Notre Dame Journal of Formal Logic
Volume V, Number 3, July 1964

THE PRAGMATICS OF TRUTH FUNCTIONS

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A sentential calculus may be conceived as a pair $<\mathbf{S}, \mathbf{R}>$, where $\mathbf{S}$ is a set of sentences generated by the operations of infixing "." and prefixing "$\sim$" from a given non-empty set of unanalyzed sentences, and $\mathbf{R}$ is the smallest equivalence relation on $\mathbf{S}$ which meets the following conditions:

1. $\mathbf{R}(s \cdot s', s' \cdot s)$,
2. $\mathbf{R}(s \cdot (s' \cdot s''), (s \cdot s') \cdot s'')$,
3. $\mathbf{R}(s \cdot \sim s', s'' \cdot \sim s'')$ *if and only if* $\mathbf{R}(s \cdot s', s)$,
4. *if* $\mathbf{R}(s,s')$, *then* $\mathbf{R}(s \cdot s'', s' \cdot s'')$,
5. *if* $\mathbf{R}(s, s')$, *then* $\mathbf{R}(\sim s \cdot \sim s')$,

for all s, s', s'' in $\mathbf{S}$. If an extra-logical axiom s_0 is adjoined to the sentential calculus, then $\mathbf{R}$ is the smallest equivalence relation on $\mathbf{S}$ which meets 1 - 5 and:

6. $\mathbf{R}(s_0, \sim(s \cdot \sim s))$,

for some $s \varepsilon \mathbf{S}$. If there exist at least two equivalence classes, then $\mathbf{B} = \mathbf{S}/\mathbf{R}$, the set of equivalence classes of $\mathbf{S}$ under $\mathbf{R}$, has the structure of a non-trivial Boolean algebra. If $p, q \varepsilon \mathbf{B}$ and $s \varepsilon p$, $s' \varepsilon q$, then $p \wedge q$ and $\bar{p}$ may be defined respectively as the equivalence classes of $s \cdot s'$ and $\sim s$. The ordered triple $<\mathbf{B}, \wedge, ->$ may be seen to be a Boolean algebra.

A Boolean logic, or logic of truth functions, may be conceived as a Boolean algebra $<\mathbf{B}, \wedge, ->$ together with a sum ideal (or filter) $\mathbf{I}$ of $\mathbf{B}$. For in the case that $\mathbf{I}$ is a maximal proper sum ideal, the induced algebra on $\mathbf{B}/\mathbf{I}$ is simple and may be considered as the Boolean algebra of truth values. Hence the homomorphism from $<\mathbf{B}, \wedge, ->$ to the induced algebra on $\mathbf{B}/\mathbf{I}$ may be considered as an interpretation of the elements of $\mathbf{I}$ as true (in the present case, the proposition containing sentences equivalent to some tautology), and of the remaining elements of $\mathbf{B}$ as false. Thus where $\mathbf{I}$ is maximal and proper, we may say that $<\mathbf{B}, \mathbf{I}, \wedge, ->$ is complete and consistent.

These considerations suggest that if a set $\mathbf{S}$ of sentences, such as is described above, is given together with a set of performances of the users of $\mathbf{S}$, then one might find a characterization of subsets of performances which will induce on $\mathbf{S}$ the structure appropriate for a truth-functional logic.

Received August 31, 1963

More precisely let:

- **U** = the set of users of sentences of **S**, over which range variables "u", "u'", "u''", . . . ;
- **T** = the set of times at which the sentences of **S** are valued, over which range variables "t", "t'", "t''", . . . ;
- **C** = the set of conditions under which the sentences of **S** are valued, over which range variables "c", "c'", "c''", . . . ;
- **V** = the set of pragmatic values which the sentences of **S** assume for their users, over which range variables "v", "v'", "v''", ...

A pragmatic condition $c \varepsilon \mathbf{C}$ may be taken as of the form $<f,o,o'>$, where f is a class of similar operations, or kinds of behavior, performable by users of **S**, and o, o' are classes of similar objects upon which such operations may be performed, or which are the results of such operations. In a given pragmatic condition, $<f,o,o'>$, an element of o' may be taken as an *index*, relative to an operation of f performed upon an element of o, for fixing a pragmatic value to some sentence.

By means of a condition c, a user u may be considered at a time t to fix a pragmatic value v to a sentence s. Among the elements of **V**, it is assumed that there are values *accept*, *reject*, and *neither accept nor reject*. These values are called *1*, *0*, and *2* respectively. The value *2*, neither accept nor reject, is given to a sentence by a user at a time when the condition is not germane to the acceptance or rejection of the sentence.

In order to obtain via a pragmatics the sentential calculus, $<\mathbf{S},\mathbf{R}>$, one must characterize subsets of $\mathbf{U} \times \mathbf{T} \times \mathbf{C} \times \mathbf{S} \times \mathbf{V}$, called interpretations of **S**, which will induce on **S** a set of equivalence relations which meet conditions 1 - 5 (or 1 - 6). The smallest such relation is the one sought.

D1. ψ *is a normal interpretation of* **S** (*or part of* **S**) *if and only if* $\psi \subset \mathbf{U} \times \mathbf{T} \times \mathbf{C} \times \mathbf{S} \times \mathbf{V}$, ψ *is univocal in the last place, and for every* u, t, u', t', c, s, s', s'' *in the field of* ψ, *the following six conditions hold:*

a. $\psi(u,t,c,s.s') = \psi(u,t,c,s'.s)$,
b. $\psi(u,t,c,s.(s'.s'')) = \psi(u,t,c,(s.s').s'')$;
c. $\psi(u,t,c,s.\sim s') = \psi(u,t,c,s''.\sim s'')$ *if and only if* $\psi(u,t,c,s.s') = \psi(u,t,c,s)$;
d. if $\psi(u,t,c,s) = \psi(u,t,c,s')$, *then* $\psi(u,t,c,s.s'') = \psi(u,t,c,s'.s'')$;
e. if $\psi(u,t,c,s) = \psi(u,t,c,s')$, *then* $\psi(u,t,c,\sim s) = \psi(u,t,c,\sim s')$;
f. $\psi(u,t,c,s) = \psi(u',t',c,s)$.

A normal interpretation ψ is a subset of $\mathbf{U} \times \mathbf{T} \times \mathbf{C} \times \mathbf{S} \times \mathbf{V}$ such that different or same users at same or different times, under the same conditions value the same sentence in the same way. Hence requirement f and the requirement that ψ be univocal in the last place. The requirements a - e parallel 1 - 5. The motivation for a - e is obvious, since one may define an equivalence relation on **S** (or part of **S**) as follows.

D2. $\mathbf{R}_\psi(s, s')$ *if and only if* $\psi(u,t,c,s) = \psi(u,t,c,s')$ *for all* u,t,c *in the field of* ψ.

In general the pair $<\mathbf{S},\mathbf{R}_\psi>$ will not form a sentential calculus for arbitrary ψ. $\mathbf{R}_\psi$ may equate too many or too few sentences of **S**. What is needed is an interpretation of all of **S** to which further quintuples may not be added without violating *D1*. Such an interpretation would induce on **S** an equivalence relation which would satisfy the requirements 1 - 5, and which would be at most too large. Thus the equivalence relation sought is the intersection of all the equivalence relations induced on **S** by such non-extensible interpretations. The equivalence relations so obtained would characterize a calculus for a variety of interpretations of **S**, which is what is expected of a logical calculus. These ideas may be made precise as follows.

D3. ψ and ϕ are compatible normal interpretations if and only if $\psi \cup \phi$ is a normal interpretation.

If ψ and ϕ are both normal interpretations and ϕ is a subset of ψ, then ψ and ϕ are clearly compatible interpretations. A non-extensible interpretation may be thought of as one whose union with any subset of its complement in $\mathbf{U} \times \mathbf{T} \times \mathbf{C} \times \mathbf{S} \times \mathbf{V}$ fails to be normal.

D4. ψ is a non-extensible normal interpretation if and only if ψ is not compatible with any subset of the complement of ψ in $\mathbf{U} \times \mathbf{T} \times \mathbf{C} \times \mathbf{S} \times \mathbf{V}$.

The following theorem concerns non-extensible interpretations.

T1. If ψ is a non-extensible interpretation, then the domain of ψ is $\mathbf{U} \times \mathbf{T} \times \mathbf{C} \times \mathbf{S}$.

The converse of *T1* is obvious. For the proof of *T1*, it is first shown that every sentence has a ψ-value for some condition c; next it is shown that if a sentence has a ψ-value for some c, it has a value for every c.

Suppose there is a sentence s_0 with no ψ-value for any c, i.e. $\langle u,t,c,s_0,v\rangle \notin \psi$, for all u,t,c,v. A function ϕ may be constructed containing ψ, which assigns to $\langle u,t,c_0,s_0\rangle$ some value v, for some condition c_0, so that ϕ satisfied *D1*.

Case 1: c_0 is not a condition for ψ. Then there is clearly a function ψ' such that $\phi = \psi \cup \psi'$ is a normal interpretation, ψ' satisfying the conditions *D1* a - f for c_0 and s_0 and for all sentences in the field of ψ.

Case 2: c_0 is a condition for ψ. Then ϕ may be constructed as follows. According to *D1*, there is an atomic constituent s_1 of s_0 which is not valued at all. Then $s_1 = s_0$ if s_0 is atomic. Now let ϕ be such that for all s,s' in the field of ψ.

$\phi(u,t,c_0,s_1) = \phi(u,t,c_0,s_1 \cdot s)$ if and only if $\phi(u,t,c_0,s_1 \cdot {\sim}s) = \psi(u,t,c_0,s' \cdot {\sim}s')$,

if $\phi(u,t,c_0,s_1) = \psi(u,t,c_0,s)$, then $\phi(u,t,c_0,s_1 \cdot s') = \psi(u,t,c_0,s \cdot s')$,

if $\phi(u,t,c_0,s_1) = \psi(u,t,c_0,s)$, then $\phi(u,t,c_0,{\sim}s_1) = \psi(u,t,c_0,{\sim}s)$,

and ϕ satisfies the remaining conditions of *D1* as well, for c_0,s_1 and all sentences in the field of ψ. By cases 1 and 2, ϕ is a normal interpretation, contrary to the hypothesis that ψ is non-extensible. Thus every sentence has a ψ-value for some condition c.

In order to show that if a sentence has a ψ-value for some c, then it does for every c, suppose there is s_0 with a value for c_0 but not for c_1. If c_1 is not a condition for ψ, then there is a function ψ' which satisfies *D1* a - f for c_1 and s_0 and all sentences in the field of ψ, so that $\phi = \psi \cup \psi'$ is a normal interpretation. If c_1 is a condition for ψ, then ϕ may be constructed as above, case 2, so that every sentence has a ψ-value for every condition if it does for some condition. This completes the proof of *T1*.

Let χ be the set of all non-extensible normal interpretations of **S**; then *T2* is as follows.

T2. If $\mathbf{R} = \bigcap \mathbf{R}_\psi$, *then* **R** *is the smallest equivalence relation on* **S** *which meets conditions* 1 - 5.

It follows from *T1* that the field of $\mathbf{R}_\psi$ for $\psi \varepsilon \chi$, and hence the field of **R**, is **S**. That $\mathbf{R}_\psi$ is an equivalence relation follows from *D1* f, *D2* and the equivalence properties of identity. That $\mathbf{R}_\psi$ satisfies requirements 1 - 5 follows from *D1* a - f, *D2*. The intersection of all such $\mathbf{R}_\psi$ is hence the smallest equivalence relation on **S** which meets the conditions 1 - 5.

If the descriptive axiom s_0 is adjoined to the sentential calculus, this may be characterized via the pragmatics of **S** by requiring that the interpretations ψ in χ be normal non-extensible and that they all satisfy, for some $s \varepsilon \mathbf{S}$, the following:

g. $\psi(u,t,c,s_0) = \psi(u,t,c,\sim(s.\sim s))$.

This last requirement is the pragmatic transcription of condition 6. The relation **R** obtained in this fashion will be the smallest equivalence on **S** which meets conditions 1 - 6.

The above method of obtaining an equivalence relation on **S** does not yield one interpretation ϕ such that $\langle \mathbf{S}, \mathbf{R}_\phi \rangle$ is a sentential calculus. Rather the method obtains a sentential calculus via a set of interpretations. Also the method abstracts from the pragmatic values which are assigned to the sentences in the various interpretations, and it abstracts from the kinds of conditions and inductive procedures by which these values are assigned. This feature of the method fulfills the expectations for a logical calculus.

Besides non-extensibility, a further condition must be placed on those interpretations which would single out an appropriate equivalence relation **R**, the sum ideal composed of the provable propositions of **S**/**R**, and the ideal composed of the refutable propositions of **S**/**R**. From the truth-functional point of view, all tautologies should be valued 1 under every condition, and therefore because of condition g, all the sentences which are members of provable propositions should also be valued 1 under every condition. It is sufficient to consider those non-extensible interpretations which assign to at least one tautology always the value *1*, and which assign to at least one counter-tautology always the value *0*. The foregoing is readily apparent from the following theorems.

T3. If ψ *is a non-extensible normal interpretation, then all tautologies are* $\mathbf{R}_\psi$ *-equivalent and all counter-tautologies are* $\mathbf{R}_\psi$ *-equivalent.*

If $\mathbf{S}/\mathbf{R}_\psi$ has only one member, then all sentences are $\mathbf{R}_\psi$-equivalent. The set of tautologies $\mathbf{T} \subset \mathbf{S}$ may be characterized as follows:

(i) if $s \varepsilon \mathbf{S}$, then $\sim(s. \sim s)\ \varepsilon \mathbf{T}$;
(ii) if $s,s' \varepsilon \mathbf{T}$, then $s.s' \varepsilon \mathbf{T}$;
(iii) if $s \varepsilon \mathbf{T}$ and $s' \varepsilon \mathbf{S}$, then $\sim(\sim s. \sim s')\ \varepsilon \mathbf{T}$;
(iv) only by (i) - (iii) may $s \varepsilon \mathbf{T}$.

The set of counter-tautologies $\mathbf{K} \subset \mathbf{S}$ may be characterized:

(i) if $s \varepsilon \mathbf{S}$, then $s. \sim s \varepsilon \mathbf{K}$;
(ii) if $s,s' \varepsilon \mathbf{K}$, then $\sim(\sim s. \sim s')\ \varepsilon \mathbf{K}$;
(iii) if $s \varepsilon \mathbf{K}$ and $s' \varepsilon \mathbf{S}$, then $s.s' \varepsilon \mathbf{K}$;
(iv) only by (i) - (iii) may $s \varepsilon \mathbf{K}$.

The proof that all tautologies are $\mathbf{R}_\psi$-equivalent is as follows.

Case (i): Since $\mathbf{R}_\psi$ meets condition 3, by putting s for s' and s'' one obtains $\mathbf{R}_\psi(s.s,s)$, for any $s \varepsilon \mathbf{S}$. Hence one obtains $\mathbf{R}_\psi(\sim(s.\sim s),\sim(s'.\sim s'))$, for any s and s' in $\mathbf{S}$.

Case (ii): Let t stand for any of the tautologies of case (i). If $\mathbf{R}_\psi(s,t)$ and $\mathbf{R}_\psi(s',t)$, then $\mathbf{R}_\psi(s,s')$ since $\mathbf{R}_\psi$ is an equivalence relation. Hence by condition 4, $\mathbf{R}_\psi(s.s',s'.s')$. Thus one obtains $\mathbf{R}_\psi(s.s',s')$, and since by hypothesis $\mathbf{R}(s',t)$, we have $\mathbf{R}_\psi(s.s',t)$.

Case (iii): If $\mathbf{R}_\psi(s,t)$, then, by 5, $\mathbf{R}_\psi(\sim s,\sim t)$. By 4, one obtains $\mathbf{R}_\psi(\sim s.\sim s',\sim t.\sim s')$. From a repeated use of 3, one obtains $\mathbf{R}_\psi(\sim\sim s,s)$, and by choosing $t = \sim(s'.\sim s')$ and employing 2 one has that $\mathbf{R}_\psi(\sim s.\sim s',s'.\sim s')$. Hence $\mathbf{R}_\psi(\sim(\sim s.\sim s'),t)$ by 5. The extremal condition (iv) yields, together with the above three cases, that if $s\varepsilon\mathbf{T}$, then $\mathbf{R}_\psi(s,t)$. All tautologies are $\mathbf{R}_\psi$-equivalent. A shorter proof of this statement is available if one notices that for the case that $\mathbf{S}/\mathbf{R}_\psi$ has more than one member, $<\mathbf{S},.,\sim>$ is a Boolean algebra with respect to $\mathbf{R}_\psi$, and $\mathbf{T}$ is a sum ideal of it. Similarly, $\mathbf{K}$ is an ideal of the algebra, the dual of $\mathbf{T}$. Thus by the duality principle, the proof of *T3* is complete.

T4. If ψ is a non-extensible normal interpretation such that there is a tautology $s \varepsilon \mathbf{T}$ for which $\psi(u,t,c,s) = 1$ for all u,t,c, then for every $s \varepsilon \mathbf{T}$ and for every u,t,c, $\psi(u,t,c,s) = 1$. Also, if ψ is a non-extensible normal interpretation such that there is a counter-tautology $s \varepsilon \mathbf{K}$ for which $\psi(u,t,c,s) = 0$ for every u,t,c, then for every $s \varepsilon \mathbf{K}$ and for every u,t,c, $\psi(u,t,c,s) = 0$

T4 is an immediate consequence of *T3* and *D2*.

Let χ_1 be the set of all non-extensible normal interpretations ψ, for which there is an $s \varepsilon \mathbf{T}$ such that $\psi(u,t,c,s) = 1$ for all u,t,c. Let χ_0 be the set of all non-extensible normal interpretations for which there is an $s \varepsilon \mathbf{K}$ such that $\psi(u,t,c,s) = 0$ for all u,t,c. The set $\chi_{01} = \chi_0 \cap \chi_1$ contains all and only the interpretations in which tautologies are always valued *1* and counter-tautologies always *0*. All the interpretations in χ_{01} are pragmatically consistent in the following sense.

T5. If $\psi \varepsilon \chi_{01}$, then $\psi(u,t,c,s) \neq \psi(u,t,c,\sim s)$, for all u,t,c,s.

For proof of *T5*, suppose that for some $\psi \varepsilon \chi_{01}$, there are u,t,c, and s such that $\psi(u,t,c,s) = \psi(u,t,c,\sim s)$. According to *D1* d, if $\psi(u,t,c,s) = \psi(u,t,c,\sim s)$, then $\psi(u,t,c,s.s) = \psi(u,t,c,\sim s.s) = \psi(u,t,c,s)$ and $\psi(u,t,c,s.\sim s) = \psi(u,t,c,\sim s.\sim s) = \psi(u,t,c,\sim s)$. According to *D1* e, since $\psi(u,t,c,\sim s.s) = \psi(u,t,c,s)$, then $\psi(u,t,c,\sim(\sim s.s)) = \psi(u,t,c,\sim s)$. Hence it follows that $\psi(u,t,c,s,\sim s) = \psi(u,t,c,\sim(s.\sim s))$. This contradicts the hypothesis that χ_{01} contains all and only interpretations in which tautologies are always valued *1* and couter-tautologies always *0*.

Even if ψ is a normal non-extensible interpretation, the converse of *T5* is not a theorem. For given that $\psi(u,t,c,s) \neq \psi(u,t,c,\sim s)$ for all u,t,c,s, it need not be the case that some tautology is valued *1* under every condition. There may be a condition c_0 under which all tautologies are valued *0* and all countertautologies are valued *1*.

If ψ is in χ_{01}, then $\mathsf{R}_\psi(s,\sim(s'.\sim s'))$ if and only if $\sim(u,t,c,s) = 1$, for all u,t,c,s,s'. Hence the class of provable sentences is characterized pragmatically as the class of all the sentences valued *1* under every condition. Let p_1 be the class of sentences valued *1* under every condition, and $\mathsf{I} = \{p_1\}$. *T6* is then as follows.

T6. If $\psi \varepsilon \chi_{01}$, then $<\mathsf{S}/\mathsf{R}_\psi, \mathsf{I}, \wedge, ->$ is a consistent Boolean logic.

$<\mathsf{S}/\mathsf{R}_\psi, \wedge, ->$ is a Boolean algebra by *D1,D2*. By hypothesis and *T5*, I fails to contain the proposition of sentences valued *0* under every condition, and moreover I is a sum ideal. For if $p \varepsilon \mathsf{I}$, then $p \vee q$ is the set of sentences equivalent to $\sim(\sim s.\sim s')$, for $s \varepsilon p$, $s' \varepsilon q$. For $p \vee q$ to be in I, it is sufficient that $\sim(\sim s.\sim s')$ be equivalent to a tautology; this is evident where s itself is equivalent to a tautology. Finally, if p and q are in I, then $p \wedge q$ is the set of sentences equivalent to $s.s'$, for $s \varepsilon p, s' \varepsilon q$. For $p \vee q$ to be in I, it is sufficient that the conjunction $s.s'$ be equivalent to a tautology; this is evident where both s and s' are equivalent to a tautology. This completes the proof of *T6*.

The question arises concerning the intuitive significance of asserting that descriptive axioms are valued *1* (accepted) under every condition. It should be remembered that, as conceived here, the truth-functional pragmatics of a language abstracts from the inductive procedures by which the descriptive axioms are selected. Moreover, since descriptive axioms are not likely to be observation statements, the present method may not have to countenance atomic observation statements which are valued *1* under every condition. For, suppose the conjunction of s_0 and s_d entails s_0', where s_d is a descriptive axiom and s_0 and s_0' are atomic observation statements. Then the hypothetical $s_0 \supset s_0'$ is construed as valued *1* under every condition, but the atomic statements themselves are not necessarily so construed. Thus the assertion that either s_0 is false or s_0' is true is pragmatically equivalent to valuing the descriptive axiom s_d as *1* under every condition. *T7* makes precise the idea of pragmatical equivalence or synonymity.

In order for $<\mathsf{S}/\mathsf{R}, \mathsf{I}, \wedge, ->$ to be complete (and consistent), i.e. for I to be maximal (and proper), every sentence must be valued either *1* or *0* under every condition. For should some sentence fail to be valued *1* or *0* under every condition, it would fail to be equivalent to either a tautology or a counter-tautology, and hence fail to be either a provable or refutable sentence. It thus appears that the present method, which limits itself to the truth-functional part of pragmatics, is inapplicable to complete descriptive languages, which contain sentences valued differently under different conditions. For it appears unacceptable to countenance atomic observation statements as valued *1* or *0* under every condition.

To each $\psi \varepsilon \chi_{01}$ and each $s \varepsilon \mathbf{S}$, there corresponds a set of quadruples $\langle u,t,c,v \rangle$ which may be called, following Peirce, the entire general intended (truth-functional) interpretant of s.

D5. $g_{\psi}(s)$ *is the set of all quadruples* $\langle u,t,c,v \rangle$ *such that* $\psi(u,t,c,s) = v$.

One may read "$g_{\psi}(s)$" as "*the* ψ*-interpretant of* $\mathbf{S}$". The ψ-interpretants of two sentences will be identical just in case the two sentences are $\mathbf{R}_{\psi}$-equivalent. This may be considered an explication of (truth-functional) pragmatical synonymity, whereby, following Peirce, two sentences which are interpretated in the same way have the same meaning.

T7. $g_{\psi}(s) = g_{\psi}(s')$ *if and only if* $\mathbf{R}_{\psi}(s,s')$.

By *D5*, $g_{\psi}(s) = \hat{u}\,\hat{t}\,\hat{c}\,\hat{v}\,\{\psi(u,t,c,s) = v\}$, *and* $\hat{u}\,\hat{t}\,\hat{c}\,\hat{v}\,\{\psi(u,t,c,s') = v\} = g_{\psi}(s')$. By *D2*, $\mathbf{R}_{\psi}(s,s')$ *if and only if* $\psi(u,t,c,s) = \psi(u,t,c,s')$. Hence *T7*.

D5 and *T7* explicate only a small portion of Peirce's full semiotic idea. That $g_{\psi}(s) = g_{\psi}(s')$ means only that s and s' have, so to speak, the same truth functional significance. If besides having truth functional significance some expressions of a language are considered as describing various objects in a given domain, then the pragmatic meta-language should be capable of mentioning these objects and also capable of describing the inductive procedures of its users. The pragmatic meta-language here employed does not contain such means.

$g_{\psi}(s)$ is of course the *entire* and *truth-functional* interpretant of s just in case $\psi \varepsilon \chi_{01}$. This consideration suggests that a logic of truth functions may be obtained via an appropriate selection of a class of interpretants. In such a case *T7* would be the definition of the desired equivalence relation.

Consider the set $\mathbf{D}$ of all mappings of $\mathbf{U} \times \mathbf{T} \times \mathbf{C}$ into $\mathbf{V}$. A class of mappings of $\mathbf{S}$ into $\mathbf{D}$ may be defined so that the ranges of these mappings are classes of normal non-extensible interpretants. More precisely:

D6. The range of g is a class of normal non-extensible interpretants of $\mathbf{S}$ *if and only if g is a mapping of* $\mathbf{S}$ *into* $\mathbf{D}$ *such that for all* s, s', s'' *in* $\mathbf{S}$:

I. $g((s.s').s) = g(s.(s'.s''))$;
II. $g(s.s') = g(s'.s)$;
III. $g(s.\sim s') = g(s''.\sim s'')$ *if and only if* $g(s.s') = g(s)$;
IV. *if* $g(s) = g(s')$, *then* $g(\sim s) = g(\sim s')$;
V. *if* $g(s) = g(s')$, *then* $g(s.s'') = g(s'.s'')$;
VI. *if* $\langle u,t,c,v \rangle \varepsilon g(s)$, *then* $\langle u',t',c,v \rangle \varepsilon g(s)$, *for all* u,t,c,v,u',t'.

The mapping g of $\mathbf{S}$ into $\mathbf{D}$ is a pairing of sentences to the appropriate sets of dispositions so that the structure induced on **S** is the same as the structure induced on **S** via some non-extensible normal interpretation.

T8. The range of g is a class of normal non-extensible interpretants of **S** *if and only if there is a normal non-extensible interpretation* ψ *such that for every* $s \varepsilon \mathbf{S}$, $g(s) = \hat{u}\,\hat{t}\,\hat{c}\,\hat{v}\{\psi(u,t,c,s) = v\}$.

If one lets $\psi(u,t,c,s) = v$ if and only if $<u,t,c,v> \varepsilon g(s)$, then *D6* I-VI are a straightforward transcription of *D1* a-f. Since **D** is the set of all mappings of $\mathbf{U} \times \mathbf{T} \times \mathbf{C}$ into **V**, and g is a mapping of **S** into **D**, ψ so defined is non-extensible.

An analogue of *T5* is available for those functions g which map at least one tautology and one counter-tautology on the *1*-valued and *0*-valued function in **D** respectively. For let s be some tautology. Then if $g(s)$ is a *1*-valued function from $\mathbf{U} \times \mathbf{T} \times \mathbf{C}$ into **V**, then $<u,t,c,1> \varepsilon g(s)$, for all u,t,c. By *T8*, $\psi(u,t,c,s) = 1$, and similarly for counter-tautologies. Thus, recalling *T4* and *T5*, the truth-functional logic obtained via corresponding g and ψ are one and the same.

Truth-functional logic has been characterized by means of pragmatic considerations with regard to the users, times of valuation, conditions of valuation, and pragmatic values of the sentences in question. The syntax for well-formed formulas in "." and "~" has been taken as given. No restrictions on the kinds of pragmatic conditions of valuation have been required. It is an interesting question whether restrictions on the set of conditions **C** are necessary in order to characterize pragmatically the complete syntax and semantics of various scientific languages.

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