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SYLLOGISTIC INFERENCE WITHIN THE PROPOSITIONAL CALCULUS

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There is a way of rendering the syllogism whereby the soundness of any syllogistic inference can be tested by the logic of truth functions, without additional formal notations for properties, predicates, or classes. Although simple, this method is not without interest, for it counters the claim often made in logic textbooks that symbolism and rules of inference beyond those provided within the propositional calculus are required for determining the validity of syllogistic inference. Quine's statement in this regard is typical.[1] At the beginning of Part Two of *Methods of Logic*, immediately following his development of the logic of truth functions, Quine asserts: "There are many simple and logically sound inferences for which the foregoing techniques are inadequate."[2] His example is a syllogism of the form EIO, figure 1, the validity of which is shown on the basis of truth functions in the fourth paragraph of this paper.

Let numerals for the numbers 1 through 7 be propositional variables, interpreted for our purpose as follows:

(1)	Something is A and B and C
(2)	Something is A and B and not C
(3)	Something is A and not B and C
(4)	Something is A and not B and not C
(5)	Something is not A and B and C
(6)	Something is not A and B and not C
(7)	Something is not A and not B and C.

Although notations for properties (or predicates, or classes, as the reader will) appear in the column to the right, only notations for formally unanalyzed propositions appear either in the left column or in any demonstration which follows. The syllogism is brought within the range of truth functions by informal interpretations of propositional variables which in no way are reflected within the operations performed upon these variables in testing the validity of the corresponding formal arguments.

Since something is A and B if and only if either something is A and B and C or something is A and B and not C, the categorical statement 'Some

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A is *B*' (I) is equivalent to the disjunction (1) $\vee$ (2).[3] Since 'No *A* is *B*' (E) is equivalent to the denial of 'Some *A* is *B*', it is equivalent to the conjunction -(1). -(2). Similarly, 'Some *A* is not *B*' (0) and 'All *A* is *B*' (A) are equivalent respectively to (3) $\vee$ (4) and -(3). -(4). In like fashion, 'All *B* is *C*' is equivalent to -(2). -(6), 'Some *A* is not *C*' to (2) $\vee$ (4), etc.

Consider the syllogism Barbara, figure 1, in the symbolism to the right:

All *C* is *B*	-(3). -(7)
All *A* is *C*	-(2). -(4)
All *A* is *B*	-(3). -(4)

The propositional form of -(3). -(7). -(2). -(4). $\supset$. -(3). -(4) is truth functionally valid, indicating that the conclusion of the syllogism cannot be false if both premises are true. Hence Barbara, figure 1, is shown valid by the propositional calculus. In like fashion, the syllogisms AII, figure 1, EAE, figure 1, and EIO, figure 1, are shown valid by the valid propositional forms respectively of:

-(3). -(7). (1) $\vee$ (3). $\supset$.(1) $\vee$ (2),
-(1). -(5). -(2). -(4). $\supset$. -(1). -(2), and
-(1). -(5). (1) $\vee$ (3). $\supset$.(3) $\vee$ (4).

An invalid syllogism is AAA, figure 2. This is shown invalid by the propositional form of -(2). -(6). -(2). -(4). $\supset$. -(3). -(4). Since it is possible for the consequence here to be false when the antecedent is true, the premises of AAA, figure 2, are shown not to entail its conclusion.

This method accords with the conception of syllogistic inference whereby no particular proposition (I or 0) is entailed by a set of premises including only universal propositions (A or E). Hence the syllogism AAI in any figure is invalid. With the additional premise 'Something is *A*', however, the syllogism AAI, figure 1, becomes valid, as shown by the valid form of:

-(3). -(7). -(2). -(4). (1) $\vee$ (2) $\vee$ (3) $\vee$ (4). $\supset$.(1) $\vee$ (2).

Moreover, if nothing is *A*, asserted by -(1). -(2). -(3). -(4), then both 'All *A* is *B*' and 'No *A* is *B*' are true, and both 'Some *A* is *B*' and 'Some *A* is not *B*' are false.

Quine lists 15 valid syllogistic forms with two premises each, and 9 forms which become valid with an additional premise asserting existence.[4] Each of these may be shown valid on the basis of truth functional relationships by the procedure illustrated above, and each of the remaining 232 invalid forms may be shown invalid thereby.

This method does not warrant the claim that the syllogism is "merely a branch of" the propositional calculus, for there are some things that some logicians might want to say about the syllogism that cannot be said in terms of the present rendition. This method, however, does establish the claim that no techniques beyond those provided by the propositional calculus are required to assess the correctness of any syllogistic inference.

NOTES

1. For similar statements, see Alice Ambrose and Morris Lazerowitz, *Fundamentals of Symbolic Logic* (Holt, Rinehart and Winston, New York, 1962), pp. 168-69; Donald Kalish and Richard Montague, *Logic: Techniques of Formal Reasoning* (Harcourt, Brace and World, New York, 1964), p. 85; and Elliott Mendelson, *Introduction to Mathematical Logic* (D. Van Nostrand Company, Inc., Princeton, 1964), p. 45.

2. W. V. O. Quine, *Methods of Logic* (Holt, Rinehart and Winston, New York, 1959, Revised Edition), p. 64. Justification of the present method rests upon proof that 'something is A and B' is equivalent to 'either something is A and B and C or something is A and B is not C'. This proof is routine within the predicate calculus, but it does require more than truth functional techniques. So Quine's statement would be correct if it were taken to mean that the propositional calculus is inadequate to *establish* canons of sound syllogistic inference. But then, justification of *any* method for testing validity requires logical techniques beyond those provided within the method itself to be justified.

3. The reader may wish to refer to Gordon Brumm's "The Method of Possibility—Diagrams for Testing the Validity of Certain Types of Inference, Based on Jevons' Logical Alphabet," in *Notre Dame Journal of Formal Logic*, Vol. III, (1962), pp. 209-233. Brumm, following Jevons, calls $AB \vee A\bar{B}$ the "development" of A, meaning thereby the equivalent of A with reference as well to B. Similarly, $ABC \vee AB\bar{C}$ is the "development" of AB, and $ABC \vee A\bar{B}C \vee AB\bar{C} \vee A\bar{B}\bar{C}$ the "development" of A, etc. The present method resembles Jevons' and Brumm's "Method of Possibility" in relying upon such "developments." Unlike their method, however, it requires neither formal notations nor rules of inference beyond those provided within the propositional calculus. This is the point of the present method.

4. Quine, *op. cit.*, p. 77.

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