $F = \cup B_n$, some B_n contains a neighborhood of zero in F.

In Section 4, our main theorems are applied for the study of quasi-invariant measures and kernels of cylindrical measures and then, we obtain the several inequalities (cf. Theorems 4.1, 4.3 and 4.4) which generalize the results of S. Chevet [1], W. Linde [2], D. Xia [5] and the author [4].

In Section 5, using these inequalities, we shall show that if a barrelled locally convex Hausdorff space E admits a cylindrical measure μ of weak p-th order (for $0 < p < \infty$) such that the kernel of μ contains E, then E is normable, and the strong dual (E^*, b) is isomorphic to a quotient space of a subspace of $L^p(\nu)$, for some probability space (Ω, Σ, ν). In particular, taking $p = 2$, we obtain that a quasi-complete barrelled locally convex Hausdorff space E is isomorphic to a Hilbert space if and only if it admits a cylindrical measure μ of weak second order such that the kernel of μ contains E.

Throughout this paper except for Section 3, we assume that all linear spaces are with real coefficients.

§ 2. Definitions and notations

Let E be a linear topological space, E^* be its topological dual space and μ be a cylindrical measure on E.

DEFINITION 2.1. For $0 < p < \infty$, the cylindrical measure μ is called of weak p-th order if for every $x^* \in E^*$, the inequality

$$\int_E \left|\langle x^*, x\rangle\right|^p d\mu(x) < \infty$$

holds. In particular, if μ is of weak 2-th order, then we shall call it of weak second order.

DEFINITION 2.2. An element x of E is called an admissible shift for the cylindrical measure μ if for every $\varepsilon>0$, there is a $\delta>0$ such that the inequality $\mu(Z)<\delta$ implies $\mu(Z-x)<\varepsilon$, for every cylindrical set Z of E. The set of all admissible shifts of μ will be denoted by M_μ.

If G is an additive subgroup of E such that $G\subset M_\mu$, then the cylindrical measure μ will be called quasi-invariant under G.

DEFINITION 2.3. An element x of E is called a partially admissible shift for the cylindrical measure μ if there are an $\varepsilon>0$ and a $\delta>0$ such that the inequality $\mu(Z)<\delta$ implies $\mu(Z-x)<1-\varepsilon$, for every cylindrical set Z of E. The set of all partially admissible shifts of μ will be denoted by $\widetilde{M}_\mu$.

We shall now define the kernel of the cylindrical measure μ. The notion of kernel has been introduced by S. Chevet [1]. Let $L: E^*\to L^0(\Omega, \Sigma, \nu)$ be a random linear functional associated with μ. (For random linear functional, we refer to [3, p. 256].)

DEFINITION 2.4. The inverse image of the topology of the convergence in probability on $L^0(\Omega, \Sigma, \nu)$ under L will be called the topology associated with μ and will be denoted by τ_μ; τ_μ is a linear topology. The topological dual of (E^*, τ_μ) will be called the kernel of μ and will be denoted by K_μ.

It is clear that τ_μ does not depend on the choice of L, and it is identical with the weakest one of all linear topologies with which the characteristic functional of μ is continuous (cf. [5] or [6].) If, for each natural number n, we put

$$U_n=\left\{x^*\in E^*;\ \mu\left\{x\in E;\ \left|\langle x^*, x\rangle\right|>\frac{1}{n}\right\}<\frac{1}{n}\right\},$$

then $\{U_n\}$ forms a fundamental system of neighborhoods of zero with respect to the topology τ_μ.

REMARK 2.1. In general, the inclusions $M_\mu\subset\widetilde{M}_\mu\subset K_\mu$ hold (cf. [4, Proposition 3.1].) It is obvious that K_μ is contained in E if τ_μ is weaker than the Mackey topology τ_k on E^*, and K_μ contains E if and only if τ_μ is stronger than the weak*-topology σ on E.

§ 3. Main theorems

In this section, we shall prove the following main theorems.

THEOREM 3.1. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and suppose that F is barrelled. Let f be a function defined on E^* (but not necessarily everywhere finite) which satisfies the following two conditions;*

(1) $0\leqq f(tx^*)\leqq tf(x^*)\leqq\infty$, *for every* $t>0$ *and every* $x^*\in E^*$,

(2) *for every* $y \in F$, *there is a* $\delta > 0$ *such that the inequality* $f(x^*) < \delta$ *implies* $|\langle x^*, T(y)\rangle| < 1$, *for every* $x^* \in E^*$.

Then there exists a neighborhood V *of zero in* F *such that for every* $x^* \in E^*$, *the inequality*

$$\sup_{y \in V} \left|\left\langle x^*, T(y)\right\rangle\right| \leqq f(x^*)$$

holds.

PROOF. We put $U = \{x^* \in E^*; f(x^*) \leqq 1\}$, and denote by U^0 the polar of U in E. Then the set U^0 is convex balanced close in E, and so $T^{-1}(U^0)$ is also convex balanced closed in F, since T is a continuous linear mapping ~~of F into E~~. Here we shall show that $T^{-1}(U^0)$ is a barrel in F. For this it is sufficient to show that it is absorbing. Let $y \in F$ be given. Then it follows from (2) that there is a $\delta > 0$ such that $f(x^*) < \delta$ (for $x^* \in E^*$) implies $|\langle x^*, T(y)\rangle| < 1$, and so from (1) we have $\delta y \in T^{-1}(U^0)$. This means that the set $T^{-1}(U^0)$ is absorbing, so that it must be a barrel in F. Since F is barrelled, there exists a neighborhood V of zero in F such that $V \subset T^{-1}(U^0)$, and this also means that for every $x^* \in E^*$ with $f(x^*) \leqq 1$, the inequality

$$\sup_{y \in V} \left|\left\langle x^*, T(y)\right\rangle\right| \leqq 1$$

holds. From this and (1), we get the desired inequality. This completes the proof.

REMARK 3.1. As is shown in this proof, if F is barrelled, then Theorem 3.1 certainly holds. Here we can show that the converse is true for every locally convex space F, that is, if F is a locally convex space for which Theorem 3.1 holds, then it must be barrelled. In fact, let F be a locally convex space for which Theorem 3.1 holds. Let $E = F$, and let T be an identity mapping of F onto E. Let A be any barrel in E, and denote by A^0 the polar of A in E^*. Define

$$f(x^*) = \inf\left\{\alpha > 0;\ \frac{1}{\alpha} x^* \in A^0\right\}, \text{ for every } x^* \in E^*.$$

Then the function f defined on E^* (but not necessarily everywhere finite) certainly satisfies the conditions (1) and (2) of Theorem 3.1, and it therefore follows from the assumption of F that there exists a neighborhood V of zero in F such that for every $x^* \in E^*$, the inequality

$$\sup_{y \in V} \left|\left\langle x^*, T(y)\right\rangle\right| \leqq f(x^*)$$

holds. This means $A^0 \subset (T(V))^0$, and so we have $T(V) \subset A$, since A is a barrel. Consequently, E is barrelled, so that F is also barrelled.

THEOREM 3.2. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and suppose that F is of the second category. Let $\{f_n\}$ be a sequence consisting of functions defined on E^* (but not necessarily everywhere finite) which satisfies the following two conditions;*

(1) $0 \leqq f_n(tx^*) \leqq tf_n(x^*) \leqq \infty$, *for every $t>0$, every natural number n and every $x^* \in E^*$,*

(2) *for every $y \in F$, there are a $\delta>0$ and a natural number n such that the inequality $f_n(x^*)<\delta$ implies $|\langle x^*, T(y)\rangle|<1$, for every $x^* \in E^*$.*

Then there exist a natural number n and a neighborhood V of zero in F such that for every $x^* \in E^*$, the inequality

$$\sup_{y \in V} \left|\left\langle x^*, T(y)\right\rangle\right| \leqq f_n(x^*)$$

holds.

PROOF. For each natural numbers m and n, we put

$$A_{m,n} = \left\{x^* \in E^* ; f_n(x^*) \leqq \frac{1}{m}\right\},$$

and denote by $A^o_{m,n}$ the polars of $A_{m,n}$ in E. Then we have $T(F) \subset \bigcup_{m,n} A^o_{m,n}$. For let $y \in F$ be given. It follows from (2) that there are a $\delta>0$ and a natural number n such that for every $x^* \in E^*$, $|\langle x^*, T(y)\rangle|<1$ if $f_n(x^*)<\delta$, so that, taking $m>\frac{1}{\delta}$, we have $T(y) \in A^o_{m,n}$.

Since $F = \bigcup_{m,n} T^{-1}(A^o_{m,n})$ and each $T^{-1}(A^o_{m,n})$ is convex balanced closed in F, and also since F is of the second category, there exist natural numbers m and n, and a neighborhood W of zero in F such that $W \subset T^{-1}(A^o_{m,n})$, so that, if we put $V = \frac{1}{m} W$, then it follows from (1) that for every $x^* \in E^*$ with $f_n(x^*) \leqq 1$, the inequality

$$\sup_{y \in V} \left|\left\langle x^*, T(y)\right\rangle\right| \leqq 1$$

holds. From this and (1), we get the desired inequality. This completes the proof.

REMARK 3.2. As is shown in this proof if F is of the second category, then Theorem 3.2 certainly holds. Here we shall consider the converse. It is shown that if F is a locally convex space for which Theorem 3.2 holds, then it must satisfy the following; for every sequence $\{B_n\}$ consisting of convex balanced closed subsets of F such that $F=\bigcup_n B_n$, there exists some B_n which contains a neighborhood of zero in F. For this we use the same method as in the one of Remark 3.1. We note that if F is a strict (LB)-space, that is, it can be represented as the strict topological inductive limit of a properly increasing sequence of Banach spaces, then it is clearly a complete barrelled space, however, Theorem 3.2 does not hold.

§ 4. Xia's inequality and Chevet's inequality

In this section, we shall apply our main theorems established in Section 3 for the study of quasi-invariant measures and kernels of cylindrical measures and then, we obtain the following inequalities which generalize the results of D. Xia [5], W. Linde [2], S. Chevet [1] and the author [4].

THEOREM 4.1. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and $0<p<\infty$. Suppose that F is barrelled, and also suppose that there exists a cylindrical measure μ on E such that $K_\mu \supset T(F)$. Then there exists a neighborhood V of zero in F such that for every $x^*\in E^*$, the inequality*

$$\sup_{y\in V}\left|\langle x^*, T(y)\rangle\right| \leqq \left(\int_E \left|\langle x^*, x\rangle\right|^p d\mu(x)\right)^{\frac{1}{p}}$$

holds.

PROOF. If we put

$$f(x^*)=\left(\int_E \left|\langle x^*, x\rangle\right|^p d\mu(x)\right)^{\frac{1}{p}}, \text{ for every } x^*\in E^*,$$

then, the function f defined on E^* (but not necessarily everywhere finite) satisfies the conditions (1) and (2) of Theorem 3.1. For this, since f clearly satisfies (1), it is sufficient to show that it satisfies (2). As mentioned in Section 2, the topology τ_μ is identical with the topology of the convergence in probability, it therefore follows from a general theorem on measure theory that for each sequence $\{x_n^*\}\subset E^*$, if the sequence $\{f(x_n^*)\}$ converges to zero, then $\{x_n^*\}$ converges to zero with respect to τ_μ, and since $K_\mu\supset T(F)$, the sequence $\{\langle x_n^*, T(y)\rangle\}$ must converges to zero, for every $y\in F$. This means that f satisfies (2), so that, from Theorem 3.1 we get the desired inequality. This completes the proof.

LEMMA 4.2. *Let E be a linear topological space, F be a linear subspace of E, and μ be a Borel probability measure on E which is quasi-invariant under F. Then, for every measurable linear subspace G of E with $\mu(G)>0$, the inclusion $F\subset G$ holds.*

The proof is easy, so we omit it.

THEOREM 4.3. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and $0<p<\infty$. Suppose that F is barrelled, and also suppose that there exists a Borel probability measure μ on E which is quasi-invariant under $T(F)$. Then, for every measurable subset A of E with $\mu(A)>0$, there exists a neighborhood V of zero in F such that for every $x^* \in E^*$, the inequality*

$$\sup_{y\in V}\left|\left\langle x^*, T(y)\right\rangle\right| \leqq \left(\int_A \left|\langle x^*, x\rangle\right|^p d\mu(x)\right)^{\frac{1}{p}}$$

holds.

PROOF. If we put

$$f(x^*) = \left(\int_A \left|\langle x^*, x\rangle\right|^p d\mu(x)\right)^{\frac{1}{p}}, \text{ for every } x^* \in E^*,$$

then, the function f defined on E^* (but not necessarily everywhere finite) satisfies the conditions (1) and (2) of Theorem 3.1. For this, since f clearly satisfies (1), it is sufficient to show that it satisfies (2). Suppose that f does not satisfy (2). Then there exist an element y of F and a sequence $\{x_n^*\}$ consisting of elements of E^* such that $|\langle x_n^*, T(y)\rangle| \geqq 1$, for all natural numbers n, and the sequence $\{f(x_n^*)\}$ converges to zero. It follows from a general theorem on measure theory that there exists a subsequence $\{x_{n_j}^*\}$ of $\{x_n^*\}$ such that it converges to zero almost surely on A. Here we put

$$G = \left\{x \in E\,;\ \lim_j \langle x_{n_j}^*, x\rangle = 0\right\}.$$

Then, G is a measurable linear subspace of E, and $\mu(G)>0$. It follows from Lemma 4.3 that $T(F) \subset G$, since μ is quasi-invariant under $T(F)$. However, this is a contradiction. Thus f satisfies (2), so that, from Theorem 3.1 we get the desired inequality. This completes the proof.

REMARK 4.1. Theorem 4.3 must be compared with Theorem 4.1. As mentioned in Section 2, if μ is quasi-invariant under $T(F)$, then the kernel of μ contains $T(F)$. But in general, the converse is not true. It can be easily verified that if the Borel probability measure μ on E is not quasi-invariant under $T(F)$, then in general, Theorem 4.3 does not hold even in the case of $K_\mu \supset T(F)$.

THEOREM 4.4. *Let E and F be linear topological spaces, T be a continuous linear mapping of F into E, and suppose that F is of the second category. Also suppose that there exists a cylindrical measure μ on E such* *that* $K_\mu \supset T(F)$. *Then there exists a neighborhood* V *of zero in* F *such that for every* $x^* \in E^*$, *the inequality*

$$\sup_{y \in V} \left| \left\langle x^*, T(y) \right\rangle \right| \leqq J_\varepsilon(\mu)(x^*)$$

holds, where $J_\varepsilon(\mu)$ *is defined as follows*;

$$J_\varepsilon(\mu)(x^*) = \inf\left\{\alpha > 0 \,;\, \mu\left\{x \in E \,;\, \left|\langle x^*, x\rangle\right| > \alpha\right\} < \varepsilon\right\}.$$

PROOF. For each natural number n, we put

$$A_n = \left\{x^* \in E^* \,;\, \mu\left\{x \in E \,;\, \left|\langle x^*, x\rangle\right| > 1\right\} < \frac{1}{n}\right\},$$

and define

$$f_n(x^*) = \inf\left\{\alpha > 0 \,;\, \frac{1}{\alpha} x^* \in A_n\right\}, \text{ for every } x^* \in E^*.$$

Then, as mentioned in Section 2, if we put $U_n = \left\{x^* \in E^* \,;\, f_n(x^*) < \frac{1}{n}\right\}$, then $\{U_n\}$ forms a fundamental system of neighborhoods of zero with respect to the topology τ_μ on E^*. Since $K_\mu \supset T(F)$, the sequence $\{f_n\}$ certainly satisfies the conditions (1) and (2) of Theorem 3. 2, so that there exist a natural number n and a neighborhood V of zero in F such that for every $x^* \in E^*$, the inequality

$$\sup_{y \in V} \left| \left\langle x^*, T(y) \right\rangle \right| \leqq f_n(x^*)$$

holds. Here, if we put $\varepsilon = \frac{1}{n}$, then $f_n(x^*) = J_\varepsilon(\mu)(x^*)$, for every $x^* \in E^*$. This completes the proof.

COROLLARY 4. 5. *Let* E *be a locally convex Hausdorff space of the second category. Suppose that there exists a cylindrical measure* μ *on* E *such that* $K_\mu \supset E$. *Then* E *is normable.*

This proof follows from Theorem 4. 4 and the fact that for every natural number n and every $x^* \in E^*$, $f_n(x^*) < \infty$.

REMARK 4. 2. If F is barrelled, then Theorem 4. 1 certainly holds, but in general, Theorem 4. 4 does not hold. Here we shall give a counter-example for which Corollary 4. 5 does not hold. Let E be the strict inductive limit of a properly increasing sequence $\{E_n\}$ of finite dimensional spaces. Then E is a complete barrelled locally convex Hausdorff space, but it is not of the second category. Let $\{e_n\}$ be a canonical basis of E, and let G be the additive subgroup of E generated by $\{e_n\}$. Since G is a countable set, we put $G=\{x_n\}$. Let $\{c_n\}$ be a sequence consisting of positive numbers such that $\sum c_n=1$. Define $\mu\{x_n\}=c_n$, for all n. Then μ is a Borel probability measure on E such that $K_\mu=E$. However, since E is not normable, Corollary 4.5 does not hold.

§ 5. Applications

In this section, using Theorem 4.1, we shall prove the following theorems.

THEOREM 5.1. *Let E be a barrelled locally convex Hausdorff space, and let $0<p<\infty$. Supppose that there exists a cylindrical measure μ on E of weak p-th order such that $K_\mu\supset E$. Then E is normable, and the strong dual (E^*, b) is isomorphic to a quotient space of a closed linear subspace of $L^p(\nu)$, for some probability space (Ω, Σ, ν).*

PROOF. First, we shall prove that E is normable. It follows from Theorem 4.1 that there exists a neighborhood V of zero in E such that for every $x^*\in E^*$, the inequality

$$(*)\qquad \sup_{x\in V}\left|\langle x^*, x\rangle\right|\leqq\left(\int_E\left|\langle x^*, x\rangle\right|^p d\mu(x)\right)^{\frac{1}{p}}<\infty$$

holds. This implies $E^*=\bigcup_n nV^0$, where V^0 is the polar of V, so that E must be normable, since it is a locally convex Hausdorff space. Next, we shall prove the second assertion. If we put

$$||x^*||_p=\left(\int_E\left|\langle x^*, x\rangle\right|^p d\mu(x)\right)^{\frac{1}{p}},\ \text{for every } x^*\in E^*,$$

then it follows from (*) that $||\cdot||_p$ is a quasi-norm on E^*, and the topology on E^* defined by $||\cdot||_p$ is stronger than the strong topology b on E^*. It is well known (cf. [3]) that each cylindrical measure determines a random linear functional, that is, there exists a probability space (Ω, Σ, ν) and a linear mapping L of E^* into $L^0(\nu)$ such that

$$\mu(Z)=\nu\left\{\omega\in\Omega\,;\ \left(L(x_1^*)(\omega), \cdots, L(x_n^*)(\omega)\right)\in B\right\}$$

for every $x_1^*, \cdots, x_n^*\in E$, and for every cylindrical set $Z=\{x\in E\,; (\langle x_1^*, x\rangle, \cdots, \langle x_n^*, x\rangle)\in B\}$, where B is a Borel subset of R^n. Consequently, for every $x^*\in E^*$, we have

$$||x^*||_p=\left(\int_E\left|\langle x^*, x\rangle\right|^p d\mu(x)\right)^{\frac{1}{p}}=\left(\int_\Omega\left|L(x^*)(\omega)\right|^p d\nu(\omega)\right)^{\frac{1}{p}}.$$

This shows that the quasi-normed space $(E^*, ||\cdot||_p)$ is linearly isometric to a subspace of $L^p(\nu)$, so that, if we denote by G the completion of $(E^*, \|\cdot\|_p)$, then G is also linearly isometric to a closed subspace of $L^p(\nu)$. Since the identity mapping of $(E^*, \|\cdot\|_p)$ onto (E^*, b) is continuous, it can be extended to a continuous linear mapping of G onto (E^*, b). Since (E^*, b) is a Banach space, it follows from the closed graph theorem that it is isomorphic to a quotient space of G. This completes the proof.

Taking $p=2$, we get the following theorem.

THEOREM 5.2. *A quasi-complete barrelled locally convex Hausdorff space E is isomorphic to a Hilbert space if and only if it admits a cylindrical measure μ of weak second order such that $K_\mu \supset E$.*

PROOF. If E is isomorphic to a Hilbert space, then the canonical Gaussian cylindrical measure γ on E certainly satisfies the desired conditions (cf. [2]). On the other hand, the converse assertion follows from Theorem 5.1. This completes the proof.

REMARK 5.1. Theorem 5.1 can be slightly generalized as follows: Let E be a locally convex Hausdorff space, F be a linear subspace of E, and μ be a cylindrical measure on E of weak p-th order $(0<p<\infty)$. If $K_\mu \supset F$, and F is barrelled with respect to the induced topology, then F is normable, and the strong dual (F^*, b) is isomorphic to a quotient space of a closed linear subspace of $L^p(\nu)$, for some probability space (Ω, Σ, ν). In this case, if $p=2$, then F is isomorphic to a pre-Hilbert space. This proof can be done by the same way as in the proof of Theorem 5.1.

COROLLARY 5.3. *Let E be a Fréchet space and F be a closed linear subspace of E. Then the space F equipped with the induced topology is isomorphic to a Hilbert space if and only if there exists a cylindrical measure μ on E of weak second order such that $K_\mu \supset F$.*

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