A CLASS OF SPACES WITH INFINITE COHOMOLOGICAL DIMENSION

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1. INTRODUCTION

There are two basic facts which can be used to easily compare the (integral) cohomological dimension and (covering) dimension of most spaces. First, if the dimension of a space is finite, then the cohomological dimension is equal to the dimension; second, the cohomological dimension of a space is greater than or equal to the cohomological dimension of each subspace. In particular, the two definitions of dimension agree for finite dimensional spaces and for infinite dimensional spaces which contain finite dimensional subsets of each dimension. It remains to compute the cohomological dimension of infinite dimensional spaces which do not contain (or are not known to contain) finite dimensional subsets of each dimension. In this paper, a method is developed for computing the cohomological dimension of a class of spaces and is used to show that many of the known examples which exhibit the above pathology have infinite cohomological dimension.

Examples of infinite dimensional compacta which contain no n-dimensional closed subsets ($n \geq 1$) were constructed first by D. Henderson ([6], [7]) in 1967 and, subsequently, by R. H. Bing [3], Zarelua ([16],[17]), the author jointly with L. Rubin and R. Schori [11], and the author jointly with R. Schori [12]. Using the abstract approach developed in [11], the author ([14], [15]) constructed examples of infinite dimensional compacta which contain no n-dimensional ($n \geq 1$) subsets (it is not known that the previous examples contain finite dimensional subsets). The renewed interest in these types of examples is motivated by the following two problems.

Cell-Like Mapping Problem: Does there exist a cell-like dimension raising mapping?

Cohomological Dimension Problem: Does there exist an infinite dimensional compactum with finite cohomological dimension?

A consequence of the Vietoris-Begle mapping theorem [13, p. 344] and the fact that cohomological dimension and dimension agree for finite dimensional spaces is that the image of a cell-like dimension raising mapping would be infinite dimensional and would have finite cohomological dimension. Recently, R. Edwards [5] established the equivalence of these two problems by showing that a compactum with finite cohomological dimension is the cell-like image of a finite dimensional compactum.

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The remainder of the paper is organized as follows. Section 2 contains terminology, definitions, and basic facts. Section 3 contains the statements of the main results. In Section 4, the equivalence between stable mappings and essential families is established. Section 5 contains statements and proofs of "finite dimensional" versions of the main results and these are used in Section 6 to prove the main results. Section 7 contains the computation of the cohomological dimension of some of the examples constructed in [11], [12], and ([14], [15]).

A suggested plan for those interested in studying the examples constructed in ([6], [7]), [3], ([16], [17]), [11], [12], and ([14], [15]) is first to read [12] and then to read [11]; the general theory developed in Section 5 in [11] is used in ([14], [15]) and can be used to simplify many of the arguments in ([6], [7]), [3], and ([16], [17]).

2. DEFINITIONS AND BASIC CONCEPTS

By a *space* we mean a separable metric space, by a *compactum* we mean a compact space, and by a *continuum* we mean a connected compactum. The covering dimension of a space X is denoted by dim X; we refer to [9] for standard results from the theory of covering dimension. The integral cohomological dimension of a space X is denoted c-dim X; we refer to [10] for standard results from the theory of cohomological dimension.

Let S^n denote an n-sphere and let K_n denote the Eilenberg-MacLane complex (obtained from S^n by attaching cells of dimension greater or equal to $n + 2$) with $\pi_i(K_n) = \pi_i(S^n)$ for $i \leq n$ and $\pi_i(K_n) = 0$ for $i \geq n + 1$.

THEOREM 2.1. [9; p. 83]. *For a space X,* dim $X \leq n$ *if and only if for each closed subset A and mapping $f: A \to S^n$ there is an extension $\bar{f}: X \to S^n$.*

THEOREM 2.2. [10; p. 7]. *For a compactum X, c-dim $X \leq n$ if and only if for each closed subset A and mapping $f: A \to K_n$ there is an extension $\bar{f}: X \to K_n$.*

Since the $(n + 1)$-skeleton of K_n is equal to S^n, we have the following.

LEMMA 2.3. *If X is a compactum with* dim $X \leq n + 1$, *then each mapping $f: X \to K_n$ is homotopic relative to $f^{-1}(S^n)$ to a mapping $f': X \to S^n$.*

Let B^{n+1} be an $(n + 1)$-ball and denote its boundary by S^n. A mapping $f: X \to B^{n+1}$ is *unstable* if there is a mapping $\bar{f}: X \to S^n$ with $\bar{f} = f$ on $f^{-1}(S^n)$; otherwise, f is *stable.* A mapping $f: X \to B^{n+1}$ is *cohomologically unstable* if there is a mapping $\bar{f}: X \to K_n$ with $\bar{f} = f$ on $f^{-1}(S^n)$; otherwise, f is *cohomologically stable.* The next results follow from Theorems 2.1 and 2.2, respectively.

COROLLARY 2.4. *For a space X,* dim $X \geq n + 1$ *if and only if there is a stable mapping $f: X \to B^{n+1}$.*

COROLLARY 2.5. *For a compactum X, c-dim $X \geq n + 1$ if there is a cohomologically stable mapping $f: X \to B^{n+1}$.*

Remark 2.6. Clearly, a cohomologically stable mapping is stable. A standard example of a stable mapping which is not cohomologically stable is obtained by letting $\alpha: S^k \to S^n$ represent a nontrivial element of $\pi_k(S^n)$ with $k > n$ and letting $f: B^{k+1} \to B^{n+1}$ be the "cone" of α.

Remark 2.7. These corollaries are central to the approach developed to detect that certain compacta have infinite cohomological dimension; specifically, we show that a certain class of stable mappings consists of cohomologically stable mappings.

Definition 2.8. Let A and B be disjoint closed subsets of a space X. A closed subset S of X is said to *separate* A and B in X if $X - S$ is the union of two disjoint open sets, one containing A and the other containing B. A closed subset S of X is said to *continuum-wise separate* A and B in X provided every continuum in X from A to B meets S.

3. MAIN RESULTS

Let N denote the natural numbers, let $Q = \Pi\{I_k: k \in N\}$ be the Hilbert cube where $I_k = [-1, 1]$, let $\Pi_k: Q \to I_k$ be the projection, and let $A_k = \Pi_k^{-1}(1)$ and $B_k = \Pi_k^{-1}(-1)$. For a subset $N' \subseteq N$, let $\Pi: Q \to \Pi\{I_k: k \in N'\}$ be the projection and let $\Pi_X: X \to \Pi\{I_k: k \in N'\}$ be the restriction of Π to $X \subseteq Q$.

THEOREM 3.1. *Let $\{N_1, N_2\}$ be a partition of N and for each $k \in N_1$, let S_k be a separator of A_k* and *B_k. Letting $X = \cap\{S_k: k \in N_1\}$, for each finite subset $\{t_1, t_2, \ldots, t_q\} \subseteq N_2$, $\Pi_X: X \to \Pi\{I_k: k = t_1, t_2, \ldots, t_q\}$ is cohomologically stable. In particular,* c-dim *X is greater than or equal to the cardinality of N_2.*

THEOREM 3.2. *Let $N_1 = \{r_1, r_2, \ldots\}$. The preceding theorem remains true if S_{r_1} continuum-wise separates A_{r_1} and B_{r_1} in Q and, for $k \geq 2$, $S_{r_k} \cap S_{r_{k-1}}$ continuum-wise separates $A_{r_k} \cap S_{r_{k-1}}$ and $B_{r_k} \cap S_{r_{k-1}}$ in $S_{r_{k-1}}$.*

The class of spaces referred to in the title is the class consisting of compacta of type described in Theorems 3.1 and 3.2 with N_2 infinite and of compacta which contain such compacta (see Section 7 for examples of the latter type). Theorem 3.1 represents the essential new result in the paper; its proof easily generalizes to yield Theorem 3.2. It is the latter statement which is needed in Section 7 in order to show that various of the examples mentioned in the introduction have infinite cohomological dimension.

4. STABLE MAPPINGS AND ESSENTIAL FAMILIES

This section contains a method for constructing and detecting stable mappings using essential families (defined below); a development of the theory of essential families can be found in [11; Section 5].

Definition 4.1. Let X be a space and let Γ be a finite or countably infinite indexing set. A family $\{(A_k, B_k): k \in \Gamma\}$ is *essential* in X if, for each $k \in \Gamma$, (A_k, B_k) is a pair of disjoint closed sets in X such that if S_k separates A_k and B_k, then $\cap\{S_k: k \in \Gamma\} \neq \emptyset$.

Example 4.2. If $I^n = \Pi\{I_k: 1 \leq k \leq n\}$ where $I_k = [-1,1]$, $\Pi_k: I^n \to I_k$ is the projection, and $A_k = \Pi_k^{-1}(1)$ and $B_k = \Pi_k^{-1}(-1)$, then $\{(A_k, B_k): 1 \leq k \leq n\}$

is an essential family in I^n (see [9; p. 40]). The family of pairs of opposite faces of the Hilbert cube described in Section 3 is an essential family.

The next result contains a precise statement of the relation between stable mappings and essential families; the result is contained implicitly in the literature (for example, see [2; p. 20]).

PROPOSITION 4.3. *Let X be a compactum, let $\{(A'_k, B'_k): 1 \le k \le n\}$ be a family of pairs of disjoint closed subsets of X, and let $f_k: X \to I_k$ with $A'_k = f_k^{-1}(1)$ and $B'_k = f_k^{-1}(-1)$. The family $\{(A'_k, B'_k): 1 \le k \le n\}$ is essential if and only if the mapping $f: X \to I^n$ defined by $f = (f_1, \ldots, f_n)$ is stable.*

Proof. Letting $\{(A_k, B_k): 1 \le k \le n\}$ be the family of pairs of opposite faces of I^n, notice that $f^{-1}(A_k) = A'_k$ and $f^{-1}(B_k) = B'_k$. Let S^{n-1} be the boundary of I^n.

If f is unstable, then there is a mapping $f': X \to S^{n-1}$ with $f' = f$ on $f^{-1}(S^{n-1})$. Moreover, there is a mapping $\bar{f}: X \to I^n - (0, 0, \ldots, 0)$ with $\bar{f} = f$ on $f^{-1}(S^{n-1})$ and $\bar{f}^{-1}(S^{n-1}) = f^{-1}(S^{n-1})$ (consider the linear homotopy between f and f'); in particular, $\bar{f}^{-1}(A_k) = A'_k$ and $\bar{f}^{-1}(B_k) = B'_k$. For $1 \le k \le$ n, let $S_k = \Pi_k^{-1}(0)$; S_k separates A_k and B_k and, hence, $S'_k = \bar{f}^{-1}(S_k)$ separates A'_k and B'_k. Observe that

$$\cap \{S'_k: 1 \le k \le n\} = \bar{f}^{-1}(\cap \{S_k: 1 \le k \le n\}) = \emptyset;$$

therefore, the family $\{(A'_k, B'_k): 1 \le k \le n\}$ is not essential.

Conversely, if the family $\{(A'_k, B'_k): 1 \le k \le n\}$ is not essential, then let S_k be a separator of A'_k and B'_k with $\cap \{S_k: 1 \le k \le n\} = \emptyset$. Let $\bar{f}_k: X \to I_k$ be such that $A'_k = \bar{f}_k^{-1}(1)$, $B'_k = \bar{f}_k^{-1}(-1)$, and $S_k = \bar{f}_k^{-1}(0)$, and let $\bar{f}: X \to I_n$ be defined by $\bar{f} = (\bar{f}_1, \ldots, \bar{f}_n)$. Observe that $\bar{f}^{-1}(S^{n-1}) = f^{-1}(S^{n-1})$ and that

$$\bar{f}(X) \subseteq I^n - (0, 0, \ldots, 0)$$

(since $\bar{f}^{-1}((0, 0, \ldots, 0)) = \cap \{S_k: 1 \le k \le n\} \doteq \emptyset$); letting $\bar{f}' = r \circ f$ where $r: I^n - (0, 0, \ldots, 0) \to S^{n-1}$ is a retraction, we see that $\bar{f}$ is unstable. However, the restrictions of f and $\bar{f}$ to $f^{-1}(S^{n-1}) = \bar{f}^{-1}(S^{n-1})$ are homotopic as mappings into S^{n-1} (the homotopy is obtained by restricting the homotopy of f to $\bar{f}$ which is constructed from the linear homotopies between the f_k's and $\bar{f}_k$'s) and, therefore, it follows from Borsuk's extension theorem [4] that f is unstable.

For the remainder of this section, let $\{(A_k, B_k): k \in \Gamma\}$ be the family of pairs of opposite faces of either the Hilbert cube or the n-cell I^n (see 4.2). We point out that Propositions 4.4 and 4.6 below were stated in [11] for Γ countably infinite; the proofs therein are valid for Γ finite.

PROPOSITION 4.4. [11; Proposition 5.5]. Let $J \subseteq \Gamma$ *and, for each* $j \in J$, *let* S_j *be a separator of* A_j *and* B_j. Letting $X = \cap \{S_j: j \in J\}$, *the family* $\{(X \cap A_k, X \cap B_k): k \in \Gamma - J\}$ *is essential in* X.

COROLLARY 4.5. *Given the hypotheses of Proposition 4.4, for each finite subset* $\{t_1, \ldots, t_q\} \subseteq \Gamma - J, \Pi_X: X \to \Pi\{I_k: k = t_1, \ldots, t_q\}$ *is stable.*

Proof. Letting $f_{t_i} = \Pi_{t_i}$ restricted to X, we have that

$$f_{t_i}^{-1}(1) = X \cap A_{t_i}, f_{t_i}^{-1}(-1) = X \cap B_{t_i},$$

and $\Pi_X = (f_{t_1}, ..., f_{t_q})$; since $\{(X \cap A_k, X \cap B_k): k = t_1, ..., t_q\}$ is essential (a subfamily of an essential family is essential), Proposition 4.3 implies that Π_X is stable.

PROPOSITION 4.6. [11; Proposition 5.6]. *Let $J = \{j_1, j_2, ...\}$ be a finite or infinite subset of Γ. Let $\{S_j : j \in J\}$ be a collection of closed subsets with S_{j_1} continuum-wise separating A_{j_1} and B_{j_1} and with, for $i \geq 2$, $S_{j_1} \cap S_{j_{i-1}}$ continuum-wise separating $A_{j_i} \cap S_{j_{i-1}}$ and $B_{j_i} \cap S_{j_{i-1}}$ in $S_{j_{i-1}}$. Letting $X = \cap \{S_j : j \in J\}$, the family $\{(X \cap A_k, X \cap B_k): k \in \Gamma - J\}$ is essential in X.*

COROLLARY 4.7. *Given the hypotheses of Proposition 4.6, for each finite subset $\{t_1, ..., t_q\} \subseteq \Gamma - J$, $\Pi_X : X \to \Pi\{I_k : k = t_1, ..., t_q\}$ is stable.*

5. FINITE DIMENSIONAL CASE

For $1 \leq q \leq n$, let $I^q = \Pi\{I_k : 1 \leq k \leq q\}$; let S^{q-1} be the boundary of I^q; and let $\{(A_k, B_k): 1 \leq k \leq n\}$ be the family of pairs of opposite faces of I^n.

THEOREM 5.1. *For $m \leq k \leq n$, let S_k be a separator of A_k and B_k and let $X = \cap \{S_k : m \leq k \leq n\}$. For each $1 \leq q \leq m - 1$, $\Pi_X : X \to I^q$ is cohomologically stable.*

Proof (by contradiction). Suppose that there is a mapping $f: X \to K_{q-1}$ with $f = \Pi_X$ on $\Pi_X^{-1}(S^{q-1})$; extend f to $X \cup \Pi^{-1}(S^{q-1})$ using Π and let $\tilde{f}$ be an extension of f to a neighborhood U of $X \cup \Pi^{-1}(S^{q-1})$; an extension exists since the image of f is contained in a finite subcomplex which is an *ANR* (absolute neighborhood retract). For $q + 1 \leq k \leq n$, let $\hat{S}_k$ be an $(n-1)$-dimensional polyhedron separating A_k and B_k chosen so that, i) $\hat{S}_{q+1} \cap ... \cap \hat{S}_n \subseteq U$ (for $m \leq k \leq n$, choose $\hat{S}_k$ near S_k); and ii) $\dim(\hat{S}_{q+1} \cap ... \cap \hat{S}_n) \leq q$ (general position). Letting $Y = \hat{S}_{q+1} \cap ... \cap \hat{S}_n$, $\Pi_Y : Y \to I^q$ is stable (Corollary 4.5) and $Y \subseteq U$. Let $\tilde{f}_Y$ be the restriction of $\tilde{f}$ to Y; since $\tilde{f}_Y : Y \to K_{q-1}$ and $\dim Y \leq q$, there is a mapping $g: Y \to S^{q-1}$ with $g = \tilde{f}_Y$ on $\tilde{f}_Y^{-1}(S^{q-1})$ (Lemma 2.3) and, therefore, $g = \Pi_Y$ on $\Pi_Y^{-1}(S^{q-1})$. This contradicts that Π_Y is stable.

Remark 5.2. If S separates A_k and B_k, then boundaries of "small" polyhedral neighborhoods are $(n-1)$-dimensional polyhedra which separate A_k and B_k and which "approximate" S. If S continuum-wise separates A_k and B_k, then "approximating" $(n-1)$-dimensional polyhedra can be constructed as follows. Let $\delta = d(A_k, B_k)$, let $S_A = \{x \in S: d(x, A_k) \leq \delta/3\}$, let $S_B = \{x \in S: d(x, B_k) \leq \delta/3\}$, and let $S_M = \mathrm{cl}(S - S_A \cup S_B)$. Let P_A, P_B, P_M be the boundaries of "small" polyhedral neighborhoods of S_A, S_B, S_M, respectively, each chosen to meet S^{n-1} in an $(n-2)$-dimensional polyhedron. The $(n-1)$-dimensional polyhedron $P = P_A \cup P_B \cup P_M$ continuum-wise separates A_k and B_k and "approximates" S. If Z is a closed subset of I_n and we only assume that $S \cap Z$ continuum-wise separates $A_k \cap Z$ and $B_k \cap Z$ in Z, then the polyhedron P has the property that $P \cap Z$ continuum-wise separates $A_k \cap Z$ and $B_k \cap Z$ in Z.

THEOREM 5.3. *Let $\{S_k: m \leq k \leq n\}$ be a collection of closed subsets of I^n with S_m continuum-wise separating A_m and B_m and, for $m+1 \leq k \leq n$, $S_k \cap S_{k-1}$ continuum-wise separating $A_k \cap S_{k-1}$ and $B_k \cap S_{k-1}$ in S_{k-1}.* Let $X = \cap \{S_k: m \leq k \leq n\}$. For $1 \leq q \leq m-1$, $\Pi_X: X \to I^q$ *is cohomologically stable.*

Proof. Make the following modifications to the proof of Theorem 5.1. Let $\hat{S}_n$ be an $(n-1)$-dimensional polyhedron "approximating" S_n as described in Remark 5.2. Since $\hat{S}_n$ is the union of boundaries of polyhedral neighborhoods, if $\hat{S}_{n-1}$ is chosen "sufficiently close" to S_{n-1}, then $\hat{S}_n \cap \hat{S}_{n-1}$ continuum-wise separates $A_n \cap \hat{S}_{n-1}$ and $B_n \cap \hat{S}_{n-1}$ in $\hat{S}_{n-1}$. For $k = n-2, \ldots, j+1$, successively, choose $\hat{S}_k$ so that $\hat{S}_{k+1} \cap \hat{S}_k$ continuum-wise separates $A_{k+1} \cap \hat{S}_k$ and $B_{k+1} \cap \hat{S}_k$ in $\hat{S}_k$. Finally, use Corollary 4.7 in place of Corollary 4.5.

6. PROOFS OF MAIN RESULTS

For convenience we assume that $\{t_1, \ldots, t_q\} = \{1, \ldots, q\}$ in the statement of Theorem 3.1. If Π_X is not cohomologically stable, then there exists $f: X \to K_{q-1}$ with $f = \Pi_X$ on $\Pi_X^{-1}(S^{q-1})$; extend f to $X \cup \Pi^{-1}(S^{q-1})$ using Π and let $\tilde{f}: U \to K_{q-1}$ be an extension of f to a neighborhood U of $X \cup \Pi^{-1}(S^{q-1})$. Let $\{r_1, \ldots, r_s\} \subseteq N_1$ be such that $\cap \{S_k: k = r_1, \ldots, r_s\} \subseteq U$; let $n = \max\{r_1, \ldots, r_s\}$ and by reordering assume that $\{r_1, \ldots, r_s\} = \{n-s+1, \ldots, n\}$. For $n-s+1 \leq k \leq n$, let $S'_k = S_k \cap I^n (I^n = I^n \times (0, 0, \ldots) \subseteq Q)$ and let $X' = \cap \{S'_k: n-s+1 \leq k \leq n\}$. Letting $\tilde{f}_{X'}$ be the restriction of $\tilde{f}$ to X', $\tilde{f}_{X'}: X' \to K_{q-1}$ and $\tilde{f}_{X'} = \Pi_{X'}$ on $\Pi_{X'}^{-1}(S^{q-1})$; therefore, $\Pi_{X'}$ is not cohomologically stable; but this contradicts Theorem 5.1.

The proof of Theorem 3.2 is the same using Theorem 5.3 instead of Theorem 5.1.

7. APPLICATIONS

We are interested in using Theorem 3.2 to show that many of the examples constructed in ([6], [7]), [3],([16, 17]), [11], ([14], [15]), and [12] have infinite cohomological dimension. Each of the constructions can be done so that Theorem 3.2 applies immediately (i.e., N_2 is infinite); however, the partition can be chosen so that $N_1 = \{2, 3, \ldots\}$ and $N_2 = \{1\}$. Even in this case it is often possible to find a subset of the example which Theorem 3.2 implies has infinite cohomological dimension; we give two illustrations. The first involves the construction in [11] (and can easily be adapted to handle the constructions in ([14], [15]) and ([16], [17]); and, the second involves the construction in [12] where the desired subset is more difficult to find.

In this paragraph we adopt the notation in Section 6 of [11]; in particular, we are interested in the example $Z = \cap \{Z_k: k \geq 2\}$ constructed in the final paragraph of Section 6 where Z_k continuum-wise separates A_k and B_k. Our goal is to show that $Z \cap \Pi_1^{-1}(1)$ has infinite cohomological dimension (in fact, if y is an endpoint of an interval from any $\mathscr{W}_k$, then $Z \cap \Pi_1^{-1}(y)$ has infinite cohomological dimension). Observe that the sub-Hilbert cube

$$\Pi_1^{-1}(1) \subseteq \cap \{Z_k: k \in N_1\}$$

and that $\{(A_k \cap \Pi_1^{-1}(1), B_k \cap \Pi_1^{-1}(1)): k \geq 2\}$ is the family of pairs of opposite faces of $\Pi_1^{-1}(1)$. Letting $N_2' = N_1$ and $N_1' = \{k: k \geq 2 \text{ and } k \notin N_2'\}$, $\{N_1', N_2'\}$ is a partition of $\{k: k \geq 2\}$. Since

$$Z \cap \Pi_1^{-1}(1) = \cap \{Z_k \cap \Pi_1^{-1}(1): k \in N_1'\}$$

and N_2' is infinite, Theorem 3.2 applied to $\Pi_1^{-1}(1)$ yields that $Z \cap \Pi_1^{-1}(1)$ has infinite cohomological dimension.

In this paragraph, we adopt the notation in Section 3 of [12]; the example $Y = \cap \{X_k: k \geq 2\}$ where X_k continuum-wise separates A_k and B_k. For $k \geq 0$, $\Pi_{3^k}^{-1}(1) \subseteq X_{3^{k+1}-1} \cap X_{3^{k+1}}$ and, therefore, the sub-Hilbert cube

$$Q' = \cap \{\Pi_{3^k}^{-1}(1): k \geq 0\} \subseteq \cap \{X_{3^{k+1}-1} \cap X_{3^{k+1}}: k \geq 0\};$$

the family of pairs of opposite faces of Q' is

$$\{(A_q \cap Q', B_q \cap Q'): q \neq 3^k \text{ for some } k \geq 0\}.$$

Partition the set $\{q: q \neq 3^k \text{ for some } k \geq 0\}$ by letting $N_1 = \{q: q \neq 3^k$ or $3^{k+1} - 1$ for some $k \geq 0\}$ and $N_2 = \{q: q = 3^{k+1} - 1 \text{ for some } k \geq 0\}$. Then $Y \cap Q' = \cap \{X_q \cap Q': q \in N_1\}$ and, since N_2 is infinite, Theorem 3.2 implies that $Y \cap Q'$ has infinite cohomological dimension.

Remark 7.1. The techniques developed in this paper can be used to analyze examples constructed by intersecting continuum-wise separators of opposite faces of the Hilbert cube. The various constructions can be done starting with any strongly infinite dimensional compactum; these techniques do not seem to apply in this general setting.

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