Michigan Math. J. 59 (2010), 411–417

An Elementary Proof of the Cross Theorem in the Reinhardt Case

Marek Jarnicki & Peter Pflug

1. Introduction and Main Result

The problem of continuation of separately holomorphic functions defined on a cross has been investigated in several papers (e.g., [B; S1; S2; AkR; Za; S3; Sh; NS; NZ1; NZ2; N; AZ; Z]) and may be formulated in the form of the following *cross theorem.*

Theorem 1.1. *Let $D_j \subset \mathbb{C}^{n_j}$ be a domain of holomorphy and let $A_j \subset D_j$ be a locally pluriregular set, $j = 1, \dots, N$, $N \ge 2$. Define the* cross

$$\boldsymbol{X} := \bigcup_{j=1}^{N} A_1 \times \cdots \times A_{j-1} \times D_j \times A_{j+1} \times \cdots \times A_N.$$

Let $f : \boldsymbol{X} \to \mathbb{C}$ be separately holomorphic*—that is, for any $(a_1, \dots, a_N) \in A_1 \times \cdots \times A_N$ and $j \in \{1, \dots, N\}$, the function*

$$D_j \ni z_j \longmapsto f(a_1, \dots, a_{j-1}, z_j, a_{j+1}, \dots, a_N) \in \mathbb{C}$$

is holomorphic. Then f extends holomorphically to a uniquely determined function $\hat{f}$ on the domain of holomorphy

$$\hat{\boldsymbol{X}} := \left\{(z_1, \dots, z_N) \in D_1 \times \cdots \times D_N : \sum_{j=1}^{N} h^*_{A_j, D_j}(z_j) < 1\right\}, \qquad (*)$$

where $h^*_{A_j, D_j}$ is the upper regularization of the relative extremal function h_{A_j, D_j}, *$j = 1, \dots, N$.*

Recall that $h_{A,D} := \sup\{u \in \mathcal{PSH}(D) : u \le 1, u|_A \le 0\}$.

Observe that in the case where A_j is open, $j = 1, \dots, N$, the cross $\boldsymbol{X}$ is a domain in $\mathbb{C}^n$ with $n := n_1 + \cdots + n_N$. Moreover, by the classical Hartogs lemma, every separately holomorphic function on $\boldsymbol{X}$ is simply holomorphic. Consequently, the formula $(*)$ is nothing more than a description of the envelope of holomorphy of $\boldsymbol{X}$. Thus, it is natural to conjecture that in this case the formula $(*)$ may be obtained without the cross theorem machinery. Unfortunately, we do not know of any such simplification.

Received February 5, 2009. Revision received June 5, 2009.
The research was partially supported by grant no. N N201 361436 of the Ministry of Science and Higher Education and DFG-grant 436POL113/103/0-2.

The aim of this note is to present an elementary geometric proof of Theorem 1.1 in the case where D_j is a Reinhardt domain and A_j is a nonempty Reinhardt open set, $j = 1, \dots, N$. The proof (Section 4) will be based on well-known interrelations between the holomorphic geometry of a Reinhardt domain and the convex geometry of its logarithmic image. Moreover, the cross theorem for the Reinhardt case may be taught in any lecture on several complex variables; its proof needs only some basic facts for Reinhardt domains (see [JP]).

2. Convex Geometry

We begin with some elementary results related to the convex domains in $\mathbb{R}^n$.

DEFINITION 2.1. Let $\emptyset \neq S \subset U \subset \mathbb{R}^n$, where U is a convex domain. Define the *convex extremal function*

$$\Phi_{S,U} := \sup\{\varphi \in \mathcal{CVX}(U),\ \varphi \le 1,\ \varphi|_S \le 0\},$$

where $\mathcal{CVX}(U)$ stands for the family of all convex functions $\varphi\colon U \to [-\infty, +\infty)$.

REMARK 2.2. (a) $\Phi_{S,U} \in \mathcal{CVX}(U)$, $0 \le \Phi_{S,U} < 1$, and $\Phi_{S,U} = 0$ on S.

(b) $\Phi_{\operatorname{conv}(S),U} \equiv \Phi_{S,U}$.

(c) If $\emptyset \neq S_k \subset U_k \subset \mathbb{R}^n$, U_k is a convex domain, $k \in \mathbb{N}$, $S_k \nearrow S$, and $U_k \nearrow U$, then $\Phi_{S_k,U_k} \searrow \Phi_{S,U}$.

(d) For $0 < \mu < 1$, let $U_\mu := \{x \in U : \Phi_{S,U}(x) < \mu\}$ (observe that U_μ is a convex domain with $S \subset U_\mu$). Then $\Phi_{S,U_\mu} = (1/\mu)\Phi_{S,U}$ on U_μ.

Indeed, the inequality "$\ge$" is obvious. To prove the opposite inequality, let

$$\varphi := \begin{cases} \max\{\Phi_{S,U}, \mu\Phi_{S,U_\mu}\} & \text{on } U_\mu, \\ \Phi_{S,U} & \text{on } U \setminus U_\mu. \end{cases}$$

Then $\varphi \in \mathcal{CVX}(U)$, $\varphi < 1$, and $\varphi = 0$ on S. Thus $\varphi \le \Phi_{S,U}$ and hence $\Phi_{S,U_\mu} \le (1/\mu)\Phi_{S,U}$ in U_μ.

(e) Let $\emptyset \neq S_j \subset U_j \subset \mathbb{R}^{n_j}$, where U_j is a convex domain, $j = 1, \dots, N$, $N \ge 2$. Put

$$W := \left\{(x_1, \dots, x_N) \in U_1 \times \cdots \times U_N : \sum_{j=1}^{N} \Phi_{S_j,U_j}(x_j) < 1\right\}$$

(observe that W is a convex domain with $S_1 \times \cdots \times S_N \subset W$). Then

$$\Phi_{S_1 \times \cdots \times S_N, W}(x) = \sum_{j=1}^{N} \Phi_{S_j,U_j}(x_j), \quad x = (x_1, \dots, x_N) \in W.$$

Indeed, the inequality "$\ge$" is obvious. To prove the opposite inequality we use induction on $N \ge 2$.

Let $N = 2$. To simplify notation write $A := S_1$, $U := U_1$, $B := S_2$, and $V := U_2$. Observe that $T := (A \times V) \cup (U \times B) \subset W$; then directly from the definition we get

$$\Phi_{A \times B, W}(x, y) \le \Phi_{A,U}(x) + \Phi_{B,V}(y), \quad (x, y) \in T.$$

Fix a point $(x_0, y_0) \in W \setminus T$. Let

$$\mu := 1 - \Phi_{A,U}(x_0) \in (0,1], \qquad V_\mu := \{y \in V : \Phi_{B,V}(y) < \mu\},$$

$$\varphi := \frac{1}{\mu}(\Phi_{A\times B,W}(x_0,\cdot) - \Phi_{A,U}(x_0)).$$

Then φ is a well-defined convex function on V_μ, $\varphi < 1$ on V_μ, and $\varphi \le 0$ on B. Thus, by (d), $\varphi(y_0) \le \Phi_{B,V_\mu}(y_0) = (1/\mu)\Phi_{B,V}(y_0)$, which finishes the proof.

Now, assume that the formula is true for $N-1 \ge 2$. Put $S' := S_1\times\cdots\times S_{N-1}$ and

$$W' := \left\{(x_1,\dots,x_{N-1}) \in U_1 \times \cdots \times U_{N-1} : \sum_{j=1}^{N-1} \Phi_{S_j,U_j}(x_j) < 1\right\}.$$

Then, by the inductive hypothesis, we have

$$\Phi_{S',W'}(x') = \sum_{j=1}^{N-1} \Phi_{S_j,U_j}(x_j), \quad x' = (x_1,\dots,x_{N-1}) \in W'.$$

Consequently,

$$W = \{(x', x_N) \in W' \times U_N : \Phi_{S',W'}(x') + \Phi_{S_N,U_N}(x_N) < 1\}.$$

Hence, using the case $N = 2$ (to $S' \subset W'$ and $S_N \subset U_N$), we get

$$\Phi_{S_1\times\cdots\times S_N,W}(x) = \Phi_{S',W'}(x') + \Phi_{S_N,U_N}(x_N) = \sum_{j=1}^{N} \Phi_{S_j,U_j}(x_j),$$
$$x = (x', x_N) = (x_1,\dots,x_N) \in W.$$

Notice that properties (d) and (e) correspond to analogous properties of the relative extremal function (cf. e.g. [S3]).

PROPOSITION 2.3. *Let $\emptyset \ne S_j \subset U_j \subset \mathbb{R}^{n_j}$, where U_j is a convex domain and* $\operatorname{int} S_j \ne \emptyset$, $j = 1,\dots,N$, $N \ge 2$, *and define the cross*

$$T := \bigcup_{j=1}^{N} S_1 \times \cdots \times S_{j-1} \times U_j \times S_{j+1} \times \cdots \times S_N.$$

Then

$$\operatorname{conv}(T) = \left\{(x_1,\dots,x_N) \in U_1 \times \cdots \times U_N : \sum_{j=1}^{N} \Phi_{S_j,U_j}(x_j) < 1\right\} =: W.$$

(It seems to us that this "convex cross theorem" is so far nowhere in the literature.)

Proof. We may assume that S_j is convex, $j = 1,\dots,N$ (cf. Remark 2.2(b)). The inclusion "$\subset$" is obvious. Let

$$T_j := S_1 \times \cdots \times S_{j-1} \times U_j \times S_{j+1} \times \cdots \times S_N, \quad j = 1,\dots,N,$$

$$T' := \bigcup_{j=1}^{N-1} S_1 \times \cdots \times S_{j-1} \times U_j \times S_{j+1} \times \cdots \times S_{N-1}, \quad S' := S_1 \times \cdots \times S_{N-1}.$$

Recall (cf. [Ro, Thm. 3.3]) that

$$\operatorname{conv}(T) = \bigcup_{\substack{t_1,\dots,t_N \ge 0 \\ t_1+\dots+t_N=1}} t_1T_1 + \dots + t_NT_N$$
$$= \operatorname{conv}((\operatorname{conv}(T') \times S_N) \cup (S' \times U_N)). \qquad (**)$$

We use induction on N. Suppose $N = 2$. To simplify notation write $A := S_1$, $U := U_1$, $p := n_1$, $B := S_2$, $V := U_2$, and $q := n_2$. Using Remark 2.2(c), we may assume that U and V are bounded.

Since $\operatorname{conv}(T)$ is open and $\operatorname{conv}(T) \subset W$, we only need to show that for every $(x_0, y_0) \in \partial(\operatorname{conv}(T)) \cap (U \times V)$ we have $\Phi_{A,U}(x_0) + \Phi_{B,V}(y_0) = 1$. Since U, V are bounded, we have $\overline{\operatorname{conv}(T)} = \operatorname{conv}(\bar{T})$ (cf. [Ro, Thm. 17.2]) and therefore $(x_0, y_0) = t(x_1, y_1) + (1 - t)(x_2, y_2)$, where $t \in [0, 1]$, $(x_1, y_1) \in \bar{A} \times \bar{U}$, and $(x_2, y_2) \in \bar{U} \times \bar{B}$. First observe that $t \in (0, 1)$.

Indeed, suppose for instance that $(x_0, y_0) \in U \times (\bar{B} \cap V)$. Take an arbitrary $x_* \in \operatorname{int} A$ and let $r > 0$ and $\varepsilon > 0$ be such that the Euclidean ball $\mathbb{B}((x_*, y_0), r)$ is contained in $A \times V$ and $x_{**} := x_* + \varepsilon(x_0 - x_*) \in U$. Then

$$(x_0, y_0) \in \operatorname{int}(\operatorname{conv}(\mathbb{B}((x_*, y_0), r) \cup \{(x_{**}, y_0)\}))$$
$$\subset \operatorname{int}(\operatorname{conv}(\bar{T})) = \operatorname{int}(\overline{\operatorname{conv}(T)}) = \operatorname{conv}(T);$$

a contradiction.

Let $L\colon \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}$ be a linear form such that $L(x_0, y_0) = 1$ and $L \le 1$ on T. Since $1 = L(x_0, y_0) = tL(x_1, y_1) + (1 - t)L(x_2, y_2)$, we conclude that $L(x_1, y_1) = L(x_2, y_2) = 1$. Write $L(x, y) = P(x) + Q(y)$, where $P\colon \mathbb{R}^p \to \mathbb{R}$ and $Q\colon \mathbb{R}^q \to \mathbb{R}$ are linear forms.

Put $P_C := \sup_C P$ with $C \subset \mathbb{R}^p$ and $Q_D := \sup_D Q$ with $D \subset \mathbb{R}^q$. Since $L \le 1$ on T and $L(x_1, y_1) = L(x_2, y_2) = 1$, we conclude that

$$P_A + Q_V = 1,$$
$$P_U + Q_B = 1.$$

In particular, $P_A = P_U$ if and only if $Q_B = Q_V$. Consider the following two cases.

(i) $P_A < P_U$ and $Q_B < Q_V$: Then

$$\frac{P - P_A}{P_U - P_A} \le \Phi_{A,U}, \qquad \frac{Q - Q_B}{Q_V - Q_B} \le \Phi_{B,V}.$$

Hence

$$\Phi_{A,U}(x_0) + \Phi_{B,V}(y_0) \ge \frac{P(x_0) - P_A}{1 - Q_B - P_A} + \frac{Q(y_0) - Q_B}{1 - P_A - Q_B} = 1.$$

(ii) $P_A = P_U$ and $Q_B = Q_V$: Then $P_U + Q_V = 1$, which implies that $(x_0, y_0) \in U \times V \subset \{L < 1\}$—a contradiction.

Now, assume that the result is true for $N - 1 \ge 2$. In particular,

$$\operatorname{conv}(T') = \left\{(x_1, \dots, x_{N-1}) \in U_1 \times \dots \times U_{N-1} : \sum_{j=1}^{N-1} \Phi_{S_j,U_j}(x_j) < 1\right\} =: W'.$$

Using $(**)$, the case $N = 2$, and Remark 2.2(e), we get

$$\begin{aligned}\operatorname{conv}(T) &= \operatorname{conv}((W' \times S_N) \cup ((S' \times U_N)) \\ &= \{(x', x_N) \in W' \times U_N : \Phi_{S',W'}(x') + \Phi_{S_N,U_N}(x_N) < 1\} = W. \qquad \square\end{aligned}$$

3. Reinhardt Geometry

Now we recall basic facts related to Reinhardt domains.

DEFINITION 3.1. We say that a set $A \subset \mathbb{C}^n$ is a *Reinhardt set* if for every $(a_1, \dots, a_n) \in A$ we have

$$\{(z_1, \dots, z_n) \in \mathbb{C}^n : |z_j| = |a_j|,\ j = 1, \dots, n\} \subset A$$

(cf. [JP, Def. 1.5.2]). Put

$$\boldsymbol{V}_j := \mathbb{C}^{n-j-1} \times \{0\} \times \mathbb{C}^{n-j}, \qquad \boldsymbol{V}_0 := \boldsymbol{V}_1 \cup \dots \cup \boldsymbol{V}_n,$$

$$\log A := \{(\log|z_1|, \dots, \log|z_n|) : (z_1, \dots, z_n) \in A \setminus \boldsymbol{V}_0\}, \quad A \subset \mathbb{C}^n,$$

$$\exp S := \{(z_1, \dots, z_n) \in \mathbb{C}^n \setminus \boldsymbol{V}_0 : (\log|z_1|, \dots, \log|z_n|) \in S\}, \quad S \subset \mathbb{R}^n,$$

$$A^* := \operatorname{int}(\overline{\exp(\log A)}), \quad A \subset \mathbb{C}^n.$$

We say that a set $A \subset \mathbb{C}^n$ is *logarithmically convex* (*log-convex*) if $\log A$ is convex (cf. [JP, Def. 1.5.5]).

THEOREM 3.2 [JP, Thm. 1.11.13]. *Let $\Omega \subset \mathbb{C}^n$ be a Reinhardt domain. Then the following conditions are equivalent*:

(i) Ω *is a domain of holomorphy*;
(ii) Ω *is log-convex and* $\Omega = \Omega^* \setminus \bigcup_{\substack{j \in \{1,\dots,n\} \\ \Omega \cap \boldsymbol{V}_j = \emptyset}} \boldsymbol{V}_j$.

THEOREM 3.3 [JP, Thm. 1.12.4]. *For every Reinhardt domain $\Omega \subset \mathbb{C}^n$, its envelope of holomorphy $\hat{\Omega}$ is a Reinhardt domain.*

COROLLARY 3.4. *Let $\Omega \subset \mathbb{C}^n$ be a Reinhardt domain and let $\hat{\Omega}$ be its envelope of holomorphy. Then*:

(a) $\boldsymbol{V}_j \cap \hat{\Omega} = \emptyset$ *if and only if* $\boldsymbol{V}_j \cap \Omega = \emptyset$;
(b) $\log \hat{\Omega} = \operatorname{conv}(\log \Omega)$.

Consequently, by Theorem 3.3,

$$\hat{\Omega} = \operatorname{int}(\overline{\exp(\operatorname{conv}(\log \Omega))}) \setminus \bigcup_{\substack{j \in \{1,\dots,n\} \\ \Omega \cap \boldsymbol{V}_j = \emptyset}} \boldsymbol{V}_j =: \tilde{\Omega}.$$

Proof. (a) If $\boldsymbol{V}_j \cap \Omega = \emptyset$, then the function $\Omega \ni z_j \mapsto 1/z_j$ is holomorphic on Ω. Thus, it must be holomorphically continuable to $\hat{\Omega}$, which means that $\boldsymbol{V}_j \cap \hat{\Omega} = \emptyset$.

(b) First observe that, by [JP, Rem. 1.5.6(a)], we get $\log \tilde{\Omega} = \operatorname{conv}(\log \Omega)$. Consequently, $\tilde{\Omega}$ is a domain of holomorphy with $\Omega \subset \tilde{\Omega}$. Hence, $\hat{\Omega} \subset \tilde{\Omega}$. Finally, $\log \Omega \subset \log \hat{\Omega} \subset \log \tilde{\Omega} = \operatorname{conv}(\log \Omega)$. $\square$

PROPOSITION 3.5 [JP, Prop. 1.14.20]. *Let Ω be a log-convex Reinhardt domain.*

(a) *Let $u \in \mathcal{PSH}(\Omega)$ be such that*

$$u(z_1,\dots,z_n) = u(|z_1|,\dots,|z_n|), \quad (z_1,\dots,z_n) \in \Omega.$$

Then the function

$$\log\Omega \ni (x_1,\dots,x_n) \overset{\varphi}{\longmapsto} u(e^{x_1},\dots,e^{x_n})$$

is convex.

(b) *Let $\varphi \in \mathcal{CVX}(\log\Omega)$. Then the function*

$$\Omega \setminus V_0 \ni z \overset{u}{\longmapsto} \varphi(\log|z_1|,\dots,\log|z_n|)$$

is plurisubharmonic.

COROLLARY 3.6. *Let $\emptyset \neq A \subset \Omega$, where Ω is a log-convex Reinhardt domain and A is a Reinhardt open set. Then*

$$h^*_{A,D}(z) = \Phi_{\log A, \log\Omega}(\log|z_1|,\dots,\log|z_n|), \quad z = (z_1,\dots,z_n) \in \Omega \setminus V_0$$

(*cf. Definition 2.1*).

Proof. Since A and Ω are invariant under rotations, we easily conclude that

$$h^*_{A,D}(z) = h^*_{A,D}(|z_1|,\dots,|z_n|), \quad z = (z_1,\dots,z_n) \in \Omega.$$

Thus, by Proposition 3.5,

$$h^*_{A,D}(z) = \varphi(\log|z_1|,\dots,\log|z_n|), \quad z = (z_1,\dots,z_n) \in \Omega \setminus V_0,$$

where $\varphi \in \mathcal{CVX}(\log\Omega)$. Clearly, $h^*_{A,D} = 0$ on A. Thus $\varphi = 0$ on $\log A$. Finally, $\varphi \le \Phi_{\log A,\log\Omega}$.

To prove the opposite inequality, observe that by Proposition 3.5, the function

$$\Omega \setminus V_0 \ni z \overset{u}{\longmapsto} \Phi_{\log A,\log\Omega}(\log|z_1|,\dots,\log|z_n|)$$

is plurisubharmonic, $u < 1$, and $u = 0$ on $A \setminus V_0$. Consequently, u extends to a $\tilde{u} \in \mathcal{PSH}(\Omega)$. Clearly, $\tilde{u} \le 1$ and $\tilde{u} = 0$ on A. Thus $\tilde{u} \le h^*_{A,D}$. □

4. Proof of the Cross Theorem When D_j Is a Reinhardt Domain of Holomorphy and A_j Is an Open Reinhardt Set, $j = 1,\dots,N$

We have to prove that the envelope of holomorphy $\hat{X}$ of the domain X coincides with

$$\tilde{X} := \left\{(z_1,\dots,z_N) \in D_1 \times \cdots \times D_N : \sum_{j=1}^{N} h^*_{A_j,D_j}(z_j) < 1\right\}.$$

First, observe that $\tilde{X}$ is a domain of holomorphy containing X. Thus $\hat{X} \subset \tilde{X}$. On the other hand, by Proposition 2.3 and Corollary 3.6, $\log\tilde{X} = \operatorname{conv}(\log X) = \log\hat{X}$. Thus, using Corollary 3.4, we only need to show that if $V_j \cap \tilde{X} \neq \emptyset$, then

$\boldsymbol{V}_j \cap \boldsymbol{X} \neq \emptyset$. Indeed, let for example $a = (a_1, \ldots, a_N) \in \boldsymbol{V}_n \cap \tilde{\boldsymbol{X}} \neq \emptyset$. Take arbitrary $b_j \in A_j$, $j = 1, \ldots, N-1$. Then $(b_1, \ldots, b_{N-1}, a_N) \in \boldsymbol{V}_n \cap \boldsymbol{X}$. □

References

[AkR] N. I. Akhiezer and L. I. Ronkin, *Separately analytic functions of several variables and "edge of the wedge" theorems,* Uspekhi Mat. Nauk 28 (1973), 27–44.

[AZ] O. Alehyane and A. Zeriahi, *Une nouvelle version du théorème d'extension de Hartogs pour les applications séparément holomorphes entre espaces analytiques,* Ann. Polon. Math. 76 (2001), 245–278.

[B] S. N. Bernstein, *Sur l'ordre de la meilleure approximation des fonctions continues par des polynomes de degré donné,* Bruxelles, 1912.

[JP] M. Jarnicki and P. Pflug, *First steps in several complex variables: Reinhardt domains,* EMS Textbk. Math., Eur. Math. Soc., Zurich, 2008.

[N] Nguyen Thanh Van, *Separate analyticity and related subjects,* Vietnam J. Math. 25 (1997), 81–90.

[NS] Nguyen Thanh Van and J. Siciak, *Fonctions plurisousharmoniques extrémales et systèmes doublement orthogonaux de fonctions analytiques,* Bull. Sci. Math. 115 (1991), 235–244.

[NZ1] Nguyen Thanh Van and A. Zeriahi, *Une extension du théorème de Hartogs sur les fonctions séparément analytiques,* Analyse Complexe Multivariables, Récents Dévelopements (Guadeloupe, 1988), Sem. Conf., 5, pp. 183–194, EditEl, Rende, 1991.

[NZ2] ———, *Systèmes doublement othogonaux de fonctions holomorphes et applications,* Topics in complex analysis (Warsaw, 1992), Banach Center Publ., 31, pp. 281–297, Polish Acad. Sci. Inst. Math., Warsaw, 1995.

[Ro] R. T. Rockafellar, *Convex analysis,* Princeton Univ. Press, Princeton, NJ, 1970.

[Sh] B. Shiffman, *Separate analyticity and Hartogs theorems,* Indiana Univ. Math. J. 38 (1989), 943–957.

[S1] J. Siciak, *Analyticity and separate analyticity of functions defined on lower dimensional subsets of* $\mathbb{C}^n$, Zeszyty Nauk. Uniw. Jagiello. Prace Mat. Zeszyt 13 (1969), 53–70.

[S2] ———, *Separately analytic functions and envelopes of holomorphy of some lower dimensional subsets of* $\mathbb{C}^n$, Ann. Polon. Math. 22 (1969/1970), 147–171.

[S3] ———, *Extremal plurisubharmonic functions in* $\mathbb{C}^N$, Ann. Polon. Math. 39 (1981), 175–211.

[Za] V. P. Zahariuta, *Separately analytic functions, generalizations of Hartogs theorem, and envelopes of holomorphy,* Mat. Sb. (N.S.) 30 (1976), 51–67.

[Z] A. Zeriahi, *Comportement asymptotique des systemes doublement orthogonaux de Bergman: Une approche élémentaire,* Vietnam J. Math. 30 (2002), 177–188.

M. Jarnicki
Institute of Mathematics
Jagiellonian University
Łojasiewicza 6
30-348 Kraków
Poland

Marek.Jarnicki@im.uj.edu.pl

P. Pflug
Institut für Mathematik
Carl von Ossietzky Universität Oldenburg
Postfach 2503
D-26111 Oldenburg
Germany

pflug@mathematik.uni-oldenburg.de