LOCALLY COHERENT MINIMAL SETS

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Let (X, T) be a transformation group with phase space X and phase group T. In a recent paper [4] Chu proved: *If* X *is compact, minimal, with* dim X = n *and if* T *acts trivially on* $H^n(X)$, *then* $H^n(A) = 0$ *for every proper closed subset* A *of* X. Thus an n-dimensional minimal set is "like" a manifold. In this paper I introduce and study the notion of local n-coherence which is a "localization" of the property "T acts trivially on $H^n(X)$." If dim X = n and if X is locally n-coherent, then X has more of the properties of a manifold than the one given by Chu's theorem (see for example Theorems 3 and 4). As an application of the notions developed here I show that if T is the group of real numbers and if (X, T) is minimal with X compact, one dimensional, and locally connected, then X is homeomorphic to a circle.

In what follows X is assumed to be locally compact with dim X = n. The notation used is that of [3]. Thus $H_c^*(X)$ denotes the cohomology of X with compact supports with coefficients in the principal ring L, and dim X is the L-dimension of X.

Definition 1. Let V be an open subset of X. Then V is *n-coherent* if the diagram

$$(1)\qquad \begin{array}{ccc} & & H_c^n(V) \\ & \nearrow & \uparrow t^* \\ H_c^n(V \cap Vt) & & \\ & \searrow & H_c^n(Vt) \end{array}$$

is commutative for all $t \in T$. Of course if $V \cap Vt = \phi$ for some $t \in T$, the condition is vacuously satisfied for that t.

To say that X itself is n-coherent is another way of saying that T acts trivially on $H_c^n(X)$.

Whether or not V is n-coherent depends upon the coefficients used. Thus when necessary, the coefficients will be explicitly mentioned, for example, we may write V is Z-n-coherent.

(X, T) is said to be *n-coherent at* $x \in X$ if x has a neighborhood base of open n-coherent sets; (X, T) is *locally n-coherent* if it is n-coherent at all $x \in X$.

Diagrams of the type 1 will occur frequently, so that it will be convenient to represent it by the symbol $(V \cap Vt, V, Vt)$.

LEMMA 1. *Let* V *be open,* M *a submodule of* $H_c^n(V)$, $(V_\alpha / \alpha \in I)$ *a family of open subsets of* V *such that* $V = \bigcup V_\alpha$, *and* $\operatorname{im} j^n_{V,V_\alpha} \subset M$. *Then* $M = H_c^n(V)$.

Proof. Let $\alpha, \beta \in I$. Then the exact sequence

$$H_c^n(V_\alpha \cap V_\beta) \to H_c^n(V_\alpha) \oplus H_c^n(V_\beta) \to H_c^n(V_\alpha \cup V_\beta) \to H_c^{n+1}(V_\alpha \cap V_\beta) = 0$$

Received September 24, 1962.
The author is a National Science Foundation Senior Postdoctoral Fellow.

shows that $\operatorname{im} j^n_{V,V_\alpha \cup V_\beta} \subset M$. Then by induction $\operatorname{im} j^n_{V,V_F} \subset M$, where $V_F = \bigcup[V_\alpha/\alpha \in F]$ and F is a finite subset of I. Finally,

$$H^n_c(V) = \varinjlim [H^n_c(V_F)/F \text{ finite} \subset I]$$

implies that $M = H^n_c(V)$.

LEMMA 2. *Let* V *be an* n-*coherent open subset of* X, W *an open subset of* X *and* $t \in T$ *with* $W \cup Wt \subset V$. *Then* $\operatorname{im} j^n_{V,W} = \operatorname{im} j^n_{V,Wt}$.

Proof. Let $u \in H^n_c(Wt)$. Then $u/V = (u/Vt)t^*$ since V is n-coherent and $Wt \subset V \cap Vt$.

The commutative diagram

$$\begin{array}{ccc} H^n_c(Wt) & \rightarrow & H^n_c(Vt) \\ \downarrow t^* & & \downarrow t^* \\ H^n_c(W) & \rightarrow & H^n_c(V) \end{array}$$

shows that $(ut^*)/V = (u/Vt)t^* = u/V$. Since t^* maps $H^n_c(Wt)$ isomorphically onto $H^n_c(W)$, the proof is complete.

LEMMA 3. *Let* V *be open and* n-*coherent,* W *open and contained in* V *with* $V \subset WT$. *Then* $j^n_{V,W}$ *is surjective.*

Proof. Let $x \in V$. Then there exists a $t \in T$ with $xt^{-1} \in W$. Let N be an open neighborhood of x such that $x \in N \subset V$ and $Nt^{-1} \subset W$. Set $U = Nt^{-1}$. Thus $U \subset W \subset V$ and $x \in Ut \subset V$. Therefore

$$\operatorname{im} j^n_{V,U} \subset \operatorname{im} j^n_{V,W} \quad \text{and} \quad \operatorname{im} j^n_{V,U} = \operatorname{im} j^n_{V,Ut}.$$

Set $N_x = Ut$. Then $x \in N_x \subset V$ and $\operatorname{im} j^n_{V,N_x} \subset \operatorname{im} j^n_{V,W}$. Since $\bigcup[N_x/x \in V] = V$, $H^n_c(V) = \operatorname{im} j^n_{V,W}$ by Lemma 1.

THEOREM 1. *Let* (X, T) *be minimal,* V *an open* n-*coherent subset of* X. *Then* $H^n_c(A) = 0$ *for every proper closed subset* A *of* V. (The transformation group (X, T) is said to be *minimal* if $\operatorname{cls}(xT) = X$ $(x \in X)$.)

Proof. Let $W = V - A$. Then W is a non-null open subset of V. Moreover, $WT = X$ since (X, T) is minimal. Hence $j^n_{V,W}$ is surjective by Lemma 3. Theorem 1 now follows from the exactness of the sequence

$$H^n_c(W) \to H^n_c(V) \to H^n_c(A) \to H^{n+1}_c(W) = 0.$$

Theorem 1 is a generalization of Chu's Theorem 1 [4] since to say that T acts trivially on $H^n_c(X)$ is equivalent to saying that X is n-coherent.

THEOREM 2. *Let* (X, T) *be minimal,* V *an open* n-*coherent subset of* X *with* $H^n_c(V) \neq 0$, *and let* A *be a closed subset of* V *with* $\dim A \leq n - 2$. *Then* V - A *is connected. In particular,* V *is connected.*

Proof. Let B, C be closed subsets of V with $B \cup C = V$ and $B \cap C = A$. Then the exact sequence

$$0 = H^{n-1}_c(B \cap C) \to H^n_c(B \cup C) \to H^n_c(B) \oplus H^n_c(C) \to H^n_c(B \cap C) = 0$$

together with Theorem 1 shows that both B and C cannot be proper.

LEMMA 4. *Let* (X, T) *be minimal and locally* n-*coherent. Then every point of* X *has a neighborhood base consisting of* n-*coherent open sets* V *with* $H_c^n(V) \neq 0$.

Proof. Since dim X = n, there exists an n-coherent open set U with $H_c^n(U) \neq 0$. Let $x \in X$, and let W be open with $x \in W$. Choose V to be n-coherent and open, and choose a t in T such that $x \in V \subset W$ and $Vt \subset U$. Then $H_c^n(Vt) \neq 0$ by Lemma 3. Hence $H_c^n(V) \neq 0$.

LEMMA 5. *Let* (X, T) *be minimal and locally* n-*coherent, let* A *be a locally compact subset of* X, *and let* $x \in A$. *Then the following statements are equivalent.* (i) $x \in$ *int* A. (ii) *There exists an open* A-*neighborhood* N *of* x *such that* $H_c^n(M) \neq 0$ *for every open* A-*neighborhood* M *of* x *contained in* N.

Proof. Let $x \in$ int A. Then by Lemma 4, there exists an n-coherent open set N with $x \in N \subset$ int A and $H_c^n(N) \neq 0$. If M is open with $M \subset N$, then $H_c^n(M) \neq 0$ by Lemma 3. Thus (i) implies (ii).

Assume (ii). Now $A = U \cap F$ with U open and F closed. Let V be an n-coherent open subset of X with $x \in V \subset U$ and $V \cap A \subset N$. Then $V \cap F = V \cap A$ is an A-neighborhood of x contained in N. Thus $H_c^n(V \cap F) \neq 0$, whence $V \cap F = V$ by Theorem 1. Therefore $V \subset A$.

THEOREM 3. *Let* (X, T) *be minimal and locally* n-*coherent. Let* U *be open in* X, *and let* f *be a homeomorphism from* U *into* X. *Then* f(U) *is open.*

Proof. Apply Lemma 5 to f(U).

THEOREM 4. *Let* (X, T) *be minimal, connected, and locally* n-*coherent. Let* A *be a locally compact subset of* X. *Then* dim A = n *if and only if* int $A \neq \emptyset$.

Proof. If int $A \neq \emptyset$, then of course dim A = n.

Conversely if int $A = \emptyset$, then, by Lemma 5, every point of A has an A-neighborhood base consisting of open sets N with $H_c^n(N) = 0$. Hence dim $A \leq n - 1$.

LEMMA 6. *Let* V *be an open* n-*coherent subset of* X. *Then* Vt *is* n-*coherent for each* $t \in T$.

Proof. Let $s \in T$ and $u \in H_c^n(Vt \cap Vts)$. Then $ut^* \in H_c^n(V \cap Vtst^{-1})$, whence $ut^*/V = (ut^*/Vr)r^*$, where $r = tst^{-1}$, since $(V \cap Vr, V, Vr)$ is commutative. Now

$$ut^*/V = (u/Vt)^* \quad \text{and} \quad (ut^*/Vr) = (u/Vts)t^*.$$

Therefore

$$(u/Vt)t^* = ut^*/V = (ut^*/Vr)r^* = (u/Vts)t^*r^* = (u/Vts)s^*t^*$$

since $r^* = t^{-1*}s^*t^*$. Thus $u/Vt = (u/Vts)s^*$.

LEMMA 7. *Let* V *be an open* n-*coherent subset of* X, *and let* t *be an element of* T *for which* $j^n_{V, V \cap Vt}$ *is surjective. Then* $V \cup Vt$ *is* n-*coherent.*

Proof. Suppose $s \in T$, and let $W = V \cup Vt$. We must show that $(W \cap Ws, W, Ws)$ is commutative. Since

$$W \cap Ws = V \cap Vs \cup Vt \cap Vs \cup V \cap Vts \cup Vt \cap Vts,$$

it suffices to show that (i) $(V \cap Vs, W, Ws)$, (ii) $(Vt \cap Vs, W, Ws)$,

(iii) $(V \cap Vts, W, Ws)$, (iv) $(Vt \cap Vts, W, Ws)$ are commutative. The proofs for diagrams (i) and (iv) are similar as are those for (ii) and (iii). Hence we need only show that (i) and (ii) are commutative.

Consider

$$\begin{array}{ccccc} & & H_c^n(V) & \longrightarrow & H_c^n(W) \\ & \nearrow & & & \\ H_c^n(V \cap Vs) & \text{I} & \uparrow s^* & \text{II} & \uparrow s^* \\ & \searrow & & & \\ & & H_c^n(Vs) & \longrightarrow & H_c^n(Ws)\ . \end{array}$$

I is merely $(V \cap Vs, V, Vs)$, which is commutative by assumption. Since II is commutative, so is $(V \cap Vs, W, Ws)$.

With respect to $(Vt \cap Vs, W, Ws)$, let $a \in H_c^n(Vt \cap Vs)$. Then $(a/Vt)t^* \in H_c^n(V)$. Hence, by assumption, there exists a $b \in H_c^n(V \cap Vt)$ with $b/V = (a/Vt)t^*$. Since $(V \cap Vt, V, Vt)$ is commutative by assumption, $b/V = (b/Vt)t^*$, whence $b/Vt = a/Vt$.

Now $(Vt \cap (Vt)t^{-1} s, Vt, (Vt)t^{-1} s)$ is commutative by Lemma 6. Hence $a/Vt = (a/Vs)(t^{-1} s)^*$, whence $b/V = (a/Vt)t^* = (a/Vs)s^*$.

Also

$$\begin{array}{ccc} H_c^n(V) & \rightarrow & H_c^n(W) \\ \uparrow s^* & & \uparrow s^* \\ H_c^n(Vs) & \rightarrow & H_c^n(Ws) \end{array}$$

is commutative. Thus

$$a/W = (a/Vt)/W = (b/Vt)/W = (b/V)/W = (a/Vs)s^*/W = ((a/Vs)/Ws)s^* = (a/Ws)s^*;$$

that is, $(Vt \cap Vs, W, Ws)$ is commutative.

LEMMA 8. *Let* V *be an* n-*coherent open subset of* X. *Then* $j^n_{V \cup Vt, V}$ *is injective for each* $t \in T$.

Proof. Let $a \in H_c^n(V)$ with $a/V \cup Vt = 0$. Since

$$H_c^n(V \cap Vt) \rightarrow H_c^n(V) \oplus H_c^n(Vt) \rightarrow H_c^n(V \cup Vt)$$

is exact, there exists a $b \in H_c^n(V \cap Vt)$ with $b/V = a$ and $b/Vt = 0$. Hence $a = 0$ because $(b/Vt)t^* = b/V$ since V is n-coherent.

LEMMA 9. *Let* V *be an* n-*coherent subset of* X, *and let* (X, T) *be minimal and* X *connected. Then* $j^n_{X,V}$ *is an isomorphism and* X *is* n-*coherent.*

Proof. Let $\mathcal{E}$ be the collection of open subsets W of X having the following properties: (i) $V \subset W$, (ii) W is n-coherent, (iii) $j^n_{W,V}$ is injective. Let $(W_\alpha / \alpha \in I)$ be a simply ordered family of elements of $\mathcal{E}$, and let $W = \bigcup W_\alpha$. Then $V \subset W$, and since $H_c^n(W) = \varinjlim H_c^n(W_\alpha)$, W satisfies (iii). Let $t \in T$. Since $(W_\alpha \cap W_\alpha t, W_\alpha, W_\alpha t)$ is commutative ($\alpha \in I$), so is $(W_\alpha \cap W_\alpha t, W, Wt)$. Therefore $(W \cap Wt, W, Wt)$ is also commutative. Thus $\mathcal{E}$ is a non-vacuous inductive set if ordered by inclusion. Let W be a maximal element of $\mathcal{E}$. Let $x \in \overline{W}$. Then there exists a $t \in T$ with $xt^{-1} \in W$. It follows that $Wt \cap W \neq \phi$. Since (X, T) is minimal, $j^n_{W,Wt \cap W}$ is surjective by Lemma 3. Hence $W \cup Wt$ is n-coherent by Lemma 7. Finally by Lemma 8, $j^n_{W \cup Wt, W}$ is injective, whence $j^n_{W \cup Wt, V}$ is injective. Thus $W \cup Wt \in \mathcal{E}$ with $W \subset W \cup Wt$. This implies that $W = W \cup Wt$, that is, $Wt \subset W$. Consequently, $x \in W$. Thus W is an open and closed subset of X. The connectedness of X implies that W = X. Hence X is n-coherent, and $j^n_{X,V}$ is injective. Then Lemma 3 implies that $j^n_{X,V}$ is surjective.

LEMMA 10. *Let* V *be an* n-*coherent open subset of* X, *and let* W *be an open subset of* V *with* $j^n_{V,W}$ *injective. Then* W *is* n-*coherent.*

Proof. Let $u \in H^n_c(W \cap Wt)$, $a = (u/Wt)t^*$, $b = u/W$. Then

$$a/V = (u/Vt)t^* = u/V$$

by the n-coherence of V. Further $a/V = b/V$, whence $a = b$; that is, $(W \cap Wt, W, Wt)$ is commutative.

LEMMA 11. *Let* (X, T) *be minimal and locally* n-*coherent, and suppose* X *is connected. Let* V, W *be* n-*coherent open subsets of* X *with* $V \cap W \neq \emptyset$. *Then* $V \cup W$ *is* n-*coherent.*

Proof. By Lemma 9, X is n-coherent. Hence by Lemma 10, it suffices to show that $j^n_{X,V \cup W}$ is injective. Let $a \in H^n_c(V \cup W)$ with $a/X = 0$. Choose $b \in H^n_c(V)$ and $c \in H^n_c(W)$ with $b/V \cup W - c/V \cup W = a$. Let U be an n-coherent open subset of X with $U \subset V \cap W$. By Lemma 3, there exist $u_1, u_2 \in H^n_c(U)$ with $u_1/V = b$ and $u_2/W = c$. Then

$$(u_1 - u_2)/V \cup W = a \quad \text{and} \quad (u_1 - u_2)/X = 0.$$

Thus $u_1 - u_2 = 0$ by Lemma 9, whence $a = 0$.

THEOREM 5. *Let* (X, T) *be minimal, locally* n-*coherent, and let* X *be connected. Let* V *be open in* X. *Then* V *is* n-*coherent if and only if* V *is connected.*

Proof. Since dim X = n, there exists an n-coherent open subset W with $H^n_c(W) \neq 0$. By Lemma 9, $H^n_c(X) \neq 0$, and X is n-coherent. Consequently, by Lemma 3, $H^n_c(U) \neq 0$ for every open set U.

Let V be an n-coherent open subset of X, and let U be an open and closed subset of V. If U were proper, $H^n_c(U) = 0$ by Theorem 1. This is impossible. Hence V is connected.

Conversely, let V be an open connected subset of X. Let $\mathcal{E}$ be the collection of open subsets W of V such that $j^n_{X,W}$ is injective. Then $\mathcal{E} \neq \emptyset$ by Lemma 9. Also $\mathcal{E}$ is inductive if ordered by inclusion. Let W be a maximal element of $\mathcal{E}$. Then W is open. Let $x \in V \cap \overline{W}$, and suppose N is an n-coherent open subset of X with $x \in N \subset V$. Then W is n-coherent by Lemma 10. Moreover, $W \cap N \neq \emptyset$ implies that $W \cup N$ is also n-coherent by Lemma 11. Hence $W \cup N \in \mathcal{E}$. Thus $x \in W \cup N \subset W$, and so W is open-closed in V. Hence W = V. The proof is complete.

THEOREM 6. *Let* X *be an* n-*cm* (see [3]) *such that* X *is* n-*coherent and* (X, T) *is minimal. Then* X *is orientable if and only if* (X, T) *is locally* n-*coherent.*

Proof. Since $H^n_c(X) \neq 0$, X is connected by Theorem 2. If X is orientable, then $j^n_{X,V}$ is an isomorphism for every connected open set V. Hence in this case (X, T) is locally n-coherent by Lemma 10.

Conversely, if (X, T) is locally n-coherent, then $H_c^n(X) = L$ (that is X is orientable) by Theorem 5 and Lemma 9.

COROLLARY 1. *Let* (X, T) *be minimal, where* X *is a connected* n-*cm. Then* (X, T) *is* Z_2-*locally* n-*coherent.*

As an application of the above results let us consider the action of the reals R on a space X.

LEMMA 12. *Let* $x \in X$ *with* $xR = X$. *Then* X *is homeomorphic to a point, the real line, or a circle.*

Proof. Let $\phi: R \to X$ be such that $\phi(t) = xt$ $(t \in R)$, let I be a compact subinterval of R. Then $\phi(I)$ is a compact subset of X, and the union of countably many translates of I covers X. Hence int $\phi(I) \neq \emptyset$.

Now let N be an open subset of R, and let $t \in N$. Choose I, an interval about 0, such that $I + (t - I) \subset N$. Let $xr \in \text{int}\,\phi(I)$ for some $r \in I$. Then

$$\phi(t) = xt = xr(t - r) \in (\text{int}\,\phi(I))(t - r) \subset \text{int}\,\phi(N).$$

Thus ϕ is open as well as continuous. The proof is complete.

THEOREM 7. *Let* (X, R) *be minimal, let* dim X = 1, *and let* X *satisfy condition* (A): *Given* $x \in X$ *and* U *a neighborhood of* x, *there exists an open neighborhood* V *of* x *such that* $V \subset U$ *and* X - V *is connected. Then* X *is homeomorphic to a circle.*

Proof. Let V be open and relatively compact with X - V connected. Consider the exact sequence $H_c^0(X - V) \to H_c^1(V) \to H_c^1(X)$. If X is not compact, X - V is not compact. Hence $H_c^0(X - V) = 0$ since X - V is connected. Thus $j^1_{X,V}$ is injective. If X is compact, the above sequence with $H_c^0(X - V)$ replaced by the reduced group again shows that $j^1_{X,V}$ is injective.

Now R is connected, whence X is 1-coherent. This implies that V is 1-coherent by Lemma 10. Hence (X, R) is locally 1-coherent if Condition (A) is satisfied.

Let $x \in X$, and suppose $\phi: R \to X$ is such that $\phi(t) = xt$ $(t \in R)$. If ϕ is not injective, then $xt = x$ for some $t \in R$. Let $I = [-t, t]$. Then $xR = xI$, and

$$X = \text{cls}(xR) = \text{cls}(xI) = xI.$$

Thus if ϕ is not injective, X is homeomorphic to a circle by Lemma 12.

Now suppose ϕ is injective. Let I be a compact subinterval of R. Then xI is a closed one dimensional subset of X, whence int $(xI) \neq \emptyset$ by Theorem 4. Thus $xR = (xI)R = X$ because (X, R) is minimal. Lemma 12 now implies that X is either a point, a circle, or the real line. It cannot be a point since dim X = 1, and it cannot be the real line because the latter does not satisfy Condition (A).

COROLLARY 1. *Let* (X, R) *be minimal, let* dim X = 1, *and let* X *be a compact metric space that is semi-locally connected* [6, p. 19]. *Then* X *is homeomorphic to a circle.*

Proof. If $xR = X$ for some $x \in X$, then X is homeomorphic with a circle by Lemma 12. Let $x \in X$, and let $y \notin xR$. Then the everywhere dense connected set $yR \subset X - x$, whence X - x is connected.

By [6, p. 50], X will satisfy Condition (A) of Theorem 7 if X - x is connected $(x \in X)$. The proof is complete.

THEOREM 8. *Let* (X, R) *be minimal, let* $\dim X = 1$, *and let* X *be compact and locally connected. Then* X *is homeomorphic to a circle.*

Proof. Let $x \in X$, let U be a neighborhood of x, and let W be an open connected neighborhood of x whose closure N is contained in U.

Suppose $X - N \neq \emptyset$ and $H_c^1(X - N) = 0$. Let K be a component of $X - N$. Then K is open, and $H_c^1(K) = 0$. Let B be the boundary of K. Then $B \neq \emptyset$, and $\operatorname{cls} K - B = K$. Consider the exact sequence

$$H_c^0(\operatorname{cls} K) \to H_c^0(B) \to H_c^1(K).$$

Since $\operatorname{cls} K$ is connected and $H_c^1(K) = 0$, B is connected. Since B is 0-dimensional, it must be a single point b. Now let $y \in K$. Since $yR \cap K \neq \emptyset$ and $yR \cap K' \neq \emptyset$, $B \cap yR \neq \emptyset$. Hence $y \in bR$; that is, $K \subset bR$. The fact that K is open then implies that $bR = X$. Since X is compact and one-dimensional, Lemma 12 now implies that X is homeomorphic with a circle. However, in this case it is clear that $H_c^1(X - N) \neq 0$.

Consequently if $X - N \neq \emptyset$, $H_c^1(X - N) \neq 0$. The exactness of the sequence

$$H_c^0(N) \to H_c^1(X - N) \to H_c^1(X)$$

shows that $j^1_{X,X-N}$ is injective. Hence $X - N$ is 1-coherent by Lemma 10. Then Theorem 2 shows that $X - N$ is connected. Now $X - \operatorname{int} N = \operatorname{cls}(X - N)$, which is again connected. Thus $V = \operatorname{int} N$ will satisfy Condition (A) of Theorem 7. The proof is complete.

REMARKS

1. Theorem 6 and its corollary show that (X, T) may be locally Z_2-n-coherent and not locally Z-n-coherent.

2. If T is connected, then (X, T) is n-coherent.

3. Let L be a finite field, let X be connected, and suppose $\dim H_c^n(X)$ is finite. Let $S = [t/t \in T, t^* = \text{id.}]$. Then S is an invariant subgroup with finite index. Since X is connected, (X, S) is again minimal. Thus the various results may be applied to (X, S).

4. Let M be the universal curve of Menger. Then [1] (M, T) is minimal, with T the group of integers. Also M is connected and locally connected [2], and $\dim M = 1$. However, (M, T) is not locally 1-coherent for if it were, M would be n-coherent, which is impossible [4].

5. For each positive integer n, let X_n denote the subset $[Z/|Z| = 1]$ of the complex numbers. Let R denote the reals. The transformation group (X_n, R, π_n) is defined by the relation

$$\pi_n(X, t) = x \exp\left(\frac{it}{2^n}\right) \qquad (x \in X_n,\ t \in R).$$

Thus $t \in R$ acts as a rotation through $t/2^n$ radians on X_n. Then (X, R_n) is minimal and equicontinuous. For each n, let $\phi_n\colon X_n \to X_{n-1}$ be the map such that $\phi_n(x) = x^2$ $(x \in X_n)$. Then ϕ_n is a homomorphism of (X_n, R) onto (X_{n-1}, R). The family $[(X_n, \phi_n);\ n = 1, \cdots)]$ defines an inverse system of transformation groups. Let (X, R) denote the inverse limit of the family. Then (X, R) is minimal and equicontinuous. Also X is a solenoid, dim X = 1, and X is connected. Moreover, X is n-coherent since R is connected. However, (X, R) is not locally n-coherent, because X is not locally connected.

Let I be the unit interval, and let $x \in X$. Then xI is a closed subset of X with dim (xI) = 1, but int (xI) = $\emptyset$. Thus X does not satisfy the conclusion of Theorem 4.

6. Mostow's result [5] that the Klein bottle is the homogeneous space of a connected Lie group shows that locally euclidean does not imply local n-coherence.

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