THE BOUNDARY BEHAVIOR OF BLOCH FUNCTIONS AND UNIVALENT FUNCTIONS

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1. Introduction. The function $f(z)=\sum_{n=0}^{\infty} a_n z^n$, analytic for $z \in D=\{z:|z|<1\}$, is called a Bloch function if the norm

$$\|f\|=\sup\{(1-|z|^2)|f'(z)|: z\in D\}+|f(0)|$$

is finite. The set of such functions forms a Banach space B and the subspace of B consisting of those $f\in B$ for which $(1-|z|^2)|f'(z)|\to 0$ as $|z|\to 1-$ is denoted by B_0. The Zygmund class Λ^* consists of those (complex-valued) continuous functions $F(t)$ of period 2π for which

$$F(t+h)+F(t-h)-2F(t)=O(|h|)$$

uniformly in t. If the above second difference is $o(|h|)$ as $|h|\to 0$ then we say that $f\in\lambda^*$.

The spaces Λ^* and λ^* are also Banach spaces with the obvious norm. Moreover,

$$f(z)=\sum_{n=0}^{\infty} a_n z^n \in B \Leftrightarrow F(t)=\sum_{n=1}^{\infty}\frac{a_n}{n}e^{int}\in\Lambda^* \tag{1}$$

and similarly $f\in B_0$ if and only if $F\in\lambda^*$. Thus the space B is isomorphic under the operation of integration to that subspace of Λ^* consisting of functions whose negative Fourier coefficients vanish. In [2] we considered the spaces of real-valued functions in Λ^* and λ^*, but in the present paper we study the complex case. This eventually resolves itself into a consideration of the radial or boundary behavior of Bloch functions and univalent functions.

As in [1], an important class of examples is given by lacunary series

$$f(z)=\sum_{k=0}^{\infty} a_k z^{n_k},$$

where $n_{k+1}/n_k\ge q>1$ for all k, and

$$\|(a_k)\|_\infty=\sup\{|a_k|: k\ge 0\}$$

is finite. Such functions belong to B and also to B_0 if $a_k\to 0$ as $k\to\infty$. If, moreover,

$$\sum_{k=0}^{\infty}|a_k|^2=\infty, \tag{2}$$

then such functions $f(z)$ have finite radial limits $\lim_{r\to 1-} f(re^{i\theta})$ only for a set E of values of θ of measure zero. Also the corresponding functions $F(t)$ have finite derivatives only in such a set E.

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We introduced in [2] the concept of dyadic Λ^*, denoted by Λ_d^* for real-valued functions. Although this can be extended in an obvious way to complex-valued functions, it seems less natural when viewed in the context of analytic functions and so we shall here consider only Λ^* and λ^*.

For a real-valued function $F(t) \in \Lambda^*$ we let E, J_1, J_2 denote the sets

$$E = \{t : F'(t) \text{ exists and is finite}\},$$
$$J_1 = \{t : F'(t) \text{ exists and is } +\infty\},$$
$$J_2 = \{t : F'(t) \text{ exists and is } -\infty\}.$$

If $\phi(t) \downarrow 0$ as $t \to 0+$ we denote by $m_\phi(E)$ the Hausdorff measure of a set E with respect to the function ϕ. Of particular importance in what follows is the function

$$h(t) = t \log \frac{1}{t},$$

and the symbol $m_h(E)$ will always refer to Hausdorff measure with respect to this function.

In §2 we state our results for functions in Λ^* for which $|E| = 0$, that is, for which E has Lebesgue measure zero. We show in particular (Theorem 1) that, in this case, $m_h(J_1) = m_h(J_2) = \infty$. An application of these results is given in §3, leading to a proof of a conjecture of McMillan and Pommerenke [16, Problem 3.4] on univalent functions. Sections 4 and 5 are devoted to proofs; §6 presents various other related problems which we believe to be of interest.

Added in proof (May 6, 1988). In a recent preprint entitled "LIL for smooth measures" [LOMI preprints E-3-88], N. G. Makarov has shown that for every univalent function $g(z)$ we have $m_\phi(J) > 0$, where J is the set where $g'(z)$ has a finite radial limit and $\phi(t) = t(\log(1/t) \log\log\log(1/t))^{1/2}$, and that this result is essentially best possible. It seems likely, as suggested above, that Theorem 1 remains true with $h(t)$ replaced by $\phi(t)$. It is also interesting to determine whether the standard Weierstrass functions have σ-finite ϕ-measure for the sets J_1 and J_2.

2. Functions in Λ^* and λ^* with $|E| = 0$. We first state

THEOREM 1. *Let $F(t)$ be a real-valued function in Λ^* for which $|E| = 0$. Then, for every subinterval I of $(0, 2\pi)$ we have $m_h(J_1 \cap I) = m_h(J_2 \cap I) = \infty$.*

Theorem 1 can be thought of as saying that a function in Λ^* must possess derivatives, finite or infinite, on a set J which is of Hausdorff dimension 1 on each subinterval I. In particular, the well-known example of a nowhere (finitely) differentiable function with lacunary Fourier series

$$F(t) = \sum_{k=1}^{\infty} a_k n_k^{-1} \cos(n_k t)$$

with $\|(a_k)\|_\infty < \infty$ and $a_k \not\to 0$ as $k \to \infty$ must have the derivatives $+\infty$ or $-\infty$ on sets J_1, J_2 with $m_h(J_i \cap I) = \infty$ for each interval I.

As in [2], however, the main thrust of our work is that the class of functions $F(t) \in \Lambda^*$ to which our methods naturally apply consists of those $F \in \Lambda^*$ which are nondifferentiable almost everywhere, and that (2) is of importance only insofar as it ensures this. We remark that, for lacunary series, Theorem 1 has the following corollary.

COROLLARY 1. *If $\{n_k\}$ is a lacunary sequence and if*

$$F(t) = \sum_{k=1}^{\infty} a_k n_k^{-1} \cos n_k(t+t_k)$$

is in Λ^* with $|E| = 0$, then the partial sums of $F'(t)$,

$$S_N(t) = -\sum_{k=1}^{N} a_k \sin n_k(t+t_k)$$

tend to $-\infty$ on J_1 and $+\infty$ on J_2, where (as before) $m_h(J_1 \cap I) = m_h(J_2 \cap I) = \infty$ for every subinterval I.

The corollary follows from the fact, observed by Freud ([6], see also [10]), that

$$\frac{F(t+h)-F(t)}{h} = -\sum_{k=1}^{N} a_k \sin n_h(t+t_k) + O(1)$$

as $N \to \infty$, where $N \approx \log(1/|h|)$.

Theorem 1 will follow from more general theorems, for which we need some further notation. A real-valued function F is said to satisfy the Banach T_2-condition on an interval I if almost every value in its range is assumed at most countably often in I. For a discussion of this see [4] or, more importantly, [17, p. 277]. It has been shown in [2, §7] that if $F \in \Lambda^*$ then F satisfies the T_2-condition. This result is implicit in the work of Mauldin and Williams [13], whose methods of proof were used in [2]. We do not know of any precise determination of those functions satisfying the T_2-condition, but point out that it follows from the consideration of [11, Thm. 5, p. 274] that if $0 < \alpha < 1$ then there are functions $F \in$ Lip α which do not have the T_2-property.

The modulus of continuity $\omega(\delta)$ and the modulus of smoothness $\omega_2(\delta)$ of a continuous function $F(t)$ on an interval I are defined by

$$\omega(\delta) = \omega(\delta, F) = \sup\{|F(t+h)-F(t)| : 0 < h \leq \delta,\ t \in I\} \quad \text{and}$$

$$\omega_2(\delta) = \omega_2(\delta, F) = \sup\{|F(t+h)+F(t-h)-2F(t)| : 0 < h \leq \delta,\ t \in I\},$$

respectively. We also define, in analogy to J_1 and J_2,

$$E_1 = \{t : 0 \leq F'(t) < \infty\}, \qquad E_2 = \{t : -\infty < F'(t) \leq 0\}.$$

Theorem 1 follows from the following more general theorem.

THEOREM 2. *Let $F(t)$ be a continuous real-valued function satisfying the T_2-condition and suppose that $\omega(\delta, f) = O(\phi(\delta))$ as $\delta \to 0$ for some function $\phi(t)$. Let I be an interval on which F has unbounded variation and suppose that $|E_1 \cap I| = 0$. Then $m_\phi(J_1 \cap I) = \infty$. Similarly, if $|E_2 \cap I| = 0$ then $m_\phi(J_2 \cap I) = \infty$.*

To deduce Theorem 1 we note that if $|E|=0$ then $|E_1|=|E_2|=0$ and $F(t)$ is of unbounded variation in every interval. Moreover, it is an exercise to show (see [12, p. 53, Problem 5]) that

$$\omega(\delta)=O\left(\delta\int_\delta^\pi t^{-2}\omega_2(t)\,dt\right)+O(\delta) \tag{3}$$

as $\delta\to 0$. Thus, in particular, it follows that for $F\in\Lambda^*$ we have $\omega(\delta)=O(h(\delta))$, and for $f\in\lambda^*$ we have $\omega(\delta)=O(\phi(\delta))$ for some suitable function ϕ with $\phi(t)=o(t\log(1/t))$ as $t\to 0+$.

Theorem 2 is itself a consequence of the following theorem.

THEOREM 3. *Let $F(t)$ be a real-valued continuous function satisfying the T_2-condition, and suppose that $\omega(\delta,F)=O(\phi(\delta))$ as $\delta\to 0+$ for some function $\phi(t)$. Let I be an interval on which F has unbounded variation and suppose that*

$$\int_{E_1\cap I} F'(t)\,dt<\infty; \tag{4}$$

then $m_\phi(J_1\cap I)=\infty$. Similarly, if

$$\int_{E_2\cap I} F'(t)\,dt>-\infty \tag{5}$$

then $m_\phi(J_2\cap I)=\infty$.

Clearly (4) or (5) will hold if $|E_1|=0$ or $|E_2|=0$, and we note that if

$$E_1(\lambda)=\{t: t\in E_1, F'(t)\geq\lambda\}\quad\text{and}\quad E_2(\lambda)=\{t: t\in E_2, F'(t)\leq -\lambda\}$$

then (4) and (5) can, respectively, be written as

$$\int_0^\infty \lambda|E_1(\lambda)|\,d\lambda<\infty,\qquad \int_0^\infty \lambda|E_2(\lambda)|\,d\lambda<\infty.$$

3. Bloch functions and univalent functions. The connection between Theorem 1 and the radial behavior of Bloch functions is provided by the following two results. The first of these is elementary (see e.g. [9, p. 34]). Let $f(z)\in B$ and $F(t)\in\Lambda^*$ be related by (1) and write

$$f(z)=u(z)+iv(z),\qquad F(t)=U(t)+iV(t),$$

where $U(t)$ and $V(t)$ are real-valued and in Λ^*. We set

$$\begin{aligned}\tilde E&=\{t:\lim_{r\to 1-} f(re^{it})\text{ exists and is finite}\}\\ \tilde E_1&=\{t:\lim_{r\to 1-} u(re^{it})\text{ exists and is finite}\}\\ \tilde E_2&=\{t:\lim_{r\to 1-} v(re^{it})\text{ exists and is finite}\}.\end{aligned} \tag{6}$$

LEMMA 1. *If $U'(t)=a$ for $-\infty\leq a\leq\infty$, then $\lim_{r\to 1-}u(re^{it})=a$. Similarly, if $V'(t)=b$ for $-\infty\leq b\leq\infty$, then $\lim_{r\to 1-}v(re^{it})=b$.*

The second result is less elementary.

THEOREM 4. *Suppose that $f(z)\in B$ and that $|\tilde E|=0$. Then $|\tilde E_1|=|\tilde E_2|=0$.*

This theorem is not true in general—that is, if the condition that $f(z) \in B$ is omitted. An example showing this is readily constructed from [3, Thm. 6, p. 195]. It may be, however, that Theorem 4 remains true for a substantially wider class of functions than B. The following theorem for Bloch functions is now an immediate consequence of Theorem 1 and Theorem 4.

THEOREM 5. *Suppose that $f(z)=u(z)+iv(z)$ belongs to B and that $|\tilde{E}|=0$, where $\tilde{E}$ is defined by* (6). *Then there are four sets $J_1(u), J_2(u), J_1(v), J_2(v)$ defined by*

$$J_1(u)=\{t: \lim_{r\to 1-} u(re^{it})=+\infty\},$$
$$J_2(u)=\{t: \lim_{r\to 1-} u(re^{it})=-\infty\},$$
$$J_1(v)=\{t: \lim_{r\to 1-} v(re^{it})=+\infty\},$$
$$J_2(v)=\{t: \lim_{r\to 1-} v(re^{it})=-\infty\},$$

such that, for each interval $I \subset (0, 2\pi)$, $m_h(J\cap I)=\infty$ for $J=J_1(u), J_2(u), J_1(v), J_2(v)$.

If the function $g(z)$ is univalent in D and we set $f(z)=\log g'(z)$, then $f(z)$ is a Bloch function [15]. If we apply Theorem 5 to the function $f(z)$ and consider only the set $J_2(u)$ we obtain

THEOREM 6. *Let $g(z)$ be univalent in D and suppose that $|\tilde{E}(g')|=0$. Then there is a set J_2 with $\lim_{r\to 1-} g'(re^{it})=0$ for $t\in J_2$ and $m_h(J_2\cap I)=\infty$ for every interval $I\subset(0,2\pi)$.*

This theorem gives a strong affirmative answer to the following question of McMillan and Pommerenke [16, Problem 3.4]: Is the logarithmic capacity $\operatorname{Cap}(\tilde{E}(g'))>0$ for each univalent function g? The much stronger statement $m_h(\tilde{E}(g'))=+\infty$ is true since $J_2\subseteq \tilde{E}(g')$ and either $|\tilde{E}(g')|>0$ or $m_h(J_2)=+\infty$. A similar theorem, whose proof we omit, follows from Theorem 3 by considering the set $E_2(\lambda)$ defined after that theorem. For $0\le\lambda<\infty$ we set

$$\tilde{E}(g',\lambda)=\{t: \lim_{r\to 1-} g'(re^{it})=l \text{ exists and } |l|\le \exp(-\lambda)\}.$$

THEOREM 7. *Suppose that $g(z)$ is univalent in D and that, for some interval I, $|\tilde{E}(g_1)\cap I|<|I|$. If further,*

$$\int^{\infty} \lambda|\tilde{E}(g',\lambda)\cap I|\,d\lambda<\infty, \tag{7}$$

then $m_h(J_2\cap I)=\infty$.

Theorem 7 states, in other words, that if the set where the derivative of a univalent function $g(z)$ has a finite radial limit is of less than full measure in some interval I and if (7) is satisfied, then $g'(z)$ possesses the radial limit zero on a set J_2 with $m_h(J_2\cap I)=\infty$. It is possible that Theorem 7 remains valid without condition (7), but we are unable to show this.

4. Proof of Theorem 3. We prove the first part of Theorem 3. The proof of the second part is similar and, as indicated previously, Theorems 1 and 2 then follow.

What is important for the proof is that F satisfies the T_2-condition of Banach so that we may apply Theorem 6.6 of [17, p. 280]. We pick $a, b \in I$ with $b > a$ and $F(b) > F(a)$. It then follows from (6.7) of [17] that

$$|F([a,b] \cap (J_1 \cup E_1)| \geq F(b) - F(a)$$

and hence

$$|F([a,b] \cap J_1)| \geq F(b) - F(a) - |F([a,b] \cap E_1)|.$$

But, by Denjoy's theorem (see, e.g., the remark on p. 272 or Theorem 6.5 on p. 227 of [17]),

$$|F([a,b] \cap E_1)| \leq \int_{[a,b] \cap E_1} |F'(t)|\, dt.$$

Since $F'(t) \geq 0$ on E_1 we have

$$|F([a,b] \cap J_1)| \geq F(b) - F(a) - \int_{[a,b] \cap E_1}^{\bullet} F'(t)\, dt.$$

Thus, for any covering $\{I_n\}_1^\infty$ of $[a,b] \cap J_1$ by intervals,

$$\sum_{n=1}^{\infty} |F(I_n)| \geq F(b) - F(a) - \int_{[a,b] \cap E_1} F'(t)\, dt.$$

But $F(t)$ has modulus of continuity $O(\phi(\delta))$ and so, since I_n is an interval, there exists a constant $K > 0$ such that

$$|F(I_n)| \leq K\phi(|I_n|), \quad n = 1, 2, \dots.$$

Thus, passing to a suitable covering of $[a,b] \cap J_1$ and taking the infimum we have

$$Km_\phi([a,b] \cap J_1) \geq F(b) - F(a) - \int_{[a,b] \cap E_1} F'(t)\, dt.$$

Since $F(t)$ is of unbounded variation in I, given any $N > 0$ we may choose *disjoint* subintervals $[a_n, b_n] \subset I$ with $F(b_n) > F(a_n)$ and such that

$$\sum_{n=1}^{l} [F(b_n) - F(a_n)] \geq N.$$

Hence,

$$\begin{aligned} Km_\phi(I \cap J_1) &\geq K \sum_{n=1}^{l} m_\phi([a_n, b_n] \cap J_1) \\ &\geq \sum_{n=1}^{l} [F(b_n) - F(a_n)] - \sum_{n=1}^{l} \int_{[a_n, b_n] \cap E_1} F'(t)\, dt \\ &\geq \sum_{n=1}^{l} [F(b_n) - F(a_n)] - \int_{I \cap E_1} F'(t)\, dt, \end{aligned}$$

since the intervals $[a_n, b_n]$ are disjoint. The last term above is bounded by hypothesis and the first is at least N. Since K is fixed and N is arbitrary we obtain $m_\phi(I \cap J_1) = \infty$ as required. □

5. Proof of Theorem 4. Theorem 4 follows readily from known results. The classical theorem of Plessner [5, Thm. 8.2, p. 147] can be sharpened, in the case of Bloch functions (see e.g. [1, Prop. 2.2]), to yield that for almost every t either

$\lim_{r\to 1-} f(re^{it})$ exists (finite or infinite) or the radial cluster set $\{f(re^{it}): 0<r<1\}$ is dense in $\mathbf{C}$. The set $\tilde{E}$ has $|\tilde{E}|=0$ by hypothesis; the set where $\lim_{r\to 1-} f(re^{it})=\infty$ has measure zero by Privalov's theorem (see [5, Cor. 1, p. 146]). We are using here the fact that $f(z)$ is a Bloch function to infer that a radial limit of ∞ will, in fact, be an angular limit of ∞.

We conclude that the set of values of t for which $\{f(re^{it}): 0<r<1\}$ is dense in $\mathbf{C}$ is of measure 2π. Since $\tilde{E}_1$ and $\tilde{E}_2$ are clearly both contained in the complement of this set we have $|\tilde{E}_1|=|\tilde{E}_2|$ as required. □

6. Concluding remarks. It is interesting to compare the results of Hawkes [8] on lacunary series to Theorem 5. For example, Hawkes has shown [8, Thm. 6] that if $f(z)=\sum_{n=1}^{\infty} z^{2^n}$ and $M(r,f)=\sum_{n=1}^{\infty} r^{2^n}$, then the set

$$G(f)=\{t: f(re^{it})\sim \lambda M(r,f) \text{ as } r\to 1- \text{ for some } \lambda>0\} \tag{8}$$

has Hausdorff dimension 1. For related results see also [7]. Although it deals with more general series, Theorem 5 makes no assertion about the rate at which $f(re^{it})$ tends to infinity. On the other hand, if $F(t)=\sum_{n=1}^{\infty} a_n\lambda_n^{-1}e^{i\lambda_n t}$, where $a_n=O(1)$ as $n\to\infty$ and $\lambda_{n+1}/\lambda_n\geq q>1$ for all n, then it follows from [6, Satz IV, Formula 23] that for a.e. t,

$$\limsup_{h\to 0}\frac{|F(t+h)-F(t)|}{|h|S(h)}>0,$$

where

$$S(h)=\left(\sum_{\lambda_n\leq 1/|h|}|a_n|^2\right)^2,$$

and we assume that $S(h)\to\infty$ as $h\to 0+$.

It therefore seems that points t in $G(f)$ defined by (8) might reasonably be called "fast points" of the function $f(z)$ (cf. [10, p. 55]). An interesting question would thus be to give a suitable definition of fast points for general functions in Λ^* and to show that if $|E|=0$ then the set of fast points is large. Of particular interest here would be find analogies with the results of [14] for the Wiener function of standard Brownian motion in one dimension.

We are unable to determine whether or not Theorem 1 is sharp with regard to the function $h(t)=t\log(1/t)$. This function $h(t)$ arises from the global estimate for $\omega(\delta,F)$ for $F\in\Lambda^*$ and sometimes a better function $\phi(t)$ is immediately available. For example, the function

$$F_1(t)=\sum_{n=1}^{\infty} 2^{-n}n^{-1/2}\cos(2^n t) \tag{9}$$

belongs to λ^*, and so has the T_2-property and, moreover, $|E|=0$. Also $\omega_2(\delta,F_1)=O(\delta(\log(1/\delta))^{-1/2})$ as $\delta\to 0+$. Thus, from (3), $\omega(\delta,F_1)=O(\delta(\log(1/\delta))^{1/2})$ as $\delta\to 0+$. Hence $m_\phi(J_1)=m_\phi(J_2)=\infty$ for $\phi(t)=t(\log(1/t))^{1/2}$.

A natural candidate for discussion in this context is the standard Weierstrass function $G(t)=\sum_{n=1}^{\infty} 2^{-n}\cos(2^n t)$. However, using other considerations (to which we hope to return at a later date) we can show that all such lacunary functions in Λ^* must have $m_\phi(J_1)=m_\phi(J_2)=\infty$ for the function $\phi(t)=t(\log(1/t)\log\log(1/t))^{1/2}$.

Indeed, for the function $F_1(t)$ of (9) we may take $\phi(t) = t(\log\log(1/t))$. The methods of proof depend strongly in the lacunarity. Nevertheless, a reasonable first guess would be that such functions $G(t)$ are extremal in the context of Theorem 1, and thus it is possible that Theorem 1 is susceptible to improvement regarding the size of the sets J_1 and J_2. However, we have not been able to prove this, and it is possible that there is a non-lacunary function $F(t)$ in Λ^* satisfying the hypothesis of Theorem 1 for which J_1 and J_2 have σ-finite h-measure.

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