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EXISTENTIAL IMPORT REVISITED

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The traditional logic supposed statements of the form ‘$(x) \cdot Fx \supset Gx$’ to have *existential import,* and so licensed the inference from ‘$(x) \cdot Fx \supset Gx$’ to ‘$(\exists x) \cdot Fx \cdot Gx$’. But let ‘$F$’ be ‘a brakeless car’ (or ‘is a unicorn’), and ‘G’ be ‘is dangerous’ (or ‘is a unicorn’). Then the false statement that there exists a brakeless car (or that there exists a unicorn) can be inferred.

The inference from ‘$(x) \cdot Fx \supset Gx$’ to ‘$(\exists x) \cdot Fx \cdot Gx$’ loses its validity when (at least) ‘F’ is replaced by a general term true of nothing. So the consistency of the traditional account can be restored by limiting replacement of (at least) ‘F’ to *general terms* true of something. But there are two disadvantages to this way out of the difficulty. First, it unduly limits the range of application of logic. For example, predicates like ‘is a member of the null class’ could not replace ‘F’. Hence, logical justification of the statement that the null class is included in any class would not be forthcoming. Secondly, it does not allow discrimination of those statements having existential import from those not having existential import, and thus would fail to distinguish between inferences for whose validity the existence of the things characterized by ‘F’ is relevant and inferences for whose validity their existence is irrelevant.

The "modern" symbolic logic resolves the fallacy of existential import in another way. It allows *unlimited* substitution into the predicate placeholders ‘F’ and ‘G’, and replaces the inference from ‘$(x) \cdot Fx \supset Gx$’ to ‘$(\exists x) . Fx \cdot Gx$’ by the inference from ‘$(x) \cdot Fx \supset Gx \cdot (\exists x) \cdot Fx$’ to ‘$(\exists x) \cdot Fx \cdot Gx$’. This move amounts to changing the notion of quantificational validity from ‘true for every replacement of the predicate placeholders F, G, H. . . . by *applicative* predicates in every non-empty domain. . .’ to ‘true for every replacement of the predicate placeholders F, G, H. . . . by predicates (*applicative* or *non-applicative*) in every non-empty domain’. Consequently, the range of application of logic is not restricted — so far as its predicate terms are concerned. Hence, it now becomes possible to justify such statements as ‘The null class is included in every class’. Further, it is now possible to distinguish between statements having existential import and those not having same. ‘All brakeless cars are dangerous’[1] gets rendered as ‘$(x) \cdot Bx \quad Dx$’, whereas ‘All men are

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animals' (where the existence of men is implied) is paraphraseable as '$(x) \cdot Mx \supset Ax \cdot (\exists x) \cdot Mx$'. This, in turn, permits the "modern" symbolic logic to discriminate between inference patterns whose validity requires an existence assumption, viz. '$(x) \cdot Fx \supset Gx \cdot (\exists x) \cdot Fx \therefore (\exists x) \cdot Fx \cdot Gx$' and inference patterns whose validity is independent of such an assumption, viz. '$(x) \cdot Fx \therefore (\exists x) \cdot Fx$'.[2] Nor is the consistency of the "modern" symbolic logic endangered by this manouvre. Most contemporary systems of logic, including a rule of substitution, incorporate the above solution to the problem of existential import and are demonstrably consistent.

Curiously, when we go from quantification theory to identity theory, we find an exception to the inference pattern '$(x) \cdot Fx \supset Gx \cdot (\exists x) \cdot Fx \therefore (\exists x) \cdot Fx \cdot Gx$'. For let 'F' be the open predicate schema '$= y$'. Then, in the conventional version of identity theory, we can prove '$(x) \cdot x = y \supset Gx \cdot \supset (\exists x) \cdot x = y \cdot Gx$' which, of course, justifies the inference from '$(x) \cdot x = y \supset Gx$' to '$(\exists x) \cdot x = y \cdot Gx$'. This in turn suggests that the modern logic regards all statements of the form '$(x) \cdot x = y \supset Gx$' as having existential import.

The curiosity borders on the obtuse in view of such counter examples as 'Everything identical with Pegasus is Pegasus', and 'Everything identical with the frictionless surface S allows unrestricted movement over it'. These, and the inference pattern '$(x) \cdot x = y \supset Gx \therefore (\exists x) \cdot x = y \cdot Gx$', yield the false statements that something is identical with Pegasus and that something is the frictionless surface S.

The inference from '$(x) \cdot x = y \supset Gx$' to '$(\exists x) \cdot x = y \cdot Gx$' loses its validity when irreferential *singular terms* replace the argument variable 'y'; it remains valid when referential singular terms replace the variable 'y'. The consistency of the "modern" quantification theory with identity can be restored by restricting replacement of 'y' to referential singular terms. But this move brings with it undesirable features parallel to those noted above in restricting predicate placeholders to substituends true of something. So the "restriction" method ought to be the course of last resort.

The popular way out is to restrict variables to replacement by other variables and accomodate statements involving singular terms via the medium of description theory. This course, however, has two unsatisfactory features. First, the favored description theories, viz., the Russell theory or the Frege theory, regard singular statements, be they 'pure' or 'mixed' (= include quantified variables), as having existential import. But this position is highly debateable.[3] Secondly, it requires treating names as abbreviations for descriptions which again is a highly debateable practise.[4] In this paper, I will explain another way of resolving the problem of existential import posed by identity theory.

II

It is not difficult to find the offending agent in the "modern" logic's inconsistent attitude toward existential import. It is that old un-reliable:

Particularization. Consider:

(1) $(x) \cdot x = y \supset Gx \cdot \supset \cdot (\exists x) \cdot x = y \cdot Gx$

is deducible from the valid quantificational formula

(2) $(x) \cdot Fx \supset Gx \cdot (\exists x) \cdot Fx \colon \supset \cdot (\exists x) \cdot Fx \cdot Gx$

by substituting '$=y$' for 'F' and detaching with the help of

(3) $(\exists x) \cdot x = y.$

(3) is valid in conventional identity theory. But (3) is deducible from the unexceptional identity axiom

(4) $y = y$

and *Particularization*, viz., '$Fy \supset (\exists x) \cdot x = y$'. If '$y$' in (3) is replaced by an irreferential singular term it becomes false, though (4) is true. And, indeed, the move from (4) to (3) is a violation of existential import; 'Pegasus is Pegasus' or 'Pegasus flies' need have no existential import.[5]

Perhaps it is important to point out that rejection of *Particularization*, because it commits the fallacy of existential import, is not the same as rejecting *Particularization* because it is the source of the so-called singular existence anomaly; inferring, for example, from 'Pegasus does not exist' that 'There exists a non-existent'. The latter basis of rejection discounts such statements as 'If Pegasus does not exist, then there exists a non-existent' but not necessarily statements such as 'If Pegasus is Pegasus, then something is Pegasus'. The former basis of rejection discounts both sorts of statement. In short, *Particularization* is here viewed not as failing merely for some special cases, for example, where non-existence is being predicated of Pegasus, but rather as failing because it suffers from a deeper disorder; it presumes that singular statements have existential import.[6]

It seems clear, therefore, that the "modern" logic's questionable attitude toward existential import, so far as quantified statements are concerned, is the result of the equally questionable presumption, reflected in *Particularization*, that all singular statements have existential import.

Following suggestions[7] in much recent work on the foundations of quantification theory, I propose to amend quantification theory as follows. I shall replace the usual axiom

(5) $(x) \cdot Fx \cdot \supset Fy$

by

(6) $\text{A1} : (y) : (x) \cdot Fx \cdot \supset Fy$

We also have as an axiom

(7) $\text{A2} \colon (x) \cdot Fx \supset Gx \cdot \supset (x) \cdot Fx \cdot \supset (x) \cdot Gx.$

To obtain a quantification theory with identity, add as axioms

(8) $\text{A4} : x = x$

and

(9) $$A5 : x = y \supset \cdot\ Fx \supset Fy$$

The rules of inference are the usual rules for substitution into predicate placeholders and argument placeholders (variables),[8] modus ponens, and the Hilbert-Ackermann version of universal generalization; from $A \supset B$, if α is not free in A, infer $A \supset (\alpha) B$. To obtain a system valid for every non-empty domain, it is necessary to add as an axiom

(10) $$A3 : (\exists\, x) \cdot Fx \supset Fx;$$

otherwise, theorems like

(11) $$(x) \cdot Fx \cdot \supset (\exists\, x) \cdot Fx$$

are not forthcoming. Without (10) the system is valid for every domain (including the empty one).[9]

For the present purpose, notice that the offending

(12) $$Fx \supset (\exists\, y) \cdot Fy$$

is not deducible here. The most that can be obtained is

(13) $$Fx \cdot \mathrm{E!}\ x \cdot \supset (\exists\, y) \cdot Fy$$ [10]

where '$\mathrm{E!}\ x$' is short for '$(\exists\, x) \cdot x = y$'. ((13) follows from (9) by means of

(14) $$(x) \cdot Fx \supset Gx \cdot \supset (\exists\, x) \cdot Fx \cdot \supset \cdot (\exists\, x) \cdot Gx$$

which is a consequence of (7).) So '$(\quad x) \cdot x = y$' is not deducible. Nor, therefore, is '$(x) \cdot x = y \supset Gx \cdot \supset \cdot (\exists\, x) \cdot x = y \cdot Gx$'. And this restores the consistency of the "modern" logic's attitude toward existential import. Further, *unlimited* substitution into free argument variables is permitted. This requires a slight emendation in the notion of quantificational validity—from 'true for every replacement of a free variable by *referential* singular terms in every non-empty domain' to 'true for every replacement of a free variable by singular terms (*referential* or *irreferential*) in every non-empty domain'. Consequently, the resulting logic, which I have elsewhere called a "free" logic,[11] does not demand that names be construed as descriptions. Finally, it discriminates between singular inference patterns where an existence assumption is relevant, for example,

(15) $$Fx \cdot \mathrm{E!}\ x \ \therefore\ (\exists\, x) \cdot Fy,$$

and those where it is not, for example,

(16) $$x = y \ \therefore\ Fx \supset Fy.$$ [12]

NOTES

1. The example is from Hugues Leblanc's *Introduction To Deductive Logic*, Wiley: 1954, p. 66.

2. This point, and the further point that inference of the form 'Fx .·. $(\exists y) \cdot Fy$' commit the fallacy of existential import, have been made by H. S. Leonard in "The Logic of Existence," *Philosophical Studies*, June: 1956, pp. 49-64. But he did not show that acceptance of 'Fx .·. $(\exists y) \cdot Fy$' affects the consistency of the "modern" logic's attitude toward existential import so far as certain *universal* statements are concerned, nor that a consistent attitude toward existential import in identity theory *requires* rejection of 'Fx .·. $(\exists y) . Fy$'.
3. Ibid. p. 61. See also P. F. Strawson, "On Referring," Mind, July: 1950, pp. 320-344; K. J. J. Hintikka, "Towards a Theory of Definite Descriptions", *Analysis*, March: 1959, pp. 49-64· T. Hailperin and H. Leblanc, "Nondesignating Singular Terms," *Philosophical Review*, April: 1959, and Karel Lambert, "Notes on 'E!': III A Theory of Descriptions," *Philosophical Studies*, June: 1962, pp. 51-59.
4. Ibid. p. 54. See also, H. Hochberg, "On Pegasizing," *Philosophy and Phenomenological Research*, pp. 551-554; W. V. Quine, *Word and Object*, Wiley: 1960, p. 182; Karel Lambert, "Explaining Away Singular Nonexistence Statements," *Dialogue* (forthcoming).
5. Op. cit. "The Logic of Existence," p. 52.
6. Ibid. p. 52. See also note 3 for other pertinent references on this point.
7. See footnote 3, especially the articles by K. J. J. Hintikka, and T. Hailperin and H. Leblanc.
8. For example, A. Church, *Introduction to Mathematical Logic: I*, Princeton: 1956, pp. 218-219.
9. A1 and A2 in this paper corresponds to Hailperin's axiom-schemata QR1 and QR3, and A3 corresponds to an equivalent of his $\sim(\alpha)$f. See T. Hailperin, "A Theory of Restricted Quantification," *Journal of Symbolic Logic*, March: 1957, p. 31. It is easy to deduce the axiom-schema QR2 from the Hilbert-Ackermann version of Universal generalization. Hailperin's quantification theory (even omitting his QR4), is valid for every domain (including the empty one); with the addition of $\sim(\alpha)$f it is valid for every domain (excluding the empty domain). Further, both of Hailperin's axiom sets are complete.
10. See note 3 for reference to the relevant paper by Hailperin and Leblanc.
11. See note 3 for the reference to my paper.
12. The solution to the problem of existential import in this paper is in some aspects similar to Lesniewski's position on this matter. (Cf. for example, C. Lejewski, "On Lesniewski's Ontology," *Ratio*, Vol. 1: 1958, 150-176). I owe this observation to Sobociński.

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