Biholomorphic Transformations That Do Not Extend Continuously to the Boundary

B. L. FRIDMAN

Let $f: D_1 \to D_2$ be a biholomorphic transformation between bounded domains in a complex manifold. The general question considered by a number of authors (see, e.g., [1–11; 14; 16]) is whether f admits a continuous or even a smooth extension to the boundary ∂D_1 of D_1. Most of the known results are positive; that is, if D_1, D_2 are some special domains (strictly pseudoconvex, analytic polyhedra, etc.; see [3; 15] for review) then f can be extended to the boundary to provide homeomorphism or diffeomorphism (in case of smooth boundaries) between $\bar{D}_1$ and $\bar{D}_2$. Counterexamples are hard to come by, probably because of the rigidity of biholomorphic mappings. Only a few are known at this time (see [1; 2; 11]).

The purpose of this paper is to present several more negative results in $\mathbf{C}^n$, $n > 1$. We provide two constructions based on a new idea.

Both of our theorems below provide counterexamples to the general question of whether a biholomorphism $f: D_1 \to D_2$ can be extended continuously to the boundary. The first theorem gives an example of domains with topologically complicated boundaries, whereas the second theorem deals with a more regular case.

In the theorem below it is assumed that $\mathbf{C}^n \subset \mathbf{C}^{n+1}$ in a natural way: $z = (z_1, \dots, z_n) \in \mathbf{C}^n$ is identified with $(z_1, \dots, z_n, 0) \in \mathbf{C}^{n+1}$.

THEOREM 1. *Let G_1, G_2 be bounded domains in $\mathbf{C}^n$. Then there are bounded domains $D_1, D_2 \subset \mathbf{C}^{n+1}$ such that:*

(1) $G_i \subset \partial D_i$, $i = 1, 2$.

(2) *There is a biholomorphic transformation $f: D_1 \to D_2$ such that f can be extended as homeomorphism to $F: (\bar{D}_1 \backslash \bar{G}_1) \to (\bar{D}_2 \backslash \bar{G}_2)$. Moreover, f can be extended to a neighborhood of $\bar{D}_1 \backslash \bar{G}_1$ as a biholomorphic transformation.*

REMARK 1.1. Clearly, since G_1, G_2 are arbitrary, f in most cases cannot be extended to the boundary as homeomorphism. From the construction that follows one will see that f cannot be extended continuously to $\bar{G}_1$ in any case.

Received September 21, 1989. Revision received June 27, 1990.
Michigan Math. J. 38 (1991).

The next theorem is a generalization of Theorem 1.

THEOREM 1.1. *Let* $\{G_k\}_{k=1}^{\infty}$ *be a countable number of arbitrary bounded domains in* $\mathbf{C}^n$. *Then there exist a bounded domain* $D \subset\subset \mathbf{C}^{n+1}$ *and bounded biholomorphic imbeddings* $f_k: D \to \mathbf{C}^{n+1}$ *such that*

(1) $\bar{D} \cap \mathbf{C}^n = \bar{G}$ *for some domain* $G \subset\subset \mathbf{C}^n$;
(2) $f_k(D) = D_k \subset\subset \mathbf{C}^{n+1}$ *and* $\bar{D}_k \cap \mathbf{C}^n = \bar{G}_k$ *for all* $k \geq 1$; *and*
(3) f_k *can be extended to a homeomorphism* $F_k: (\bar{D} \setminus \bar{G}) \to (\bar{D}_k \setminus \bar{G}_k)$. *Moreover,* f_k *can be extended to a neighborhood of* $\bar{D} \setminus \bar{G}$ *as a biholomorphic transformation.*

We introduce now the following notations: $z' = (z_1, \dots, z_{n-1})$; $z = (z', z_n) \in \mathbf{C}^n$. We call a domain $D \subset \mathbf{C}^n$ a *disk domain* (with respect to z_n) if there is such a domain $D' \subset \mathbf{C}^{n-1}$, a function $c(z')$, and a nonnegative function $R(z')$, $z' \in D'$, such that $D = \{z = (z', z_n) \mid z' \in D',\ |z_n - c(z')| < R(z')\}$. We denote $R(D) = \inf_{z' \in D'} R(z')$.

The next theorem provides an example of a C^∞ disk domain in $\mathbf{C}^2$ containing only large 1-dimensional disks on the boundary, and a biholomorphic map onto an arbitrarily close and similar disk domain (so the map is almost an automorphism) which cannot be extended continuously to the boundary.

THEOREM 2. *There exists a bounded convex disk domain* $D \subset \mathbf{C}^2$, $\partial D \in C^\infty$, $R(D) = 1$ *such that for every* $\epsilon > 0$ *there exists a disk domain* $D_\epsilon \subset\subset \mathbf{C}^2$ *with the following properties:*

(1) $R(D_\epsilon) = 1$.
(2) *There exists a biholomorphism* $f_\epsilon: D \to D_\epsilon$ *that cannot be extended continuously to* ∂D.
(3) *There exists a continuous map* $\phi_\epsilon: \bar{D} \to \bar{D}_\epsilon$ *that is one-to-one and* $|\phi_\epsilon(z) - z| \leq \epsilon$ *for all* $z \in \bar{D}$.

REMARK 2.1. In addition to the properties listed we will see that f is going to be fiber- (disk-) preserving and $\phi_\epsilon \in C^\infty(D) \cap C(\bar{D})$.

REMARK 2.2. The domain D from Theorem 2 has the following neighborhood basis of disk domains D_k:

(1) $\bar{D}_{k+1} \subset \bar{D}_k$; $\bigcap_{k=1}^{\infty} D_k = D$; $R(D_k) = 1$.
(2) Every $\bar{D}_k$ is topologically equivalent to $\bar{D}$; moreover, there exists a family of homeomorphisms $\{\Phi_k\}$, $\Phi_k: \bar{D} \to \bar{D}_k$, that converges uniformly on $\bar{D}$ to the identity map.
(3) Every D_k is biholomorphically equivalent to D, and the corresponding biholomorphism does not extend continuously to the boundary.

Proof of Theorem 1

We will use the following notation: $B(a, r) = \{z \mid |z - a| < r\}$, $B = B^n = B(0, 1)$ the unit ball in $\mathbf{C}^n$; $p = (0, 0, \dots, 1) \in \partial B$; $q = -p$. $\Delta(a, r) = \{t \mid |t - a| < r\}$ a disk in $\mathbf{C}$; $\Delta = \Delta(0, 1)$ the unit disk in $\mathbf{C}$.

If X, Y are sets in a metric space with distance $d(x,y)$, then $d(X,Y) = \inf\{\epsilon > 0 \mid \forall x \in X\ \exists y \in Y,\ d(x,y) < \epsilon\ \&\ \forall y \in Y\ \exists x \in X,\ d(y,x) < \epsilon\}$. We will say that sets $M_s \to M$ as $s \to \infty$ if $d(M_s, M) \to 0$ as $s \to \infty$. We also define

$$F(\mu, z) = \left\{ \sqrt{1-\mu^2}\, \frac{z'}{1-z_n\mu},\ \frac{z_n - \mu}{1 - z_n\mu} \right\}.$$

Note that for a fixed μ, $0 < \mu < 1$, $F(\mu, z) \in \mathrm{Aut}(B)$ the holomorphic automorphism group of B.

1. LEMMA. *Let G be a bounded domain in $\mathbf{C}^n$. Then there is a linear fractional transformation $L_G\colon G \to B$ such that the following two properties hold:*

(1) *L_G^{-1} is defined in a neighborhood of $\bar{B}$.*

(2) *For any sequence $\lambda_k \to 1$, $0 < \lambda_k < 1$, there exists a subsequence $\{\mu_k\} \subset \{\lambda_k\}$ and a domain $E \subset B$ such that*

(a) *$E \supset B \backslash B(p, \frac{1}{3})$ and*

(b) *$L_G^{-1} \circ F_k[E \cap S] \to \bar{G}$ as sets when $k \to \infty$, where $F_k(z) = F(\mu_k, z)$ is defined above and S is any domain containing $B(p, \frac{1}{2})$.*

Proof of this statement can be done by constructing E in a way similar to the constructions of exhausting domains in [12; 13].

2. By using the above Lemma for $i = 1, 2$ we now find $L_i = L_{G_i}$. Fix a sequence $\lambda_k = 1 - 1/(k+1) \to 1$ and by the same lemma (where $G = G_1$) we find $\{\mu_k\} \subset \{\lambda_k\}$ and the set E_1, and consequently $\{\nu_k\} \subset \{\mu_k\}$ and the set E_2 (for $G = G_2$). $\{\nu_k\}$ form a monotone sequence, $\nu_k \to 1$. Let J be the operator of multiplying by -1, $J\colon z \to -z$. We set $E = E_1 \cap JE_2$. One can check that

(1) $E'_k = L_1^{-1} \circ F_k[E] \to \bar{G}_1$ and $E''_k = L_2^{-1} \circ F_k[JE] \to \bar{G}_2$ as $k \to \infty$,

where $F_k(z) = F(\nu_k, z)$.

3. Let $g(t) = (t+1)/2$. For $|t| < 1$ we fix a branch of $\sqrt{1-g^2}$ which will be an analytic function. Now for $|t| < 1$ and $|z_n| < 1$, $F(g(t), z)$ is a well-defined analytic transformation. If we set $\Phi\colon (z, z_{n+1}) \to (F(g(z_{n+1}), z), z_{n+1} - 1)$ one can see that $\Phi\colon B \times \Delta \to \mathbf{C}^{n+1}$ is a biholomorphic imbedding which can be extended biholomorphically into a neighborhood of $\bar{B} \times (\bar{\Delta} \backslash \{1\})$.

4. Let $\epsilon'_k = d(E'_k, G_1)$, $\epsilon''_k = d(E''_k, G_2)$, and $\epsilon_k = \max(\epsilon'_k, \epsilon''_k, 1/k)$. (1) means that $\epsilon_k \to 0$ for $k \to \infty$. One can now see that for any k, a number $\delta_k > 0$ can be found such that if $|\nu - \nu_k| < \delta_k$ then

$$M_1 = L_1^{-1} \circ F(\nu, [E]) \quad \text{and} \quad M_2 = L_1^{-1} \circ F(\nu, [JE])$$

are defined and $d(M_i, G_i) < 2\epsilon_k$ for $i = 1, 2$.

5. One can check from the definition of F that $F(g(t), [B(0, \frac{1}{2})]) \to q$ when $t \to 1$, $|t| < 1$.

6. Let $t_0 = 0$ and, for all $k \geq 1$, let $t_k = 2\nu_k - 1$ and $\eta_k = \min(\delta_k, (t_{k+1} - t_k)/3, (t_k - t_{k-1})/3)$. We set $U = \bigcup_{k=1}^{\infty} \Delta(t_k, \eta_k)$, $V = \Delta \backslash U$, and $D = (E \times U) \cup (B(0, \frac{1}{2}) \times V)$. One can see that $U \subset \Delta$ and D is a domain in $\mathbf{C}^{n+1}$.

7. We introduce consequently

$$J'\colon (z, z_{n+1}) \to (-z, z_{n+1}), \qquad T_i\colon (z, z_{n+1}) \to (L_i^{-1}(z), z_{n+1}), \quad i=1,2;$$

$$\Phi_1 = T_1 \circ \Phi, \qquad \Phi_2 = T_2 \circ \Phi \circ J'.$$

$$D_i = \Phi_i[D], \quad i=1,2; \qquad f = \Phi_2 \circ \Phi_1^{-1}\colon D_1 \to D_2.$$

One can check now that D_1, D_2 and f satisfy all conditions of Theorem 1. □

8. Theorem 1.1 can be proved in a way similar to the proof of Theorem 1. The main difference will be in constructing the set E. Above we constructed the set E by taking some sets out of B near two points p and q. For Theorem 1.1 we will need to take some sets out of B near a countable number of points on ∂B. The subsequence $\{\nu_k\}$ can be chosen by the usual diagonalization process.

Proof of Theorem 2

1. First we choose a real function $\varphi(x) \in C^\infty(-1,1) \cap C[-1,1]$ such that

(a) $\varphi(0)=2$, $\varphi(1)=1$, and φ is decreasing;
(b) $\varphi^{(k)}(0)=0$ $\forall k \geq 1$; and
(c) if $\psi = \varphi^{-1}$ then $\psi^{(k)}(1)=0$ $\forall k \geq 1$.

2. Define the domain D as $D = \{(z_1, z_2) \mid |z_1| < 1,\ |z_2| < \varphi(|z_1|)\}$. From the construction of φ one can see that $\partial D \in C^\infty$ and D is a convex disk domain with $R(D)=1$.

3. Let ν, $\frac{1}{3} > \nu > 0$, be a number that depends on ϵ and will be chosen later. Obviously there exists a $\delta > 0$ such that $\varphi(|t|) - 1 < \nu^2$ for $t \in \Delta \cap \Delta(1,\delta)$. We now can find a function $g \in A(\Delta_1) \cap C(\bar{\Delta}_1)$, where $\Delta_1 = \Delta(-1,2)$, such that:

(a) $|g(t)| < 1$ for $t \in \bar{\Delta}_1 \setminus \{1\}$, and $g(1)=1$;
(b) $|g(t)| < \nu$ for $t \in \Delta \setminus \Delta(1,\delta)$;
(c) $\varphi(|t|) - 1 < \nu(1-|g(t)|^2)^2$ for $t \in \Delta \cap \Delta(1,\delta)$; and
(d) $|\operatorname{Im} g(t)| < \nu(1-|g(t)|^2)^2$ for $|t| < 1$.

Such a function g can be constructed as a conformal map from the disk Δ_1 onto an appropriate region Ω in $\mathbf{C}$. Below is a detailed construction of g.

(1) First we introduce sequences $u_k = 1 - \delta/(k+1)$ for all integers $k \geq 0$,

$$X_k = \{t = u + iv \mid u \in [u_{k-1}, u_k]\}) \cap \Delta, \qquad m_k = \sup_{X_k}(\varphi(|t|) - 1) \quad \text{for } k \geq 1,$$

$$s_k = \left(1 - \left(\frac{2m_k}{\nu}\right)^{1/2}\right)^{1/\sqrt{2}} \quad \text{for } k \geq 2, \qquad s_1 = \min\left(\nu^{\sqrt{2}}, \left(1 - \left(\frac{2m_1}{\nu}\right)^{1/2}\right)^{1/\sqrt{2}}\right),$$

and $s_0 = -s_1$.

(2) One can see that $m_k \downarrow 0$ and $s_k \uparrow 1$ when $k \to \infty$. Also

$$(s_0, 1) = \bigcup_0^\infty (s_k, s_{k+1}].$$

(3) We consider below sequences of real numbers $\Lambda=\{v_k\}_{k=1}^{\infty}$ with $0<v_k\to 0$ for $k\to\infty$. For such a sequence we introduce a domain $\Omega(\Lambda)$ and a function g_Λ. $\Omega(\Lambda)$ is the interior of the set $\bigcup_1^\infty\{[s_{k-1},s_k]\times[-v_k,v_k]\}$. g_Λ is a conformal map from Δ_1 onto $\Omega(\Lambda)$ such that $g_\Lambda(0)=0$, $g_\Lambda(1)=1$. This last condition is appropriate since any conformal map from Δ_1 onto $\Omega(\Lambda)$ is continuously extendible to the boundary. Our goal is to produce a sequence $\Lambda=\{v_k\}$ such that $g=g_\Lambda$ satisfies the above conditions (a)–(d).

(4) In order to construct the needed sequence $\Lambda=\{v_k\}$ we start first with a sequence $\Lambda_0=\{y_k\}$ such that:

(A) $\bigcup_1^n\{[s_{k-1},s_k]\times[-y_k,y_k]\}\subset\Delta(0,s_n^{1/\sqrt{2}})$, $\forall n\geq 1$;
(B) $g_{\Lambda_0}(\Delta\backslash\Delta(1,\delta))\subset[s_0,s_1]\times[-y_1,y_1]$;
(C) $0<y_{k+1}<\frac{1}{2}y_k$, $\forall k\geq 1$; and
(D) $|\operatorname{Im} t|<\nu(1-|t|^2)^2$, $\forall t\in\Omega(\Lambda_0)$.

One can find such a sequence Λ_0 by initially constructing a sequence satisfying (A), (C), and (D). Then one can fix y_1 and choose y_2 to be so small that if y_k $(k\geq 3)$ are chosen to satisfy (C) and are smaller than the initial choice, the new sequence will satisfy all (A)–(D).

(5) We will construct the sequence $\Lambda=\{v_k\}$ by induction. We require that $\forall k\geq 1$, $0<v_k\leq y_k$ and $v_{k+1}\leq\frac{1}{2}v_k$. We also require that the following property hold for $k\geq 2$:

> Let $\Lambda_k=\{\alpha_n\}$ be any sequence where $\alpha_n=v_n$ for $n\leq k$, and, for all $n\geq k$, let $0<\alpha_{n+1}\leq\min(\frac{1}{2}\alpha_n,y_{n+1})$. Then, for all $n\leq k-1$, $g_{\Lambda_k}(X_n)\subset\bigcup_1^n\{[s_{p-1},s_p]\times[-v_p,v_p]\}$.

For $k=1$ we take $v_1=y_1$. If $v_1,\ldots,v_k$ have been chosen and the above property holds, then one can see that if $0<v_{k+1}\leq\min(v_k/2,y_{k+1})$ is chosen to be small enough the required property will also hold for any Λ_{k+1}.

(6) $\Lambda=\{v_k\}$ constructed in the previous paragraph satisfies $v_1=y_1$ and $v_k\leq y_k$ $\forall k\geq 2$. One can check now that (A)–(D) imply the properties (a), (b), and (d) for $g=g_\Lambda$. Property (c) follows from the property of all Λ_k described in the preceding paragraph. To verify this one should take into account the definitions of s_k. This completes the construction of g.

4. The map $f_\epsilon: D\to\mathbf{C}^2$ and D_ϵ are defined as follows:

$$f_\epsilon:\begin{cases} w_1=z_1, \\ w_2=\dfrac{z_2-g(z_1)}{1-z_2g(z_1)}; \end{cases}\qquad D_\epsilon=f_\epsilon(D).$$

(1) We now estimate

$$|1-z_2g(z_1)|\geq 1-|z_2|\,|g(z_1)|\geq 1-\varphi(|z_1|)\,|g(z_1)|>0$$

for $z_1\in\Delta\backslash\Delta(1,\delta)$, since $1\leq\varphi\leq 2$ and $|g|<\nu$ for these values of z_1. For $z_1\in\Delta\cap\Delta(1,\delta)$ we use the above described properties (c) and (a) of the function g to obtain

$$|1-z_2 g(z_1)| \geq 1-\varphi|g| = (1-|g|)\left(1-\frac{\varphi-1}{1-|g|}|g|\right) > (1-|g|)(1-2\nu) > 0.$$

This shows that f_ϵ is defined on D. One can also see that f_ϵ is a biholomorphic transformation of the domain D onto D_ϵ that extends as a biholomorphic imbedding at every boundary point of D except the disk $\tilde{\Delta} = \{z_1 = 1, |z_2| \leq 1\} \subset \partial D$.

(2) Since f_ϵ preserves fibers of the disk domain D and since, for every fixed z_1, f_ϵ is a linear fractional transformation, one can calculate that D_ϵ has the following structure:

$$D_\epsilon = \{(w_1, w_2) \mid |w_1| < 1, |w_2 - c(w_1)| < R(w_1)\},$$

where

$$c(w_1) = \frac{\varphi^2 \bar{g} - g}{1-\varphi^2|g|^2}, \qquad R(w_1) = \varphi \frac{|1-g^2|}{1-\varphi^2|g|^2}; \qquad \varphi = \varphi(|w_1|), \quad g = g(w_1).$$

By using inequalities (c) and (d) of the function g one can estimate that $c(w_1) \to 0$ and $R(w_1) \to 1$ for $w_1 \to 1$. This limiting disk is $\tilde{\Delta} = \{w_1 = 1, |w_2| \leq 1\}$. Also, $R(w_1) \geq \varphi(|w_1|) \geq 1$ for $|w_1| < 1$. Therefore D_ϵ is a disk domain and $R(D_\epsilon) = 1$. If $z^\circ \in \tilde{\Delta} \setminus (1,1)$ and $z' \in D$, $z' \to z^\circ$, then $f_\epsilon(z') \to (1, -1)$. One can also see that the limit set for limit points of $f_\epsilon(z')$ when $z' \to (1,1)$ is the whole disk $\tilde{\Delta} \subset \partial D_\epsilon$. So, f_ϵ cannot be extended continuously to the boundary.

(3) Let $h: D \to \mathbf{C}^2$ be the following map:

$$h: \begin{cases} z_1 \mapsto z_1, \\ z_2 \mapsto \dfrac{(z_2 + g(z_1)\varphi(|z_1|))\varphi(|z_1|)}{\varphi(|z_1|) + z_2 \bar{g}(z_1)}. \end{cases}$$

One can check that h is a C^∞ automorphism of D. We introduce now $\phi_\epsilon = f_\epsilon \circ h: D \to D_\epsilon$. One can determine that

$$\phi_\epsilon: \begin{cases} w_1 = z_1, \\ w_2 = \psi_\epsilon(z) = \dfrac{z_2(\varphi - |g|^2) + \varphi g(\varphi - 1)}{z_2(\bar{g} - \varphi g) + \varphi(1 - \varphi g^2)}, \end{cases}$$

where $\varphi = \varphi(|z_1|)$ and $g = g(z_1)$.

We will now describe the way to choose $\nu > 0$. Using property (b) of the function g one can see that $|\psi_\epsilon(z) - z_2| < \epsilon$ for $z_1 \in \Delta \setminus \Delta(1, \delta)$ if ν is chosen to be small enough. This is the first restriction on ν. By calculating explicitly we get

$$\psi_\epsilon(z) - z_2 = \frac{A\alpha_1 + B\alpha_2}{A\alpha_3 + B\alpha_4 + 1},$$

where

$$|\alpha_i(z)| < 6 \ (1 \leq i \leq 4)$$

and

$$A = \frac{\varphi - 1}{1-|g|^2}, \qquad B = \frac{\operatorname{Im} g}{1-|g|^2}.$$

Now by using properties (c) and (d) of the function g we get

$$\psi_\epsilon(z)-z_2=\frac{O(1)\nu}{1+O(1)\nu}$$

for $z_1\in\Delta\cap\Delta(1,\delta)$, and estimates on $O(1)$ do not depend on ν. Therefore, if ν is small enough one can make $|\psi_\epsilon(z)-z_2|<\epsilon$ for $z_1\in\Delta\cap\Delta(1,\delta)$. So, by choosing ν to be appropriately small, one can make $|\psi_\epsilon(z)-z_2|<\epsilon$ for $|z_1|<1$ and therefore $|\phi_\epsilon(z)-z|<\epsilon$ for $z\in D$. This concludes the proof of Theorem 2. □

References

1. D. Barrett, *Biholomorphic domains with inequivalent boundaries,* Invent. Math. 85 (1986), 373–377.
2. ———, *Boundary singularities of biholomorphic maps,* Lecture Notes in Math., 1268, pp. 24–28, Springer, Berlin, 1987.
3. E. Bedford, *Proper holomorphic mappings,* Bull. Amer. Math. Soc. (N.S.) 10 (1984), 157–175.
4. S. Bell, *Mapping problems in complex analysis and the $\bar{\partial}$-problem,* Bull. Amer. Math. Soc. (N.S.) 22 (1990), 233–259.
5. S. Bell and D. Catlin, *Boundary regularity of proper holomorphic mappings,* Duke Math. J. 49 (1982), 385–396.
6. S. Bell and E. Ligocka, *A simplification and extension of Fefferman's theorem on biholomorphic mappings,* Invent. Math. 57 (1980), 283–289.
7. S. Bell and S. Krantz, *Smoothness to the boundary of conformal maps,* Rocky Mountain J. Math. 17, (1987), 23–40.
8. K. Diederich and J. E. Fornaess, *Boundary regularity of proper holomorphic mappings,* Invent. Math. 67 (1982), 363–384.
9. C. Fefferman, *The Bergman kernel and biholomorphic mappings of pseudoconvex domains,* Invent. Math. 26 (1974), 1–65.
10. B. Fridman, *On a class of analytic polyhedra,* Soviet Math. Dokl. 19 (1979), 1258–1261.
11. ———, *One example of the boundary behaviour of biholomorphic transformations,* Proc. Amer. Math. Soc. (2) 89 (1983), 226–228.
12. ———, *A universal exhausting domain,* Proc. Amer. Math. Soc. (2) 98 (1986), 267–270.
13. ———, *An approximate Riemann mapping theorem in* $\mathbf{C}^n$, Math. Ann. 275 (1986), 49–55.
14. G. Henkin, *An analytic polyhedron is not holomorphically equivalent to a strictly pseudoconvex domain,* Soviet Math. Dokl. 14 (1973), 858–862.
15. S. Krantz, *Function theory of several complex variables,* Wiley, New York, 1982.
16. S. Pinčuk, *On proper holomorphic mappings of strictly pseudoconvex domains,* Siberian Math. J. 15 (1974), 644–649.

Department of Mathematics
Wichita State University
Wichita, KS 67208