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A LIBERALIZED SYSTEM OF QUANTIFICATIONAL DEDUCTION[1]

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In this note, we prove a liberalized version of the system of quantificational deduction of Section VI of Patton [2] to be sound.

The rules of that system were **UI** and **EI**, where the instantial variable of an EI step wasn't allowed to be one that was free in a previous line of the deduction. The system was designed to show a set of formulas to be inconsistent by deriving a truth-functionally inconsistent set of quantifierless lines from prenex normal form versions of these formulas. Soundness was proved by showing this to be impossible if the original set of formulas is consistent.

The liberal system is like this one except that the premises of a deduction—the formulas of the set to be shown inconsistent--may be expressed as disjunctions of formulas in prenex normal form and **UI** and **EI** are supplanted by a rule that lets us drop from one to all leftmost quantifiers of the disjuncts of such a disjunction and replace all the variables they bound by a single variable that doesn't become bound in the line inferred. The **EI** restriction is imposed here if an existential quantifier is dropped. (Thus every deduction of the old system is a deduction of this system too.)

Now consider the system like this liberal one except that its rules are **UI**, **EI**, and a nonformal rule that allows passage from a given line to any formula equivalent to it. Despite the nonformal character of this system, the soundness proof of Patton [2] extends to it in obvious fashion. Our liberal system will be proved sound by showing that its every deduction D has a counterpart D' in this nonformal system such that every line in D also occurs in D'.

D' will have the same premises as D. Suppose that the first k-1 lines of D also occur in D' and that line (k) in D comes from a previous line (j) thus (where Em, Fn, Gr, and Hs are formulas in which m, n, r, and s respectively occur free and a is neither free in a line previous to (k) nor bound in (k)):

1. Some features of this system, but not its rule of inference, were suggested to me in conversation by B. Dreben.

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(j) $$(m)Em \vee (\exists n)Fn \vee (r)Gr \vee (\exists s)Hs$$

(k) $$Ea \vee Fa \vee (r)Gr \vee Ha$$

By our supposition, (j) also occurs in D', and the nonformal rule (where b and c are new) lets us pass in D' from (j) to

(j') $$(\exists b)(c)(Ec \vee Fb \vee (r)Gr \vee Hb)$$

We may then obtain (k) from (j') in D' by steps of **EI** and **UI**.

BIBLIOGRAPHY

[1] B. Dreben, P. Andrews, S. Aanderaa, 'False Lemmas in Herbrand', *Bulletin of the American Mathematical Society* (to appear).

[2] T. E. Patton, 'A System of Quantificational Deduction', *Notre Dame Journal of Formal Logic*, vol. IV (1963), pp. 105-112.

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