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# UNIVERSAL VARIABLE NON-TARSKIAN FUNCTORS

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In [1] it was shown how to distinguish between Tarskian and non-Tarskian functors, the former being those whose Henkin-axioms when added to positive implication produce classical implication. It was further shown that all variable non-Tarskian functors have axioms interpretable in positive implication, alternation, conjunction (*C*-*A*-*K*) with the functors themselves interpretable in *A* and *K*. There seems therefore some point in reducing this non-Tarskian *A*-*K*-complex to a single functor which would be in composition with positive *C*, a universal functor for all variable non-Tarskian functors. Such is easy to find and can be provided with a neat set of positive axioms. We use *C*(implication) and triadic *M*(conjunction-alternation), the basis being:

(*C*) i.e. any set of positive *C*-axioms,

*M1* *CMpqrp*
*M2* *CCqsCCrsCMpqrs*
*M3* *CpCqMpqr*
*M4* *CpCrMpqr*
*Df.A* *Axy=MCxxxy*
*Df.K* *Kxy=Mxyy*

and the usual rules of substitution, detachment, definition.

The system is sound, for if we interpret *Mxyz* as *KxAyz* then *M1-4* are provable in positive *C*-*A*-*K*, and the definitions are obtainable as co-implications.

The system is complete, for (1) it is complete for positive *C*-*A*-*K*, and (2) *Mxyz* is provably equivalent to *KxAyz*. So if some *C*-*M*-thesis was unprovable, some *C*-*A*-*K*-thesis would be unprovable, against (1). We prove (1) and (2).

From (*C*) we have the theses

*C1* *Cqq*
*C2* *CCpCqrCCspCCsqCsr*.
*K1* *CKpqp* (*M1* *r*/*q*, *Df.K*)
*K2* *CKpqq* (*M2* *r*,*s*/*q*, *C1*, *Df.K*)

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| | | |
|---|---|---|
| *K3* | *CpCqKpq* | (*M3 r/q*, *Df.K*) |
| *A1* | *CCqsCCrsCAqrs* | (*M2 p/Cqq*, *Df.A*) |
| *A2* | *CpAqr* | (*M3 p/Cqq*, *C1*, *Df.A*) |
| *A3* | *CrAqr* | (*M4 p/Cqq*, *C1*, *Df.A*) |

With these last six theses (1) is proved.

| | | |
|---|---|---|
| *M5* | *CMpqrAqr* | (*M2 s/Aqr*, *A2*, *A3*) |
| *M6* | *CCspCCsqCsMpqr* | (*C2 r/Mpqr*, *M3*) |
| *2.1* | *CMpqrKpAqr* | (*M6 s/Mpqr*, *q/Aqr*, *r/Aqr*, *M1*, *M5*, *Df.K*) |
| *A4* | *CCpCqsCCpCrsCpAqrs* | (*C2 p/Cqs*, *q/Crs*, *r/CAqrs*, *s/p*, *A1*) |
| *M7* | *CpCAqrMpqr* | (*A4 s/Mpqr*, *M3*, *M4*) |
| *2.2* | *CKpAqrMpqr* | (*C2 q/Aqr*, *r/Mpqr*, *s/KpAqr*, *M7*, *K1*, *q/Aqr*, *K2 q/Aqr*) |

With *2.1*, *2.2*, (2) is proved.

That result was obtained by a composition of a constant true function with *M*. If we take a sufficiently defined constant false function, say *0*, we can get a similar result with *Lxyz* (alternation-conjunction), interpreted as *AxKyz*. The definitions *Df.A Axy* = *Lxyy*, *Df.K Kxy* = *L0xy*, are indeed creative with respect to minimal *C*-*0*-logic, for if the latter was complete we should have *C0A0p*, *CA0pKpp* (by the definitions), *CKppp*, and so the intuitionistic *C0p*. But if we adopt intuitionistic *C*-*0*, the new definitions and *L*-axioms:

*L1 CpLpqr* *L2 CqCrLpqr* *L3 CCpsCCqCrsCLpqrs*

we can obtain *A1*-*3*, *K1*-*3*, and the equivalence of *Lxyz* with *AxKyz* intuitionistically, and the definitions are no longer creative.

## BIBLIOGRAPHY

[1] Ivo Thomas, "Independence of Tarski's Law in Henkin's Propositional Fragments," *Notre Dame Journal of Formal Logic*, vol. I (1960), pp. 74-78.

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