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PURE NUMERICAL BOOLEAN SYLLOGISMS

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A. Undefined notions.

1. Let a, b, c, d, etc. be general terms (which may be empty).
2. Let m, n, and q be integers greater than or equal to zero.
3. $\mathbf{A}^+\mathrm{n}\ bc$, read: "At most n b are not c."
4. $\mathbf{I}^+\mathrm{n}\ bc$, read: "More than n b are c."

B. Definitions.

1. $\mathbf{E}^+\mathrm{n}\ bc = \mathrm{df} \sim \mathbf{I}^+\mathrm{n}\ bc$

   "At most n b are c" means "Not more than n b are c."

2. $\mathbf{O}^+\mathrm{n}\ bc = \mathrm{df} \sim \mathbf{A}^+\mathrm{n}\ bc$

   "More than n b are not c" means "It is false that at most n b are not c."

(3-6) Standard categorical forms (Boolean interpretation):

3. $\mathbf{A}\,bc = \mathrm{df}\ \mathbf{A}^+0\ bc$

   "Every b is c" means "At most zero b are not c."

4. $\mathbf{E}\,bc = \mathrm{df}\ \mathbf{E}^+0\ bc$
5. $\mathbf{I}bc = \mathrm{df}\ \mathbf{I}^+0\ bc$
6. $\mathbf{O}bc = \mathrm{df}\ \mathbf{O}^+0\ bc$
7. $\mathbf{A}^+$ and $\mathbf{E}^+$ forms are called *quasi-universal*, $\mathbf{I}^+$ and $\mathbf{O}^+$ *particular*.
8. A numerical argument is called *pure* when all its propositions have quantifiers.[1]

C. Immediate inferences.

1. Contradictories:

   a. $\mathbf{A}^+\mathrm{n}\ bc$ and $\mathbf{O}^+\mathrm{n}\ bc$ are contradictories.
   b. $\mathbf{E}^+\mathrm{n}\ bc$ and $\mathbf{I}^+\mathrm{n}\ bc$ are contradictories.

2. Contraries:

   a. $\mathbf{A}^+\mathrm{n}\ bc$ and $\mathbf{O}^+\mathrm{m}\ bc$ are contraries iff $\mathrm{m} > \mathrm{n}$.
   b. $\mathbf{E}^+\mathrm{n}\ bc$ and $\mathbf{I}^+\mathrm{m}\ bc$ are contraries iff $\mathrm{m} > \mathrm{n}$.

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3. Subcontraries:

    a. A^+n *bc* and O^+m *bc* are subcontraries iff n $>$ m.
    b. E^+n *bc* and I^+m *bc* are subcontraries iff n $>$ m.

4. Superimplication:

    a. A^+n *bc* is a superimplicant of A^+m *bc* iff m $>$ n.
    b. E^+n *bc* is a superimplicant of E^+m *bc* iff m $>$ n.
    c. I^+n *bc* is a superimplicant of I^+m *bc* iff n $>$ m.
    d. O^+n *bc* is a superimplicant of O^+m *bc* iff n $>$ m.

5. Conversion:

    a. A^+n *bc*: no valid conversion.
    b. E^+n *bc* converts to E^+n *cb*.
    c. I^+n *bc* converts to I^+n *cb*.
    d. O^+n *bc*: no valid conversion.

D. Distribution.

1. The distributed terms are bracketed, the undistributed are not bracketed:

a. A^+n (*a*)*b*	b. E^+n (*a*)(*b*)
c. I^+n *ab*	d. O^+n *a*(*b*)

2. The concept of distribution appropriate here is based on the DeMorgan-Curry principle of class substitution:[2] A term t_a naming the *a*'s is distributed in a declarative sentence *S* when, for any term t_c naming the *c*'s, given that every *c* is an *a* and that *S* is true, it would follow that the sentence is true formed from *S* by replacing t_a by t_c. For example, if it is true that at most one million Scots are not Protestants (A^+n *sp*), it follows that at most one million Highland Scots are not Protestants, but not that at most one million Scots are not Baptists.

E. The Valid forms of Pure Numerical Boolean Syllogism.

1. Fundamental (unweakened) forms

(Barbara)	(*Baroko*)	(*Bokardo*)
A^+n *bk*	A^+n *bk*	O^+n+m *ak*
A^+m *ab*	O^+n+m *ak*	A^+m *ab*
A^+n+m *ak*	O^+m *ab*	O^+n *bk*

(Celarent)	(*Disamis*)	(*Festino*)
E^+n *cf*	I^+n+m *df*	E^+n *cf*
A^+m *dc*	A^+m *dc*	I^+n+m *df*
E^+n+m *df*	I^+n *cf*	O^+m *dc*

(*Camestres*)	(Darii)	(*Ferison*)
A^+m *dc*	A^+m *dc*	E^+n *fc*
E^+n *fc*	I^+n+m *fd*	I^+n+m *fd*
E^+n+m *fd*	I^+n *fc*	O^+m *dc*

(*Cesare*)	(*Datisi*)	(**Ferio**)
$E^{+}n$ *fc*	$A^{+}m$ *dc*	$E^{+}n$ *fc*
$A^{+}m$ *dc*	$I^{+}n{+}m$ *df*	$I^{+}n{+}m$ *df*
$E^{+}n{+}m$ *df*	$I^{+}n$ *fc*	$O^{+}m$ *dc*
(*Camenes*)	(*Dimaris*)	(*Fresison*)
$A^{+}m$ *dc*	$I^{+}n{+}m$ *fd*	$E^{+}n$ *cf*
$E^{+}n$ *cf*	$A^{+}m$ *dc*	$I^{+}n{+}m$ *fd*
$E^{+}n{+}m$ *fd*	$I^{+}n$ *cf*	$O^{+}m$ *dc*

2. When all coefficients are zero, each of the above forms is equivalent to the standard Boolean syllogism named above it. Any "More-at most" syllogistic form not in the above table is equivalent to an invalid standard Boolean syllogism in the all-zeros case.

The three syllogistic forms in the same row are equivalent by indirect reduction; the three 4th-figure forms are in the last row. Excluding the first row, the four forms in the same column agreeing in middle terms are directly equivalent by simple conversion. Hence all 15 forms follow if we postulate any one in the first row together with any one in the other rows. Reduction to first-figure forms—named in bold face—can be guided by the mnemonic names.

3. Subaltern forms. The subaltern forms for the above are obtained by substituting q for the coefficient n+m where $q > n+m$.

F. Rules of Validity for Pure Numerical Syllogisms.

1. A pure numerical Boolean syllogism is valid iff it satisfies rules NS 1-7. NS 1-5 are traditional-type rules for (standard and numerical) Boolean syllogisms. NS 6-7 are peculiar to certain numerical forms. The parenthetical clauses in the coefficient rules cover the subaltern forms.

NS 1. The middle term is distributed at least once.

NS 2. Any term distributed in the conclusion is distributed in a premiss.

NS 3. If the conclusion is affirmative, both premisses are affirmative.

NS 4. If the conclusion is negative, just one premiss is negative.

NS 5. If the conclusion is particular, a premiss is particular.

NS 6. The coefficient of a quasi-universal conclusion is equal to (or greater than) the sum of the coefficients of the premisses.

NS 7. The coefficient of a particular conclusion is equal to (or less than) the coefficient of the particular premiss minus that of the other premiss.

Cor. If the premisses differ in quantity, the coefficient of the particular premiss is greater than or equal to the coefficient of the quasi-universal premiss.

2. Antilogistic rules. A pure numerical Boolean syllogism is valid iff the triad consisting of its premisses and the contradictory of its conclusion satisfies the following rules:

A1. Just one proposition is particular.

A2. Every term is distributed just once.

A3. The coefficient of the particular proposition is equal to (or greater than) the sum of the coefficients of the other two propositions.

G. Some exponible numerical forms.

1. **J**n bc = df **E**$^+$n bc and **I**$^+$n-1 bc.

   "Just n b are c" means "At most n b are c and more than n-1 b are c."

2. **J**nb**A**c = df **J**n bc and **A**cb

   "Just n b are all the c" means "Just n b are c and every c is b."

NOTES

1. The forms of this paper are called "Boolean" because the terms may apply to nothing. We do not find the syllogistic forms and assumptions of our system in De Morgan, Boole, or Jevons. The numerical syllogisms of Keynes's *Formal Logic* (4th ed.) suggest ours, but are "mixed," one proposition in each argument being in standard categorical form. W. T. Parry in "On Numerical Moods of the Syllogism," *Philosophy and Phenomenological Research*, vol. 10 (1950), 411-13, uses "A_n" for "Less than n s are not p," ("**A**$^+$n−1"), "E_n" for "Less than n s are p," "I_n" for "At least n s are p," "O_n" for "At least n s are not p," where n > 0. He generalizes Keynes's forms, giving the four fundamental forms of the first figure, e.g., $A_nA_qA_{q+n-1}$, $E_nI_{q+n-1}O_q$. This system would give 15 fundamental forms, corresponding by mood and figure to the 15 valid forms of standard Boolean syllogism, or the 15 forms of sec. E,1 below. "More-at most" forms are used in the present paper mainly because the coefficients in the syllogism are a bit simpler.

2. In an unpublished paper on distribution, Dr. Parry finds this principle in De Morgan's *Formal Logic*, in H. B. Curry (*Mind*, 1936), and in S. F. Barker's *Elements of Logic* (1965).

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