A Sufficient Condition for $\text{Proj}^1 \mathcal{X} = 0$

RÜDIGER W. BRAUN & DIETMAR VOGT

1. INTRODUCTION. In [7], Palamodov established a homological theory for projective spectra of topological vector spaces. In applications of this theory, it is crucial to decide whether, for a given projective spectrum $\mathcal{X}$ of (DFS) spaces, a certain vector space $\text{Proj}^1 \mathcal{X}$ is trivial. A topological characterization of $\text{Proj}^1 \mathcal{X} = 0$ has been given by Retakh [8]. In practical cases, its evaluation is hard. In Vogt [9; 10], more tractable conditions were given, which were motivated from the structure theory of nuclear Fréchet spaces. There is a sufficient as well as a necessary condition, but these are probably different. In the case of sequence spaces, it is shown in [9] that the necessary condition is also sufficient. Recently, Wengenroth [11; 12] has proved the sufficiency of the necessary condition also for (DFM) spectra. His proof is based on the investigation of topological properties of the dual inductive spectrum. In the present paper, we give a direct proof of Wengenroth's result for the case of (DFS) spectra. It grew out of a third condition, the sufficiency of which was shown in Braun [1].

To present an application, let $P(D)$ denote a constant coefficient partial differential operator, let $\Omega \subset \mathbb{R}^N$ be a convex domain, and denote by $A(\Omega)$ the space of all real analytic functions on Ω and by $\Gamma^d(\Omega)$ the Gevrey class of exponent d. Recall that $\Gamma^1 = A$ and that $\Gamma^d(\Omega)$ contains test functions if $d > 1$. Hörmander [5] has characterized the surjective operators $P(D): A(\Omega) \to A(\Omega)$. He used a Mittag–Leffler procedure to show the sufficiency of his condition and a Baire category argument to derive necessity. Braun, Meise, and Vogt [2; 3] used Palamodov's homological approach to extend this theorem to $\Gamma^d(\mathbb{R}^N)$, $d > 1$. To do so, they proved the equivalence of Vogt's two conditions using Fourier analysis. This failed in the case of arbitrary convex domains, which was solved in Braun [1]. The functional analysis part of this proof was distilled out of the Mittag–Leffler argument in Section 5 of Hörmander [5]. Further refinement then led to the proof presented here. A very similar proof was independently found by Frerick and Wengenroth [4] by dualizing the acyclicity argument of Wengenroth [11; 12].

The authors thank the referee for suggesting a stronger formulation of Corollary 10.

2. DEFINITION. A projective spectrum $(X_k, \iota_{k+1}^k)_k$ consists of a sequence $(X_k)_k$ of vector spaces together with linear mappings $\iota_{k+1}^k: X_{k+1} \to X_k$. Each of the spaces X_k is the inductive limit $X_k = \bigcup_{n=1}^{\infty} X_{k,n}$ of a sequence $X_{k,1} \subset X_{k,2} \subset \cdots$

Received February 28, 1996. Revision received August 23, 1996.
Michigan Math. J. 44 (1997).

of Banach spaces. The unit ball of $X_{k,n}$ is denoted by $B_{k,n}$. For $K \geq k$, we define spectral mappings by

$$\iota_K^k = \begin{cases} \mathrm{id}_{X_k}, & K = k, \\ \iota_{k+1}^k \circ \cdots \circ \iota_K^{K-1}, & K > k. \end{cases}$$

We require that

$$\iota_K^k X_{K,n} \subset X_{k,n} \quad \text{for all } k,\ K, \text{ and } n. \tag{1}$$

3. THEOREM (Palamodov [7, p. 542]). *There is a functor* Proj^1 *such that, for each exact sequence of projective spectra*

$$0 \to \mathcal{X} \to \mathcal{Y} \to \mathcal{Z} \to 0,$$

there is a natural long exact sequence of vector spaces

$$0 \to \mathrm{proj}\,\mathcal{X} \to \mathrm{proj}\,\mathcal{Y} \to \mathrm{proj}\,\mathcal{Z} \to \mathrm{Proj}^1\,\mathcal{X} \to \mathrm{Proj}^1\,\mathcal{Y} \to \mathrm{Proj}^1\,\mathcal{Z} \to 0.$$

In applications, it is crucial to have topological criteria for $\mathrm{Proj}^1\,\mathcal{X} = 0$. The following definition presents two of those.

4. DEFINITION (Vogt [9; 10]). For a projective spectrum $\mathcal{X}$ as in Definition 2, we define the following properties.

(P_2) $\mathcal{X}$ is said to satisfy condition (P_2) if, for each μ, there are n and k such that, for all K and m, there are N and S satisfying

$$\iota_k^\mu B_{k,m} \subset S(\iota_K^\mu B_{K,N} + B_{\mu,n}).$$

($\overline{P_2}$) For each μ there are k and n such that, for all K, m, and $\varepsilon > 0$, there are N and S with

$$\iota_k^\mu B_{k,m} \subset S\iota_K^\mu B_{K,N} + \varepsilon B_{\mu,n}.$$

Note that neither (P_2) nor ($\overline{P_2}$) are changed if for any k the sequence $X_{k,1} \subset X_{k,2} \cdots$ is replaced by a subsequence. Thus the assumption (1) does not restrict generality.

The spectrum $\mathcal{X}$ is said to be *reduced* if, for each μ, there is k such that, for all K, the closure of $\iota_K^\mu X_K$ in X_μ contains $\iota_k^\mu X_k$. This definition of being reduced is implied by the usual one as well as by ($\overline{P_2}$).

5. LEMMA. *If the spectrum $\mathcal{X}$ is reduced and satisfies* (P_2), *then there are, for each μ, numbers k and n such that for each K we have*

$$\iota_k^\mu X_k \subset \iota_K^\mu X_K + B_{\mu,n}. \tag{2}$$

For given μ, the number n is the same as in (P_2).

Proof. For given μ there are n and k as in (P_2). Let K be given. Then (P_2) implies that for each m there are numbers $N(m)$ and $S(m)$ with

$$\iota_k^\mu B_{k,m} \subset S(m)(\iota_K^\mu B_{K,N(m)} + B_{\mu,n}). \tag{3}$$

Since the spectrum is reduced, there exists an l such that, for all K, the space $\iota_K^k X_K$ is dense in $\iota_l^k X_l$ in the topology of X_k. We claim that

$$\iota_l^\mu X_l \subset \iota_K^\mu X_K + B_{\mu,n}.$$

Assume this to be false. Then there is an $x \in X_l$ with $\iota_l^\mu x \notin \iota_K^\mu X_K + B_{\mu,n}$. Set

$$U = \Gamma\left(\bigcup_{j=1}^{\infty} \frac{1}{S(j)} B_{k,j}\right).$$

This is a neighborhood of zero in X_k (see e.g. [6, 24.6]). Since $\iota_K^k X_K$ is dense in $\iota_l^k X_l$, there are $\xi \in X_K$ and $\sum_{j=1}^L \lambda_j b_j \in U$, that is, $\lambda_j \geq 0$ with $\sum_{j=1}^L \lambda_j = 1$ and $b_j \in S(j)^{-1} B_{k,j}$, $j = 1, \dots, L$, such that

$$\iota_l^k x = \iota_K^k \xi + \sum_{j=1}^{L} \lambda_j b_j.$$

By (3), we have

$$\iota_k^\mu b_j \in \iota_K^\mu X_K + B_{\mu,n}.$$

This implies

$$\iota_l^\mu x \in \iota_K^\mu X_K + B_{\mu,n}.$$

As this contradicts the assumption, the claim is shown. □

6. Remark. By removing some of the steps X_k, we can arrange that $(\overline{\mathrm{P}_2})$ holds in the following form.

$(\overline{\mathrm{P}_2})'$ For each $k \in \mathbb{N}$ there is $n(k) \in \mathbb{N}$ such that, for all $K, m \in \mathbb{N}$ and $\varepsilon > 0$, there are $N = N(k, K, m, \varepsilon) \in \mathbb{N}$ and $S = S(k, K, m, \varepsilon) > 0$ with

$$\iota_k^{k-1} B_{k,m} \subset S \iota_K^{k-1} B_{K,N} + \varepsilon B_{k-1,n(k)}.$$

7. Lemma. *If a spectrum $\mathcal{X}$ satisfies $(\overline{\mathrm{P}_2})$, then there is a sequence $(\tilde{n}(k))_{k\in\mathbb{N}}$ of natural numbers such that the following holds: For all $k, K, m \in \mathbb{N}$ there are $\tilde{N}(k, K, m) \in \mathbb{N}$ and $\tilde{S}(k, K, m) > 0$ with*

$$\iota_k^{k-1} B_{k,m} \subset \tilde{S}(k, K, m) \iota_K^{k-1} B_{K,\tilde{N}(k,K,m)} + \bigcap_{\nu=1}^{k-1} (\iota_{k-1}^\nu)^{-1} B_{\nu,\tilde{n}(\nu)}.$$

Proof. We may assume that $(\overline{\mathrm{P}_2})$ holds in the form $(\overline{\mathrm{P}_2})'$. First, we define $\tilde{S}$, $\tilde{N}$, and $\tilde{n}$, using S, N, and n as in Remark 6. The definition proceeds by induction over k, starting with $k = 2$:

$$\begin{aligned}
\tilde{S}(2, K, m) &= S(2, K, m, 1),\\
\tilde{S}(k, K, m) &= 2S\left(k, K, m, \frac{1}{2\tilde{S}(k-1, k, n(k))}\right),\\
\varepsilon_2 &= 1,\\
\varepsilon_k &= \frac{1}{2\big(1+\tilde{S}(k-1, k, n(k))\big)},\\
\tilde{N}(2, K, m) &= N(2, K, m, 1),\\
\tilde{N}(k, K, m) &= \begin{cases} N\left(k-1, K, \tilde{N}(k-1, k, n(k)), \varepsilon_k\right) & \text{if } m \le \tilde{N}(k-1, k, n(k)),\\ N(k-1, K, m, \varepsilon_k) & \text{if } m > \tilde{N}(k-1, k, n(k)),\end{cases}\\
\tilde{n}(2) &= n(2),\\
\tilde{n}(k) &= \max\big(n(k+1), \tilde{N}(k, k+1, n(k+1))\big).
\end{aligned}$$

It is immediate from $(\overline{\mathrm{P}_2})'$ that the claim holds for $k = 2$. To proceed inductively, let $k \ge 3$ be given and assume that the assertion is true for $k-1$. Let K and m be given and define

$$\tilde{S} = \tilde{S}(k-1, k, n(k)).$$

The induction hypothesis implies

$$\iota_{k-1}^{k-2} B_{k-1,n(k)} \subset \tilde{S}\iota_k^{k-2} B_{k,\tilde{N}(k-1,k,n(k))} + \bigcap_{\nu=1}^{k-2} (\iota_{k-2}^{\nu})^{-1} B_{\nu,\tilde{n}(\nu)}. \tag{4}$$

From $(\overline{\mathrm{P}_2})'$ we get

$$\iota_k^{k-1} B_{k,m} \subset S\iota_K^{k-1} B_{K,N(k,K,m,\varepsilon_k)} + \varepsilon_k B_{k-1,n(k)}. \tag{5}$$

Let $u \in B_{k,m}$ be given. Because of (5), there are $v_{k-1} \in SB_{K,N(k,K,m,\varepsilon_k)}$ and $u_{k-1} \in \varepsilon_k B_{k-1,n(k)}$ with

$$\iota_k^{k-1} u = \iota_K^{k-1} v_{k-1} + u_{k-1}.$$

Because of (4) there are

$$v_{k-2} \in \varepsilon_k \tilde{S} B_{k,\tilde{N}(k-1,k,n(k))} \quad \text{and} \quad \tilde{u}_{k-2} \in \varepsilon_k \bigcap_{\nu=1}^{k-2} (\iota_{k-2}^{\nu})^{-1} B_{\nu,\tilde{n}(\nu)}$$

with $\iota_{k-1}^{k-2} u_{k-1} = \iota_k^{k-2} v_{k-2} + \tilde{u}_{k-2}$. Set

$$u_{k-2} = u_{k-1} - \iota_k^{k-1} v_{k-2}.$$

Then, for $1 \le \nu \le k-2$, the following holds:

$$\begin{aligned}
\iota_{k-1}^{\nu} u_{k-2} &= \iota_{k-1}^{\nu} u_{k-1} - \iota_k^{\nu} v_{k-2}\\
&= \iota_{k-2}^{\nu}(\iota_{k-1}^{k-2} u_{k-1} - \iota_k^{k-2} v_{k-2}) = \iota_{k-2}^{\nu} \tilde{u}_{k-2} \in \varepsilon_k B_{\nu,\tilde{n}(\nu)}.
\end{aligned}$$

Because of the definition of $\tilde{n}$ and of (1), this implies

$$\begin{aligned}
u_{k-2} = u_{k-1} - \iota_k^{k-1} v_{k-2} &\in \varepsilon_k B_{k-1,n(k)} + \varepsilon_k \tilde{S} B_{k-1,\tilde{N}(k-1,k,n(k))}\\
&\subset \varepsilon_k(\tilde{S}+1) B_{k-1,\tilde{n}(k-1)}.
\end{aligned}$$

Note that $\varepsilon_k(\tilde{S}+1) = 1/2$. Thus $u_{k-2} \in (1/2)\bigcap_{\nu=1}^{k-1}(\iota_{k-1}^{\nu})^{-1}B_{\nu,\tilde{n}(\nu)}$. This shows

$$\begin{aligned}\iota_k^{k-1}u &= \iota_K^{k-1}v_{k-1} + \iota_k^{k-1}v_{k-2} + u_{k-2}\\ &\in S\iota_K^{k-1}B_{K,N(k,K,m,\varepsilon_k)} + \frac{1}{2}B_{k,\tilde{N}(k-1,k,n(k))} + \frac{1}{2}\bigcap_{\nu=1}^{k-1}(\iota_{k-1}^{\nu})^{-1}B_{\nu,\tilde{n}(\nu)}.\end{aligned}$$

Thus, for $m \geq \tilde{N}(k-1,k,n(k))$ we have

$$\iota_k^{k-1}u \in S\iota_K^{k-1}B_{K,N(k,K,m,\varepsilon_k)} + \frac{1}{2}B_{k,m} + \frac{1}{2}\bigcap_{\nu=1}^{k-1}(\iota_{k-1}^{\nu})^{-1}B_{\nu,\tilde{n}(\nu)}.$$

Inductively, for all $j \in \mathbb{N}$ we find elements x_j, y_j, and w_j in the corresponding bounded sets with

$$\begin{aligned}\iota_k^{k-1}u &= \iota_K^{k-1}x_1 + \iota_k^{k-1}y_1 + w_1,\\ \iota_k^{k-1}y_j &= \iota_K^{k-1}x_{j+1} + \iota_k^{k-1}y_{j+1} + w_{j+1}.\end{aligned}$$

Since $X_{k,m}$ and $X_{k-1,\tilde{n}(k-1)}$ are Banach spaces, the following series converge:

$$v = \sum_{j=1}^{\infty} x_j \in 2SB_{K,N(k,K,m,\varepsilon_k)}, \qquad w = \sum_{j=1}^{\infty} w_j \in B_{k-1,\tilde{n}(k-1)}.$$

For each $l \in \mathbb{N}$ we have

$$\iota_k^{k-1}u = \iota_K^{k-1}\sum_{j=1}^{l} x_j + \iota_k^{k-1}y_l + \sum_{j=1}^{l} w_j.$$

Since the spectral mappings are continuous, this implies

$$\iota_k^{k-1}u = \iota_K^{k-1}v + w.$$

Furthermore, for $1 \leq \nu \leq k-1$, we have $\iota_{k-1}^{\nu}w = \sum_{j=1}^{\infty}\iota_{k-1}^{\nu}w_j$. This sum converges absolutely in $X_{\nu,\tilde{n}(\nu)}$ to an element of $B_{\nu,\tilde{n}(\nu)}$. Thus

$$\iota_k^{k-1}u \in 2S\iota_K^{k-1}B_{K,N(k,K,m,\varepsilon_k)} + \bigcap_{\nu=1}^{k-1}(\iota_{k-1}^{\nu})^{-1}B_{\nu,\tilde{n}(\nu)},$$

which ends the induction step.

8. Theorem. *If the spectrum $\mathcal{X}$ satisfies $(\overline{\mathrm{P}_2})$, then* Proj^1 $\mathcal{X} = 0$.

Proof. We must prove the following necessary and sufficient condition (P) of [10, 4.4], which is a version of Retakh's condition [8]:

(P) There is a sequence $(\tilde{n}(k))_{k\in\mathbb{N}}$ in $\mathbb{N}$ such that for each μ there is k such that for each K there is S with

$$\iota_k^{\mu}X_k \subset \iota_K^{\mu}X_K + S\bigcap_{\nu=1}^{\mu}(\iota_{\mu}^{\nu})^{-1}B_{\nu,\tilde{n}(\nu)}.$$

We may assume that the spectrum satisfies $(\overline{\mathrm{P}_2})'$. By the remark following Definition 4 it is reduced, and by Lemma 5 it also satisfies (2) with $k = \mu + 1$. Let μ

be given, and set $k = \mu + 2$. Let m and K be arbitrary, and let $\tilde{N}$ and $\tilde{S}$ be as in Lemma 7. Then

$$\begin{aligned}\iota_k^{k-2} X_k &\subset \iota_{k-1}^{k-2}(\iota_K^{k-1} X_K + B_{k-1,n(k)})\\ &\subset \iota_K^{k-2} X_K + S(k-1, K, n(k-1))\iota_K^{k-1} B_{K,\tilde{N}(k-1,K,n(k-1))}\\ &\quad + \bigcap_{\nu=1}^{k-2} (\iota_{k-1}^{\nu})^{-1} B_{\nu,\tilde{n}(\nu)}\\ &\subset \iota_K^{\mu} X_K + \bigcap_{\nu=1}^{\mu} (\iota_{k-1}^{\nu})^{-1} B_{\nu,\tilde{n}(\nu)}.\end{aligned}$$

REMARK. Theorem 8 was independently obtained by Frerick and Wengenroth [4]. Its significance comes from Corollary 11. Wengenroth proved Corollary 11 for the larger class of (DFM) spectra in [11] (see also Wengenroth [12]). There, he derives the result from his theorem about acyclic inductive spectra.

9. LEMMA. *If a reduced (DFS) spectrum $\mathcal{X}$ satisfies* (P_2), *then it satisfies* $(\overline{\mathrm{P}_2})$.

Proof. For given μ let k and n be as in (P_2). For K and m arbitrarily given there are N and S as in (P_2). There is $l > n$ such that $B_{\mu,n}$ is relatively compact in $B_{\mu,l}$. Thus there is a finite number of points $x_1, \ldots, x_L \in X_{\mu,n}$ with $SB_{\mu,n} \subset \bigcup_{j=1}^{L} \left(x_j + (\varepsilon/2) B_{\mu,l}\right)$. The spectrum has property (2) by Lemma 5; thus, for each $x_j \in X_k$, there are elements $y_j \in X_K$ and $b_j \in (\varepsilon/2) B_{\mu,n}$ with $\iota_k^{\mu} x_j = \iota_K^{\mu} y_j + b_j$. Each y_j lies in some $X_{K,N(j)}$. Choose $N' = \max(\{N\} \cup \{N(j) \mid j = 1, \ldots, L\})$ and $S' = \max_{j=1}^{L} \|y_j\|_{K,N}$. If now $u \in B_{k,m}$ is arbitrary, then there are $v \in SB_{K,N}$ and $u_1 \in SB_{\mu,n}$ with $\iota_k^{\mu} u = \iota_K^{\mu} v + u_1$. For u_1 there are j and $u_2 \in (\varepsilon/2) B_{\mu,l}$ with $u_1 = x_j + u_2$. Thus

$$\begin{aligned}\iota_k^{\mu} u = \iota_K^{\mu} v + \iota_K^{\mu} y_j + b_j + u_2 &\in S\iota_K^{\mu} B_{K,N} + S'\iota_K^{\mu} B_{K,N'} + \frac{\varepsilon}{2} B_{\mu,l} + \frac{\varepsilon}{2} B_{\mu,l}\\ &\subset (S + S')\iota_K^{\mu} B_{K,N'} + \varepsilon B_{\mu,n}.\end{aligned}$$

□

10. COROLLARY. *For a (DFS) spectrum $\mathcal{X}$, the following are equivalent:*

(1) *$\mathcal{X}$ satisfies property* (P_2) *and is reduced*;
(2) *$\mathcal{X}$ satisfies property* $(\overline{\mathrm{P}_2})$;
(3) $\mathrm{Proj}^1 \mathcal{X} = 0$.

(For the notion of a reduced spectrum, see Definition 4.)

Proof. Lemma 9 shows that (1) implies (2). Theorem 8 gives that (2) implies (3). From (3), we get (P_2) by [10, 2.7]. In this case, $\mathcal{X}$ is reduced by condition (P), which was quoted in the proof of Theorem 8. □

In the context of (DFS) spectra, the bipolar theorem allows a dual version of (P_2), called $(\mathrm{P}_2)^*$. By the remark before Theorem 2.8 of [10], (P_2) and $(\mathrm{P}_2)^*$ are equivalent for (DFS) spaces. Condition $(\mathrm{P}_2)^*$ has the advantage that it is expressed as an inequality for Minkowski functionals $\|y\|^*_{\mu,n} = \sup\{\, |y(x)| \mid x \in B_{\mu,n} \}$ for $y \in X'_\mu$.

11. Corollary (Wengenroth [11, 3.10]). *If the spectrum $\mathcal{X}$ is a reduced (DFS) spectrum, then the following are equivalent:*

(1) $\operatorname{Proj}^1 \mathcal{X} = 0$;
(2) *the following condition* $(\mathrm{P}_2)^*$ *is satisfied*:
$(\mathrm{P}_2)^*$ *for each μ, there are n and k such that, for all K and m, there are N and S such that*

$$\|y\|^*_{k,m} \le S \max(\|y\|^*_{K,N}, \|y\|^*_{\mu,n}).$$

for all $y \in X'_\mu$.

To investigate more closely what condition $(\mathrm{P}_2)^*$ means, assume that we are given a set V, a linear space A of functions on V, and a family $(b_{k,n})_{k,n}$ of weight functions on V such that

$$b_{k,n} \le b_{k+1,n} \quad \text{and} \quad b_{k,n} \ge b_{k,n+1} \quad \text{for } k, n \in \mathbb{N}.$$

We assume

$$X'_{k,n} = \{\, f \in A \mid \|f\|^*_{k,n} = \sup_{t \in V} |f(t)| \exp(-b_{k,n}(t)) < \infty \} \tag{6}$$

and hence

$$X'_k = \{\, f \in A \mid \|f\|^*_{k,n} < \infty \text{ for all } n \}.$$

In this context, condition $(\mathrm{P}_2)^*$ becomes:

(PL_2) For each μ, there are n and k such that, for all K and m, there are N and S such that for each $f \in X'_\mu$ that satisfies estimates (a) and (b) also condition (c) is valid, where
(a) $\log|f| \le b_{\mu,n}$,
(b) $\log|f| \le b_{K,M}$,
(c) $\log|f| \le b_{k,m} + S$.

In the investigation of partial differential operators $P(D)$, a version of the Ehrenpreis fundamental principle can often be used to show that the kernel spectrum of $P(D)$ is of the form (6), with A the space of holomorphic functions on an algebraic variety V. Thus the condition $(\mathrm{P}_2)^*$ is an extension of the Phragmén–Lindelöf condition of Hörmander [5]. In this paper, he characterizes when $P(D)\colon A(\Omega) \to A(\Omega)$ is surjective, where $A(\Omega)$ denotes the space of all real analytic functions on a convex domain $\Omega \subset \mathbb{R}^n$. Predecessors of Corollary 10 were applied by Braun, Meise, and Vogt [2; 3] and Braun [1] to characterize the surjective operators on Roumieu classes. In concrete settings, complex analytic methods are used to reformulate (PL_2) in more suitable terms. In particular, condition (b) can often be replaced by a condition involving only the real points of V. This makes the analogy to the classical principle of Phragmén and Lindelöf more obvious.

References

[1] R. W. Braun, *Surjektivität partieller Differentialoperatoren auf Roumieu–Klassen,* Habilitationsschrift, Düsseldorf, 1993.

[2] R. W. Braun, R. Meise, and D. Vogt, *Applications of the projective limit functor to convolution and partial differential equations,* Advances in the theory of Fréchet spaces (T. Terzioğlu, ed.) (Istanbul 1987, NATO ASI Series C, vol. 287), pp. 29–46, Kluwer, Dordrecht, 1989.

[3] ———, *Characterization of the linear partial differential operators with constant coefficients which are surjective on nonquasianalytic classes of Roumieu type on* $\mathbb{R}^N$, Math. Nachr. 168 (1994), 19–54.

[4] L. Frerick and J. Wengenroth, *A sufficient condition for vanishing of the derived projective limit functor,* Arch. Math. (Basel) 67 (1996), 296–301.

[5] L. Hörmander, *On the existence of real analytic solutions of partial differential equations with constant coefficients,* Invent. Math. 21 (1973), 151–183.

[6] R. Meise and D. Vogt, *Einführung in die Funktionalanalysis,* Vieweg Stud. 62, Vieweg, Braunschweig, 1992.

[7] V. P. Palamodov, *The projective limit functor in the category of linear topological spaces,* Math. USSR-Sb. 4 (1968), 529–559.

[8] V. S. Retakh, *Subspaces of a countable inductive limit,* Soviet Math. Dokl. 11 (1970), 1384–1386; translation of Dokl. Akad. Nauk SSSR 194 (1970), no. 6.

[9] D. Vogt, *Lectures on projective spectra of (DF) spaces,* Seminar lectures, AG Funktionalanalysis, Düsseldorf/Wuppertal, 1987.

[10] ———, *Topics on projective spectra of (LB)-spaces,* Advances in the theory of Fréchet spaces (T. Terzioğlu, ed.) (Istanbul 1987, NATO ASI Series C, vol. 287), pp. 11–27, Kluwer, Dordrecht, 1989.

[11] J. Wengenroth, *Retractive (LF) spaces,* Dissertation, Trier, 1995.

[12] ———, *Acyclic inductive spectra of Fréchet spaces,* Studia Math. 120 (1996), 247–258.

R. W. Braun
Mathematisches Institut
Heinrich-Heine-Universität
Universitätsstraße 1
D-40225 Düsseldorf
Germany

braun@cs.uni-duesseldorf.de

D. Vogt
Fachbereich Mathematik
Bergische Universität
Gaußstraße 20
D-42097 Wuppertal
Germany

vogt@math.uni-wuppertal.de