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CORRIGENDUM AND ADDENDUM TO MY PAPER "A GENERALIZATION OF SIERPIŃSKI'S THEOREM ON STEINER TRIPLES AND THE AXIOM OF CHOICE"

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§1. *Corrigendum*. A manuscript mix-up in the composition of [1] resulted in the appearance of an incorrect proof of Theorem 1 based on inadequate formulations of Definitions 2 and 3 used in preliminary researches. To remedy this difficulty:

1) Replace Definitions 2 and 3 on page 164 with the following

Definition 2: *Let γ be any ordinal number less than ω_λ. Then set* $F_\gamma^{(n-1)} = \{\alpha_\gamma^{(1)}, \ldots, \alpha_\gamma^{(n-1)}\}$.

Definition 3: *Let* $S(F_1^{(n-1)}) = (\sum_{i=1}^{n-1} \alpha_1^{(i)}) + 1$. *Let $\gamma < \omega_\lambda$ and suppose $S(F_\xi^{(n-1)})$ is defined for all $\xi < \gamma$. Then set $S(F_\gamma^{(n-1)})$ to be the first element of E not contained in the set* $\bigcup \{F_\xi^{(n-1)} : \xi \leqq \gamma\} \cup \bigcup \{S(F_\xi^{(n-1)}) : \xi < \gamma\}$.

2) On page 164, omit lines 1-3 after "*Proof:*" and lines 23-24.
3) On page 167, omit lines 26-31.
4) On page 168, omit lines 1-19 and replace with

From (29), (30) and Definition 3 we obtain $S(F_{\varphi_\xi}^{(n-1)}) \notin \{F_{\varphi_\eta}^{(n-1)} \cup S(F_{\varphi_\eta}^{(n-1)})\}$ and consequently $\{\alpha^{(1)}, \ldots, \alpha^{(n-1)}\} = F_{\varphi_\xi}^{(n-1)}$. But with (29) this implies $\{\alpha_{\varphi_\xi}^{(1)}, \ldots, \alpha_{\varphi_\xi}^{(n-1)}\} \subset \{F_{\varphi_\eta}^{(n-1)} \cup S(F_{\varphi_\eta}^{(n-1)})\}$ which, in virtue of the fact that $\varphi_\eta < \varphi_\xi$, contradicts the construction of φ_ξ. Hence (29) and (30) cannot both obtain and $\mathfrak{J}_n$ is a Steiner family of order n for the set E.

5) A remark on certain notations used in [1] is in order. Frequently in that work there appears expressions of the form $x \not\subset \{y_i : i \in I\}$ where x is a set and y_i is a set for each i in some index set I. Such an expression in [1] should be interpreted to mean $x \not\subset y_i$ for each $i \in I$. Likewise for $x \overline{\subset} \{y_i : i \in I\}$.

§2. *Addendum*. The author wishes to take this opportunity to announce that all results given in [1] have been further generalized to the higher cardinal numbers.[1] To indicate the direction these generalizations take we will

1. These generalizations constitute a segment of the author's thesis, "Block designs on infinite sets," which was written under the direction of Professor B. Sobociński and accepted by the Graduate School of the University of Notre Dame in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mathematics, February, 1966. This work will appear in forthcoming issues of the *Notre Dame Journal of Formal Logic*.

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state our main result. To do this we introduce

Definition A: *Let M be any non-finite set and F a family of subsets of M. Let $\mathfrak{p}$ be any cardinal number such that $\mathfrak{p} \leqq \overline{\overline{M}}$. Then F is said to be a non-nested $\mathfrak{p}$-tuple family of M if and only if* a) $\overline{\overline{x}} = \mathfrak{p}$ *for each* $x \in F$ *and* b) $x \not\subset y$ *and* $y \not\subset x$ *whenever* $x, y \in F$ *and* $x \neq y$.

Definition B: *Let M be any non-finite set and F and G two families of subsets of M. Let $\mathfrak{p}$ be any cardinal number such that $\mathfrak{p} \leqq \overline{\overline{M}}$. Then F is said to be a Steiner cover of degree $\mathfrak{p}$ for the family G if and only if for every $x \in G$ the set $\{y \in F: x \subset y\}$ has cardinality $\mathfrak{p}$.*

With these definitions we are able to prove

Theorem C: *Let M be any non-finite set and suppose $\mathfrak{n}$, $\mathfrak{m}$, $\mathfrak{p}$ are non-zero cardinal numbers such that* a) $\mathfrak{n} < \mathfrak{m} < \overline{\overline{M}}$ *and* b) $\mathfrak{p} \leqq \overline{\overline{M}}$. *Let F be any non-nested $\mathfrak{n}$-tuple family of M. Then there exists a family G of subsets of M such that* (i) *G is a Steiner cover of degree $\mathfrak{p}$ for the family F and* (ii) $\overline{\overline{x}} = \mathfrak{m}$ *for each* $x \in G$.

It is clear that Theorem C subsumes Theorem 1 of [1] as a special case. One notices, moreover, that Theorem C asserts, in particular, that all the well-known generalizations of Steiner triple systems in combinatorial analysis (i.e. tactical configurations and incomplete block designs) always exist on every non-finite set.

Bibliography

[1] Frascella, W. J. "A generalization of Sierpiński's theorem on Steiner triples and the axiom of choice," *Notre Dame Journal of Formal Logic,* Vol. 6 (1965) pp. 163-179.

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