

## Introduction to the Special Issue on Vaught's Conjecture

Peter Cholak

Vaught's Conjecture, stated in Vaught's 1961 paper [3], "Denumerable models of complete theories," says that for a countable complete elementary first-order theory, the number of countable models, up to isomorphism, is either countable or  $2^{\aleph_0}$ . The conjecture has attracted a great deal of attention over the past 40 years, inspiring work in model theory and in other branches of logic as well.

In May 2005, the Logic Group at the University of Notre Dame held a workshop to discuss the ongoing forward progress of resolving Vaught's Conjecture. The idea was to bring together a number of people from different areas of mathematical logic and allow them to focus together on Vaught's Conjecture. The result was, by all accounts, an outstanding workshop.

This refereed special issue is one of the outcomes of this workshop. Not only does this issue contain the writeup of some of the talks given at the workshop but two of the papers (Marker [2], p. 93 and Hjorth [1], p. 49) solve questions which arose during the workshop.

Some of the talks (for example, by Marker and by Buechler) focused on settings in which Vaught's Conjecture holds. Some of the talks and resulting papers focused on the features that a counterexample would necessarily have, and on examples illustrating at least some of those features. The "Scott rank" of a countable structure is a measure of internal model theoretic complexity. Morley showed that if  $T$  is a counterexample to Vaught's Conjecture, then  $T$  has only countably many models of any given countable rank, and it has models of arbitrarily large countable rank. Thus, the number of countable models of  $T$ , up to isomorphism, is  $\aleph_1$ . Sacks's talk described a chain of models that a counterexample would have, of increasing Scott rank, preserving satisfaction for increasingly large admissible fragments.

The speakers at the workshop were, in the order in which they spoke, Gerald Sacks (2 presentations), Su Gao, David Marker (2 presentations), Wesley Calvert, Valentina Harizanov, Robin Knight, Łudomir Newelski (2 presentations), Nate

Printed February 26, 2007

Keywords: Vaught's conjecture

©2007 University of Notre Dame

Ackermann, Steven Buechler (2 presentations), Reed Solomon, Robert Soare, Greg Hjorth, John Baldwin, Jessica Young, Greg Hjorth (2 presentations), Bektur Baizhanov, Alexander Kechris (2 presentations), Howard Becker, Sławomir Solecki, and Chris Laskowski.

The workshop was organized by Steven Buechler, Peter Cholak, Julia Knight, and Sergei Starchenko with the help of postdocs Noam Greenberg and David Lippel, and our graduate students at the time, Logan Axon, Prerna Bihani, Wesley Calvert, Joshua Cole, Panteleimon Eleftheriou, Jacob Heidenreich, Christina Maher, Sara Miller, Christopher Porter. The workshop was funded by the National Science Foundation through Grant DMS-0516576. We were also supported by the Logic Group at Notre Dame Research Training Grant, NSF-EMSW21-RTG-03-53748, Notre Dame's College of Science, Notre Dame's Office of Research, our own individual NSF grants, and Knight's Charles L. Husking Chair of Mathematics.

I would like to thank all involved, including the authors and the referees, for their help. I am particularly thankful to one anonymous referee whose efforts were far beyond the norm.

### References

- [1] Hjorth, G., "A note on counterexamples to the Vaught Conjecture," *Notre Dame Journal of Formal Logic*, vol. 48 (2007), pp. 49–51 (electronic). [1](#)
- [2] Marker, D., "The Borel complexity of isomorphism for theories with many types," *Notre Dame Journal of Formal Logic*, vol. 48 (2007), pp. 93–97 (electronic). [1](#)
- [3] Vaught, R. L., "Denumerable models of complete theories," pp. 303–321 in *Infinitistic Methods (Proceedings of the Symposium on the Foundations of Mathematics, Warsaw, 1959)*, Państwowe Wydawnictwo Naukowe and Pergamon, Warsaw and Oxford, 1961. [Zbl 0113.24302](#). [MR 0186552](#). [1](#)

Department of Mathematics  
University of Notre Dame  
Notre Dame IN 46556  
[cholak.1@nd.edu](mailto:cholak.1@nd.edu)