Notre Dame Journal of Formal Logic
Volume III, Number 4, October 1962

THE METHOD OF POSSIBILITY–DIAGRAMS FOR TESTING THE VALIDITY OF CERTAIN TYPES OF INFERENCES, BASED ON JEVONS' LOGICAL ALPHABET

GORDON L. BRUMM

In *The Principles of Science*,[1] and other works, W. S. Jevons expounded a method for testing elementary deductions, similar to that using Venn Diagrams or similar devices. Jevons' method was based on what he called the "Logical Alphabet." This paper will be devoted to outlining Jevons' method, revising it with regard to certain types of deduction, and applying it to areas beyond those with which Jevons dealt.[2] The name "possibility diagrams" will be applied to what is, essentially, the device which Jevons called the "Logical Alphabet".

Part I will be the outline of Jevons' scheme. Part II will consist of a revised version of the scheme, the revision applying within the area with which Jevons was concerned, *viz.*, inferences involving truth-functionally simple quantifications with monadic predicates. The revisions will consist mainly of the replacement of Jevons' way of notation by the better (or at least currently more popular) terminology of variables, predicate-letters, and quantifiers, and the improvement of Jevons' treatment of particular propositions. When applied in the original area, the method–both as originally invented by Jevons and as revised–is similar to that involving Venn Diagrams; however, the former is suitable for a wider range of inferences.

In Part III and subsequent parts, the application of possibility–diagrams will be extended beyond the basic area dealt with in Part II. The following uses will be described: III. Truth-functional inferences. IV. The use (in favorable cases) of more than one diagram for one inference, so as to reduce the number of predicate-letters which need be treated in any one diagram. V. Inferences involving truth-functional compounds of quantifications with monadic predicates.

1. W. Stanley Jevons, *The Principles of Science; a Treatise on Logic and Scientific Method*, 2d ed., 1877. *Cf.* also Jevons' *Elementary Lessons in Logic*, and *The Substitution of Similars*. All references are to *The Principles of Science*.

2. The author wishes to express his gratitude to Messrs. Peter Kugel and Joe W. Swanson, and to the referee, for their aid in preparing this article.

Received June 23, 1961

I. *Jevons' Use of the Logical Alphabet.* For Jevons, the type of proposition most basic to logic was that asserting identity of objects, classes, or qualities. The most important type of identity was the "simple identity", of the form $A = B$. "Partial identities" and "limited identities" were also defined. Partial identities are of the form $A = AB$, meaning that A is identical with the conjunction of A and B (or, in other words, all A's are B.) Two partial identities, each the converse of the other, are of course equivalent to a simple identity—e.g. "$A = AB$" and "$B = AB$" are the equivalent of "$A = B$". Limited identities are of the form $AB = AC$. (I.e., this one is limited to within the class A.)

Identity-propositions derived much of their importance, to Jevons, from their intimate connection with the rule of substitution. This rule, thought Jevons, was the foundation of deductive logic, and he expressed it as follows: "For any term occurring in any proposition substitute the term which is asserted in any premise to be identical with it."[3] The most straightforward use of the rule is found in what was called "direct deduction", i.e. deduction accomplished by making substitutions in the premisses themselves. E.g., from "$A = B$" and "$B = C$" is deduced "$A = C$", since the second premiss (itself an identity-proposition) allows the substitution of "C" for "B" in the first premiss. Or, "$A = B$" and "$B = BC$" vield "$A = AC$"; and so on. Jevons considered direct deduction sufficient for many types of argument, but in order to prove certain other types of argument valid, he felt obliged to introduce what he called "indirect deduction".

Indirect deduction is described as the method which "points out what a thing is, by showing that it cannot be anything else."[4] Jevons develops this method by use of the Law of Duality (i.e., Law of the Excluded Middle), according to which any class, object, or quality is either identical to its conjunction with some particular class etc., or is identical to something distinct from that class etc. I.e., $A = AB$ or Ab, where the lower case letter, "b" stands for the negative of B. (The alternation "AB or Ab" comprises the "development" of A.) It also follows that $A = ABC$ or ABc or Abc or AbC—and so on for greater numbers of terms. A proposition of this form, of course, need not be premissed; it is true in virtue of the necessarily-true Law of Duality. And when premisses are given, these being identity-propositions, certain terms may be substituted into the alternatives of the disjunctive proposition so developed. Such substitution would, perhaps, transform some of these alternatives into contradictions. The alternation of whichever alternatives do not become contradictions may be equated with the term developed, to form an identity-proposition which is a a valid conclusion to the premisses given. If the alternation has only one member, that member is, of course, equated with the term developed. If the alternation has two or more members, it may be reduced to that group of terms which all the members have in common. For example, suppose the

3. Chap. IV. p. 49.
4. Chap. VI. p. 81.

premisses are "$A = B$" and "$B = c$". The development of A is: "$A = ABC$ or ABc or Abc or AbC". Substituting "B" for "A" in the third and in the fourth alternative (as warranted by the first premiss) results in contradictions. Substituting "c" for "B" in the first alternative (second premiss) likewise results in a contradiction. The second alternative remains. Therefore, a valid conclusion is: "$A = ABc$"; from this and the given two premisses it follows also that $A = c$. If, after the substitution-process there remained both "ABC" and "ABc", then "$A = AB$" would be a valid conclusion, but not "$A = ABC$". The entire process is summarized by Jevons as follows:

"1. By the Law of Duality develop the utmost number of alternatives which may exist in the description of the required class or term as regards the terms involved in the premises.

2. For each term in these alternatives substitute its description as given in the premises.

3. Strike out every alternative which is then found to break the Law of Contradiction.

4. The remaining terms may be equated to the term in question as the desired description."[5] (To which must be added: If there are two or more alternatives remaining, the term in question may be equated to that group of terms which appears in every alternative. Thus, if "A" is the term in question, and "ABC" and "ABc" remain, then "$A = AB$" is a valid conclusion. This step is implied by the process which Jevons calls "abstraction of indifferent circumstances.")[6]

The Logical Alphabet is introduced as a means of simplifying this process. The Logical Alphabet is formed, for any group of terms, by producing the complete series of all combinations of the terms in the group, indicating the absence of a term by its negative. Thus, the logical alphabet for "A" and "B" is: "AB, Ab, aB, ab." The logical alphabet for "A", "B", and "C" is: "ABC, ABc, AbC, Abc, aBC, aBc, abC, abc." (Jevons adds that properly speaking an "X" should be appended before any combination, for any group of terms, in order to denote that some higher class is being developed. Thus, for "A" and "B": "XAB, XAb, XaB, Xab." Since the "X" is always understood to be present, however, it may be omitted.)

The procedure for carrying out deductions by means of the Logical Alphabet is as follows· First, develop the logical alphabet for the terms which appear in the argument in question. Then consider all the identity-propositions which serve as premisses for the argument. Within the combinations making up the logical alphabet for the argument, make all appropriate substitutions in accordance with the premisses. When any such substitution transforms a combination into a self-contradiction (as, e.g., "Aa"), eliminate that combination from consideration. Consider all the combinations which remain. Any term in one or more remaining combinations may be equated with the *alternation* of the combinations in which it

5. Chap. VI. pp. 89-90.

6. Chap. VI. p. 97.

appears. (For example, if "AB" and "aB" are the remaining combinations, then $B = A$ or a.) That term may therefore be equated with the group of terms appearing in each of those alternative combinations.

As an example, consider the treatment of the syllogism, "All A is B; all B is C; therefore, all A is C." The premisses would be expressed as "$A = AB$" and "$B = BC$"; the conclusion would be "$A = AC$". The logical alphabet would be: 1) "ABC", 2) "ABc", 3) "AbC", 4) "Abc", 5) "aBC", 6) "aBc", 7) "abC", 8) "abc". The first premiss transforms combinations 3) and 4) into contradictions; the second premiss so transforms combinations 2) and 6). The remaining combinations are "ABC", "aBC", "abC", and "abc". Of these, only "ABC" is a combination in which "A" appears. Therefore "A" may be equated with "ABC"; from this conclusion and the first premiss, we may deduce "$A = AC$". However, we may not, for example, draw the conclusion "$C = CA$", because "C" appears in the following remaining combinations: "ABC", "aBC", and "abC". "C" must therefore be equated with the alternation of all three of these combinations, and "CA" does not appear in all of the alternatives. In fact, the only group of terms appearing in all three "C"–including combinations is simply "C" itself, and so the only conclusion to be drawn with reference to "C" is "$C = C$".

The process involving the Logical Alphabet is of course derived from the somewhat more primitive method, described previously, which is based on the development of terms according to the Law of Duality. The development of each term in an argument, with respect to the other terms in the argument, is comprised in the logical alphabet for those terms. E.g., the logical alphabet for "A" and "B" (*viz.*, "AB", "Ab", "aB", "ab") contains the development of "A" with respect to "B", *viz.*, "AB or Ab", and contains the development of "B" with respect to "A", *viz.*, "BA or Ba." (There may also be other combinations in the logical alphabet, not included in the developments of the terms–in the example given, "ab"–which are superfluous.) In both procedures, certain combinations are transformed into contradictories, by substitution in accordance with the premisses, and are thus eliminated. Any remaining combination in the logical alphabet, then, represents, for any term appearing in the combination, one of the alternatives in the development for that term, which alternative is not contradicted by the premisses. Therefore, the set of alternative combinations in which any one term appears is the set of alternatives to which that term may be equated. And so that term may be equated with the group of terms which appears in every one of that set of alternative combinations–and this is just what is dictated by the procedure laid down in connection with the Logical Alphabet. For example, the logical alphabet for "A", "B", and "C" is the development of "A", "B", and "C" with respect to each other (plus the superfluous combination, "abc".) Suppose that, after substitution in accordance with the premisses, only "ABC" and "ABc" remain. According to the procedure stemming from the development of terms, "A" may therefore be equated with "ABC or ABc". This means that it may be equated with "AB", the group of terms which appears in both alternative combinations. And this is just what is prescribed by the procedure laid down for use with the Logical Alphabet.

Jevons recognized that the Logical Alphabet could be used to show the logical equivalence, consistency, or contradiction of propositions or sets of propositions. Two propositions are equivalent, he pointed out, "when they remove the same combinations from the Logical Alphabet, and neither more nor less." Consistent propositions "jointly allow each term and the negative of each term to remain somewhere in the Logical Alphabet". Contradictories, then, are those propositions which "taken together remove any one or more letter-terms from the Logical Alphabet."[7]

It may be noticed that the description of Jevons' method has so far been confined exclusively to universal propositions. Jevons said little about particular propositions, and the treatment he proposed for them is clearly inadequate. He proposed to treat the term (or, quantifier) "some" in the same way as any other term.[8] I.e., if "A" stands for "triangles", then "P" might stand for "some", and "some triangles" would be represented as "AP", this combination being treated in exactly the same way as, e.g., "AB" would be if it stood for "big triangles". (Or else, the term "A" by itself might stand for "some triangles".) The inadequacy of this treatment is shown by the following example, in which a false conclusion is derived from true premisses in accordance with Jevons' rules: Let "P" stand for "some", "A" for "triangle", and "B" for "large". Take as premisses these two propositions: "Some triangles are large" and "some triangles are not large". These become, in Jevons' notation, "$PA = PAB$" and "$PA = PAb$". They may both be realistically assumed to be true. By the rule of substitution, we may draw the conclusion, "$PAB = PAb$". This conclusion is surely false. For it must have one of two meanings. Either the "PA" ("some triangles") in "PAB" refers to the *same* class of triangles as the "PA" in "PAb", or it ("PA") refers to a *different* class of triangles. If the former, then the proposition in question (the conclusion to the argument) is that the same triangles are both large and not large (both B and b.) This, being self-contradictory, is certainly false. If the latter is meant, and the "PA"'s on the two sides of the identity refer to different classes of triangles, then the proposition in question is that different classes of triangles are identical. This, being self-contradictory, is also certainly false. In the revised version of Jevons' method, to be expounded in the next part, particular propositions will be given a treatment completely different from Jevons', though universal propositions will be treated in essentially the same way as Jevons treated them.

II. *The Basic Method as Adapted.* The basic method, as revised, is concerned with the same area as was Jevons' method, *viz.*, inferences which involve quantifications with monadic predicates, such as "$(x)\ (Ax \supset Bx)$", but not truth-functional compounds of such quantifications. Moreover, the quantifications involved must not contain any buried quantifiers; i.e., a

7. Chap. VI. p. 116.
8. Chap. IV. p. 56-7.

schema such as "(x) $(Fx \supset (\exists x)$ (Fx)" is excluded. The revision will differ slightly from Jevons' procedure, with regard to the notation used to indicate the fundamental combinations of predicates. Also, it will differ in the treatment of universal premisses and conclusions in two ways: other types of sentences, besides identities, will be considered (all sentences being put in the form of quantifiers, variables, predicate-letters, and truth-functional connectives); and the testing of conclusions will be accomplished by a search for counter-instances among uncrossed combinations of predicates. Finally, there will be a radically different treatment of particular propositions. These of course have existential import, and they will be signified by putting circles around the appropriate combinations of predicates. Existential conclusions will be tested by a special procedure.

Fundamental diagram: Each deduction will be accomplished by means of a diagram, to be called a "possibility diagram". This will consist, basically, of the exhaustive list of combinations of predicate-letters involved in the argument to be tested—which is to say, the logical alphabet for those predicate-letters. As opposed to Jevons' way, however, the absence of a predicate-letter from a combination will be indicated by a blank space, instead of a lower-case letter. Also, the combinations of predicate-letters will be called "descriptions". Each description will be numbered. There will be, of course, 2^n descriptions for each diagram, where n is the number of predicate-letters involved. As an example, consider an argument with predicate-letters "A", "B", and "C". The diagram for this will have the following basic form:

	A	*B*	*C*
1	2	3	4
AB	*AC*	*BC*	*ABC*
5	6	7	8

The other basic notational devices are crosses and circles. A cross is placed over a description to mean the non-existence of any objects which meet that description. A circle is put around a description to mean the existence of at least one object which meets the description.

Procedure: Universal premisses. It will be understood that universal premisses have no existential import. To transcribe a universal premiss on to the diagram, therefore, one simply places a cross over every description which is a counter-instance to that premiss. By "counter-instance" is meant a description such that the existence of an object meeting that description would contradict the premiss. For example, the universal premiss "(x) $(Ax \supset Bx)$" would be transcribed as follows:

	~~*A*~~	*B*	*C*
1	~~2~~	3	4
AB	~~*AC*~~	*BC*	*ABC*
5	~~6~~	7	8

In case there are both universal and existential premisses, the universal should always be transcribed first.

Existential premisses. To transcribe an existential premiss on to the diagram, one puts a circle around every description which satisfies the premiss. By saying that a description “satisfies” a sentence or schema (whether it be premiss or conclusion), I mean that the description is such that the sentence or schema is true if there are any objects which meet that description. To continue the example mentioned, transcribing the existential premiss “$(\exists x)\ (Cx \cdot Ax)$” would make the diagram appear as follows:

	~~A~~	B	C
1	~~2~~	3	4
AB	~~AC~~	BC	ABC
5	~~6~~	7	8

Universal conclusions. Once the premisses are transcribed, one tests the validity of the inference by finding whether or not the resulting diagram has yielded the transcription of the conclusion. In the case of a universal conclusion, this is done by seeking counter-instances among the un-crossed descriptions. If there are no such counter-instances to the conclusion, then the inference is valid; if there is at least one such counter-instance, then the inference is invalid. For example, the argument

$$\begin{array}{c} (x)\ (Ax \supset Bx) \\ \underline{(x)\ (Bx \supset Cx)} \\ (x)\ (Ax \supset Cx) \end{array}$$

has premisses which would be transcribed as follows:

	~~A~~	~~B~~	C
1	~~2~~	~~3~~	4
~~AB~~	~~AC~~	BC	ABC
~~5~~	~~6~~	7	8

The inference is valid, because there are no counter-instances, among the uncrossed descriptions, to the conclusion: *#1* is not a counter-instance because it is not an *A*; *#8* is not a counter-instance because it is an *A* but also a *C*; etc.

However, if the conclusion to that argument were: “$(x)\ (Cx \supset Ax)$”, then the inference would be invalid. The uncrossed counter-instances to the conclusion would be *#4* and *#7* (either of which would be sufficient to invalidate the argument.) For both *#4* and *#7* are descriptions of objects which are *C* but are not *A*; therefore to say that any such objects exist would contradict “$(x)\ (Cx \supset Ax)$”.

Existential conclusions. When the conclusion is an existential sentence or schema, one tests for validity by finding whether there is any set of circled descriptions which together satisfy all the existential premisses without satisfying the conclusion. This may be done in two steps. First, find whether among the circled descriptions (assuming there are any) there is at least one description which satisfies the conclusion. If there is none, then of course the inference is invalid. If, however, there is at least one description, among those circled, which satisfies the conclusion, then the second step must be followed (although the outcome may be obvious), *viz.*, one must tabulate the numbers of the descriptions which satisfy each existential premiss and the conclusion; then find whether or not there is any set of numbers such that each existential premiss is satisfied by at least one member of the set, while no members of the set satisfy the conclusion. If there *is* such a set of description-numbers, the inference is invalid; if there is *not*, the inference is valid. E.g., in the argument:

$$\begin{array}{c} (x)\ (Ax \supset Bx) \\ \underline{(\exists\, x)\ (Cx \cdot Ax)} \\ (\exists\, x)\ (Cx \cdot Bx) \end{array}$$

the one and only description circled was #8. This also satisfies the conclusion. Therefore, the tabulation of description-numbers would be as follows:

$$\frac{8}{8}$$

There is obviously no set of description-numbers of which one satisfies the existential premiss, without any satisfying the conclusion; so the inference is valid. But consider this argument:

$$\begin{array}{c} (x)\ (Ax \supset Bx) \\ (\exists\, x)\ (Ax \cdot -\ Cx) \\ \underline{(\exists\, x)\ (Bx \cdot Cx)} \\ (\exists\, x)\ (Ax \cdot Cx) \end{array}$$

The diagram with transcription of the premisses would appear as follows:

1	~~A 2~~	B 3	C 4
AB 5	~~AC 6~~	BC 7	ABC 8

There is one description—*viz.*, #8—among the circled descriptions, which does satisfy the conclusion. The tabulation of description-numbers satisfying the two existential premisses and conclusion would be as follows:

First existential premiss:	5	
Second existential premiss:	7	8
Conclusion:		8

Here, there is one set of description-numbers—viz., #5 and #7—such that each existential premiss is satisfied by at least one member of the set, without any member satisfying the conclusion. Therefore, this inference is invalid.

Justification of the method. The justification for this method is essentially the same as that for a method such as Venn Diagrams. Consider any argument whatsoever for whose predicates the predicate-letters are substituted. When the universal premisses of the argument have been transcribed as shown, the result is a symbolization (in the crossed descriptions) of what types of things cannot exist if those premisses are true. If the remaining descriptions symbolize nothing which could falsify the conclusion of the inference, then the inference is valid; otherwise it is invalid. In the case of existential premisses, the procedure insures the circling of descriptions of all types of things such that the existence of something of any of the types would make one or more of those premisses true. If there are none of these such that the existence of something of the type would make the existential premisses all true without also making the existential conclusion true, then the argument is valid; otherwise it is invalid.

Inconsistency, consistency, and tautology. These situations are easily symbolized or recognized in possibility-diagrams. (Cf. Jevons' treatment of consistency and "contradiction".) The inconsistency of a set of sentences (or likewise, of a set of schemata representing sentences)—i.e., the fact of their all being true in no possible situations—may be symbolized in two ways. First, the set is inconsistent if, in order to transcribe any of the existential sentences, a circle must be put around any description which is crossed. Such a necessity would mean that the set of sentences would be true all together only if there existed something of a certain description, but also nothing existed of that description; thus the set of sentences could not be true all together. Second, a set of sentences is inconsistent in any non-empty universe if its transcription leads to the crossing of every description. For this would indicate that the set of sentences can all be true only if nothing exists of any description; thus they cannot be true in any situation in a non-empty universe.

Accordingly, the consistency of a sentence or set of sentences is indicated by the fact that the transcription of these sentences has not necessarily led to the crossing and circling of the same description, and (in the case of a non-empty universe) the fact that not all descriptions are crossed.

The tautology of a set of sentences—i.e., the fact of its being true in all possible situations—is symbolized, in case only universal sentences are involved, by the diagram in which no descriptions are crossed. In the case of a set of sentences including at least one existential sentence (assuming a non-empty universe), tautology is symbolized by the circling of *all* descriptions (which involves, obviously, the crossing of none of them), in virtue of *each* existential sentence.

III. *Truth-functional inference.* The applicability of the system of notation of possibility-diagrams is even more evident in the case of inferences involving truth-functional compounds of sentences, represented by sentence-letters. In fact, each description, when possibility-diagrams are so adapted, becomes the equivalent simply of one particular line on a truth-table.

The fundamental notation involves all the sentence-letters which appear in the argument, arranged in all possible combinations—on the model of the quantificational inference just described. The other notational device will be the cross, placed over appropriate descriptions. Circles are not used.

In the fundamental diagram, there are 2^n descriptions (where n is the number of sentence-letters), and each description corresponds to a distinct situation, given any interpretation of the sentence-letters, which is defined by the truth of the sentences represented by the sentence-letters present in the description, and the falsity of the sentences represented by the sentence-letters which are absent. Thus, for example, the description "*pq*" corresponds to the situation in which any sentence represented by "*p*", as well as any represented by "*q*", are true, and all other sentences represented by letters involved in the diagram are false.

The procedure for testing the validity of an inference involves considering each premiss in turn and simply crossing all descriptions which are counter-instances to the premiss. Then the conclusion is considered. If there are no counter-instances to the conclusion, the inference is valid; if there is at least one counter-instance, the inference is invalid. A counter-instance is a description which contradicts the conclusion or the premiss, as the case may be. For example, consider the argument:

$$\begin{array}{c} (p \lor q) \supset r \\ (r \supset s) \lor q \\ \underline{p \cdot s} \\ q \cdot (r \supset s) \end{array}$$

This involves 16 descriptions:

~~1~~	~~*p* 2~~	~~*q* 3~~	~~*r* 4~~	~~*s* 5~~	~~*pq* 6~~
~~*pr* 7~~	~~*ps* 8~~	~~*qr* 9~~	~~*qs* 10~~	~~*rs* 11~~	~~*pqr* 12~~
~~*pqs* 13~~	*prs* 14	~~*qrs* 15~~	*pqrs* 16		

In virtue of the first premiss, descriptions numbered *2, 3, 6, 8, 10,* and *13* are crossed. In virtue of the second premiss, descriptions numbered *4* and *7* are crossed. In virtue of the third premiss, descriptions numbered *1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12,* and *15* are crossed (or would be, if not crossed already.) Number *14* and No. *16* remain uncrossed. Number *16* is not a counter-instance to the conclusion, but number *14* is, and so the inference is invalid.

When applied to truth-functional inference, possibility-diagrams are obviously merely a different method of writing out a truth-table. In some cases—especially those involving long premisses—these diagrams may be more clumsy and less safe than the conventional truth-table arrangement. In other cases, possibility-diagrams may be considerably more convenient, and at any rate, they provide a general decision-technique applicable to both truth-functional and quantificational inferences of the types mentioned.

IV. *The use of two small diagrams in one inference.* It should be apparent that the convenience of possibility-diagrams decreases rapidly—indeed, geometrically—with the number of different predicates involved in the argument tested. For example, a diagram adequate to an argument with seven predicates must contain 128 descriptions, and in the absence of any pre-constructed diagrams, the effort of composing such a variety of descriptions might well be prohibitive. This limitation can be largely overcome, under certain favorable conditions, by the use of two diagrams in combination, so that each diagram may be of a convenient size.

This technique is useful, of course, only in case the premisses of the argument can be divided into two groups such that the premisses in each group involve together only a manageable number of predicate-letters.

Procedure. To begin, the premisses of the argument are divided into two groups in some way which divides the predicate-letters between the two groups. It is permissible—and in most cases unavoidable—to have one or more predicate-letters shared by the two groups of premisses. To give an example, the following set of premisses all together involve six predicates: "$(x)\ (Ax \supset Bx)$, $(x)\ (Bx \supset Cx)$, $(x)\ (Cx \supset Dx)$, $(x)\ (Dx \supset Ex)$, $(x)\ (Ex \supset Fx)$, $(\exists\ x)\ (Ax)$". It might be split into two groups as follows, each involving the predicate-letters appearing below it.

$(x)\ (Ax \supset Bx)$	$(x)\ (Dx \supset Ex)$
$(x)\ (Bx \supset Cx)$	$(x)\ (Ex \supset Fx)$
$(\exists\ x)\ (Ax)$	$(x)\ (Cx \supset Dx)$
A, B, C	C, D, E, F

With this division decided, each group of premisses is transcribed on to the appropriate possibility-diagram, in accordance with the basic procedure. After this transcription, the two diagrams are combined, as follows.

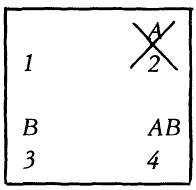
For universal premisses: Make an overall list of crossed descriptions, in the following way.

> Consider *each* crossed description, in *either* diagram, and include it in the overall list, plus the conjunction of it with *any* description in the *other* diagram besides its own, *just so long as* that

other description does *not* include any predicate-letter which is involved in the diagram of the original, crossed description.[9]

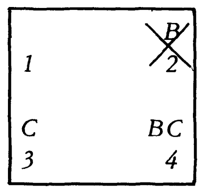
In talking about one description in "conjunction" with another, I mean simply the combining of the predicate-letters which constitute the one, with the predicate-letters which constitute the other (eliminating repeated letters, which are redundant.) For example, the conjunction of "*AB*" and "*CD*" is "*ABCD*". The conjunction of "*AB*" and "*BC*" is "*ABC*". An example of the conjoining of crossed descriptions is as follows:

"$(x)(Ax \supset Bx)$" "$(x)(Bx \supset Cx)$"



Combined List of Crossings

A
B
AC
AB



"*A*" is included in the combined list of crossings—which I shall call the "overall list of crossed descriptions"—simply in virtue of its being crossed in its own diagram, and likewise for "*B*". "*AC*" is included as the conjunction of "*A*" and "*C*". "*AB*" is included as the conjunction of "*B*" and "*A*". The conjunction of "*A*" and "*BC*" is *not* included, because of the fact that "*B*" (part of "*BC*") is involved in the left-hand diagram; likewise for the conjunction of "*B*" and "*AB*" (although that conjunction—*viz.*, "*AB*" itself —is included for another reason.) As for description *#1*, its conjunction with any other description is always that other description itself, so in this diagram it may be considered to be conjoined with "*A*" and "*B*".

For existential premisses: Make an overall list of descriptions circled. This will include every description circled in either diagram, *plus* the conjunction of every such description with every uncrossed description in

9. This procedure, if completely followed, will result in placing some descriptions on the overall list twice. *Viz.*, if "*A*" is a crossed description in the first diagram containing no predicate-letters involved in the second, and "*C*" is a crossed description in the second diagram containing no predicate-letters involved in the first, then "*AC*" will be included *both* in virtue of the inclusion of the conjunction of the crossed "*A*" with the appropriate descriptions in the second diagram, *and* in virtue of the inclusion of the crossed "*C*" with the appropriate descriptions in the first diagram. To avoid this, refrain from placing on the overall list any conjunctions formed by a crossed description *in the second diagram considered* and any *crossed* description in the first diagram considered. *It is easily seen that all conjunctions thus precluded* from the list will be on the list already.

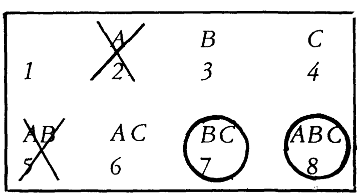
the other diagram, *plus* those conjunctions resulting from the following: conjoining a circled description which contains letters common to both diagrams, with a crossed description (in the other diagram) containing no letters common to both diagrams. If there are universal premisses in the argument, any descriptions on the overall list of crossed descriptions must subsequently be eliminated from the overall list of circled descriptions. (In the case of an argument with nothing but existential premisses, then, the overall list of circled descriptions will be made up of the conjunction of every circled description with every description in the other diagram.)

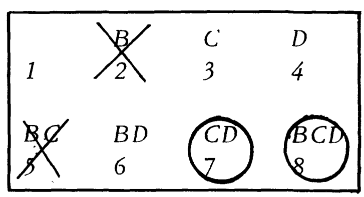
A simple example of the diagramming of existential premisses, along with universal premisses, is shown in the case of

$$(x)\,(Ax \supset Cx)$$
$$(x)\,(Bx \supset Dx)$$
$$(\exists x)\,(Bx \cdot Cx)$$
$$(\exists x)\,(Cx \cdot Dx)$$

This would be diagrammed as follows:

$(x)\,(Ax \supset Cx)$ $(\exists x)\,(Bx \cdot Cx)$	Letters in Common: $\left.\begin{matrix}B\\C\end{matrix}\right\}$	$(x)\,(Bx \supset Dx)$ $(\exists x)\,(Cx \cdot Dx)$





Overall list, crossed descriptions

A	*AD*	*BC*	*ABC*
B	*AB*	*ABD*	

Overall list, circled descriptions

~~*BC*~~	*BCD*
CD	~~*ABC*~~
ACD	*ABCD*

The descriptions on the overall list of crossed descriptions are entered in accordance with the above procedures. Considering the overall list of circled descriptions, "*BC*", "*CD*", "*BCD*", and "*ABC*" are entered because they themselves are circled in one or the other of the diagrams. "*ABCD*" is entered as the conjunction of "*ABC*" and "*D*". (Some of these descriptions would also be eligible to be on the list as conjunctions of other descriptions.) "*ACD*" also is entered on the list as the conjunction of "*CD*" (containing "*C*", which is common to both diagrams) with "*A*" (which is crossed but contains no letter common to both diagrams). Finally, there are two casualties: "*BC*" and "*ABC*" are crossed off this list because they are also on the overall list of crossed descriptions.

Justification for the treatment of the premisses. The aim of the procedures just described is, of course, to determine which descriptions, involving *all* predicate-letters in the entire argument, deserve to be crossed and circled, on the basis of *all* the premisses. Justification of the procedure involves separate consideration of universal and existential premisses.

For the case of universal premisses, consider any two diagrams used in connection with an argument. Let "*A*" refer to any combination of predicate-letters which are involved in the first diagram but not the second. Let "*B*" represent any set of predicate-letters which are involved in both diagrams; let "*C*" represent any one or group of the predicate-letters which are involved in the second argument but not the first. (These three categories exhaust the possibilities.) Suppose that either "*A*" alone is crossed, or "*A*" in conjunction with "*B*" is crossed. The procedure then implies three specifications:

(1) "*A*" or "*AB*" is included in the overall list of crossed descriptions,
(2) the conjunction of "*A*" or "*AB*" with "*C*" is included in the overall list, whether "*C*" is uncrossed or not, and
(3) the conjunction of "*A*" or "*AB*" with "*B*" (in the second diagram) or "*BC*" is *not* included.

The justification for (1) is easily seen—"*A*" or "*AB*" can validly be crossed only on the basis of premisses which contain as predicate-letters "*A*", or "*A*" *and* "*B*", as the case may be. Therefore, nothing pertaining to the second diagram can affect the validity of their being crossed, and they deserve to be on the overall list of crossed descriptions. As for (2), "*A*" or "*AB*" has been crossed in virtue of being *A*, or being *A* and *B*, as the case may be. This means that, among *all* possible descriptions involving *all* predicate-letters of the entire argument, all descriptions which include "*A*" (or, "*AB*") must be crossed. This requires putting the conjunction of "*A*" or "*AB*" and "*C*" on the overall list of crossed descriptions, whether "*C*" has already been crossed, or not. However, as stated in specification (3), this does *not* require putting the conjunction of "*A*" or "*AB*" with "*B*" or "*BC*" on the overall list.

The justification for (3) involves consideration of two possibilities: (a) that "*B*" or "*BC*" is *not* crossed, and (b) that "*B*" or "*BC*" *is* crossed. (a) Consider the possibility that "*B*" or "*BC*" is not crossed; and take the case of "*BC*". The conjunction of either "*A*" or "*AB*" with "*BC*" will be "*ABC*". Suppose (i) that "*ABC*" deserves to be crossed on the basis of being simply *A* or *A and B* (the other alternative is its being crossed on the basis of being *A* and not-*B*); then it will be crossed anyway as the conjunction of "*AB*" and "*C*", hence it is superfluous to cross it as the conjunction of either "*A*" or "*AB*" and "*BC*". This takes care of all cases in which "*A*" is crossed *not* in virtue of being *A* and not-*B*. Now consider (ii) the cases in which "*A*" *is* crossed in virtue of being *A and* not-*B*. In such instances, it would be *invalid* to cross "*ABC*" as the conjunction of

"A" and "BC"; for since "A" has been crossed in virtue of being A and not-B, no conjunction is permitted in which "A" appears *along with* "B". So in summary, in any instance in which "A" or "AB", already crossed, were conjoined with "BC", not crossed, to put the resulting conjunction on the overall list of crossed descriptions would be either superfluous or invalid. The corresponding conclusion holds true also if we are considering "B" instead of "BC".

(b) Now consider the possibility that "B" or "BC" *is* crossed. In such a case, the inclusion of the conjunction of "A" or "AB" with "B" or "BC" is not warranted, because it would be in all instances superfluous. It would be superfluous in the case of "A" conjoined with "B" or "BC", because the conjunction will be included anyway when the crossed descriptions from the second diagram are considered, and "BC" plays the same role which has heretofore been assigned to "AB", while "A" then plays the role heretofore played by "C". In the case of "AB", likewise, the resulting conjunctions will be included in virtue of other parts of the procedure, as shown by the following:

(i) Suppose first that "AB" is being conjoined with "B". The resulting conjunction will be "AB", and this will have been included in virtue of the crossing of "AB", itself, in the first diagram.

(ii) Suppose, then, that "AB" is being conjoined with "BC". The resulting conjunction will be "ABC", and this will have been included, already, because of specification (2), which prescribes that the conjunction of "AB" and "C" be included.

Thus the procedure is justified for cases in which "A" is the predicate-letter (or group of predicate letters) being considered. It can be given an exactly analogous justification for cases in which "C" is the predicate-letter (or group) being considered, for "C" is in exactly the same relation to the second diagram as "A" is to the first. There remains those cases in which "B" alone is the predicate-letter (or group) being considered. At this point it must be recognized that there may be more than one letter (or group of letters) "B"—i.e. more than one predicate-letter (or group) which appears in both diagrams. Since it will be necessary to talk about the conjunction of one of these with some other, let us use "B" and "B'" to refer to any two letters or groups of letters which appear in both diagrams. Now the rules given above make three specifications with regard to any letter or group, "B", in the first diagram ("B'", here, referring to an appropriate letter or group as it appears in the *second* diagram):

(1) "B" is included in the overall list, by itself, if crossed in either diagram;
(2) the conjunction of "B" (if crossed) and "C" is included.
(3) the conjunction of "B" with "B'" or "$B'C$" is *not* included.

These specifications are justified as follows: (1) If crossed in either diagram, "B" deserves to be on the overall list in virtue of the premisses involved with that diagram; the premisses involved with the other diagram can have no effect in nullifying its being crossed. (2) If "B" deserves to be on the overall list in virtue of the premisses in the first diagram, then so does any conjunction of "*B*" and other letters which are not affected by the premisses involved in that diagram—which is to say, the conjunction of "*B*" and "*C*".

As for (3): Consider first the conjunction of "*B*" and "*B'C*", *viz.*, "*BB'C*". This might deserve to be on the overall list in either of two relevant circumstances: (a) in case there is some premiss involved in the second diagram which would require "*BB'C*" to be crossed; or (b) in case there is some premiss involved in the first diagram which would require "*BB'*" to be crossed, and would therefore require any conjunction of "*BB'*" with "*C*" to be crossed. If (a) holds true, then the inclusion of "*BB'C*" in the overall list is accomplished anyway, since that combination would appear in the second diagram, would be crossed in virtue of the premiss hypothesized, and would therefore be placed on the overall list. If (b) holds true, then since "*B'*" constitutes a description which appears in the first diagram, the conjunction of "*BB'*" with "*C*" would be assured a place on the overall list due to specification (2) above. Thus, in any case in which "*BB'C*" deserves to be on the overall list, it will appear there without having to be entered as the conjunction of "*B*" from the first diagram and "*B'C*" from the second.

The case of the conjunction of "*B*" and "*B'*" is somewhat simpler. Both letters (or groups of letters) appear in both diagrams. Hence, if there is a premiss involved in either diagram which requires "*BB'*" to be crossed, that conjunction will appear on the overall list in virtue of specification (1) above; it would be superfluous to put it on the list for any other reason. (This justification has involved the supposition that "*B*" is in the first diagram. Exactly similar considerations would apply if "*B*" were in the second diagram, in which case "*A*" would replace "*C*", and "*B'*" would be considered as it appeared in the first diagram.)

For the procedure involving existential premisses, the justification is less complex. As in the case of the basic procedure, the aim of the procedure here is simply to compile a list of descriptions all of which satisfy one existential premiss or another. The descriptions circled in either of the diagrams all satisfy one existential premiss or another. Thus all such descriptions are put on the overall list of circled descriptions. In addition, all conjunctions of circled descriptions with any other description will satisfy some existential premiss. But these are not all put on the overall list, for those conjunctions must be eliminated which would be crossed. In order to eliminate the appropriate conjunctions, no conjunction is permitted with a description already crossed, with some exceptions—*viz.*, those cases in which the original circled description contains some letter involved in both diagrams, and the crossed description in the other diagram contains only letters not found in the diagram of the original circled description. These are not eliminated for this reason: it may not be required to cross the conjunction of the letters included in the original circled description with those crossed letters involved in the other diagram. For example, suppose "*C*" has been crossed in the second diagram, in virtue of the premiss, "$(x)\ (Cx \supset Bx)$". If "AB" is circled, then the result of its conjunction with "C"—*viz.*, "ABC"—should also be circled, because "ABC" would not be crossed in virtue of the premiss that required the crossing of "C" by itself. However, the qualification just mentioned will still result in some circlings which should not be permitted—*viz.*, in those cases in which the crossed description (the "C" just mentioned) has been crossed simply, i.e., *not* in virtue of the absence of any letter which is present in the original circled description. For example, if "C" has been crossed in virtue of the premiss, "$(x)\ (-\ Cx)$", then indeed the conjunction "ABC" should be crossed. But in such cases, the illicit circled description will always be removed from the list of circled descriptions when the time comes to remove from that list all descriptions which also appear on the overall list of *crossed* descriptions.

The justification of this last step—*viz.*, removing from the overall list of circled descriptions any which appear on the overall list of crossed descriptions—is obvious. In most cases, if not always, there will be descriptions crossed as a result of combining the two diagrams (in accordance with the processes described above) which do not appear in either of the diagrams themselves. These must be eliminated at the last stage, after the overall list of circled descriptions has been compiled.

Procedure for testing the conclusion. Once the premisses have been transcribed in their respective diagrams, and the diagrams have been combined in the way described, the conclusion can be tested against the result in order to find whether the inference is valid or invalid. As in the case of the basic procedure, the testing is done differently in the case of existential conclusions and universal conclusions.

Existential conclusions: With these, the procedure is not essentially different from the corresponding part of the basic procedure. If an argument with an existential conclusion is to be valid at all, of course, there must be some description(s) circled as a result of transcribing the premisses. There will therefore be an overall list of circled descriptions. The conclusion is checked against these. If *none* of the circled descriptions satisfies the conclusion, then the inference is invalid. If there is some description on the list which satisfies the conclusion, then the tabulation, used in the basic procedure, must be undertaken. It is determined whether there is a set of descriptions which all together satisfy all the existential premisses, without satisfying the conclusion. If there is, the inference is invalid. If there is not, the inference is valid.

The justification for this test is apparent. The overall list of circled descriptions represents all the descriptions which would be circled, were all the premisses of the argument transcribed in one large diagram. Therefore this list has exactly the same relation to the conclusion as does the list of premisses circled in any *single* diagram symbolizing the premisses of an argument.

Universal premisses: Validity of an argument with a universal conclusion of course requires one or more universal premisses in the argument. Consequently there will be an overall list of crossed descriptions, representing all descriptions which would be crossed in one large diagram in which all premisses were transcribed. The universal conclusion cannot be immediately tested against this list, however; for the test of validity is the absence of any crossed counter-instances to the conclusion, and the list of crossed descriptions allows us no way of *identifying all* the counter-instances so that we may know whether or not there are any which would remain uncrossed. Nor can we find all the possible counter-instances merely by examining the two diagrams.

The simplest way of solving the problem seems to be as follows: (1) Find the *number* of counter-instances to the conclusion. To do this, first find the number of descriptions, in terms *only* of the term-letters *in the conclusion*, which are counter-instances to the conclusion. "(x) $(Ax \cdot Bx)$", for example, has three such counter-instances, *viz.*, "A", "B", and "___". Then multiply this number by 2^n, where n is the number of term-letters involved in the entire argument which do *not* appear in the conclusion. This gives the total number of counter-instances. For example, if "(x) $(Ax \cdot Bx)$" were the conclusion to an argument with six terms, ("A", "B", "C", "D", "E", and "F"), then there would be 48 counter-instances in all—the three counter-instances solely in terms of "A" and "B", multiplied by 2^4, i.e. *16*. (2) Identify the counter-instances to the conclusion which appear on the overall list of crossed descriptions. Then count these counter-instances on the list. If there are *fewer* distinct counter-instances on the overall list of crossed descriptions than there are counter-instances *in toto*, then the inference is invalid. But if there are the *same* number of counter-instances on the list of crossed descriptions, as there are counter-instances *in toto*, then the inference is valid. (Of course, if we want to know *which* counter-instances remain uncrossed, we must follow a longer procedure: Identify all the counter-instances to the conclusion by first finding the counter-instances solely in terms of the letters in the conclusion, and then combining with each of these descriptions all possible combinations of the remaining term-letters; finally eliminating from this list all counter-instances which appear on the overall list of crossed descriptions.)

The justification for this procedure is, again, apparent. If there are more possible counter-instances to the conclusion than there are counter-instances which are crossed on the basis of the premisses, then there must by *some* counter-instance(s) which remain uncrossed even after the premisses are transcribed; and this defines the invalidity of the inference. Otherwise, the inference is valid.

V. *Inferences involving truth-functional compounds of quantifications.* With an extension of the basic procedure, possibility-diagrams may be used to test the validity of arguments involving premisses and/or conclusion which are truth-functional compounds having quantifications with monadic predicates as their components. The most convenient way of making such a test involves negating the conclusion of the argument and testing the conjunction formed by that negation, along with the various premisses, for consistency.

Overall Procedure. The overall procedure for testing such arguments consists of the following steps:

a. Negate the conclusion of the argument.

b. Transform all the truth-functionally compound sentences involved into conjunctions or disjunctions (thus eliminating the negations of quantifications, as well as all other forms except for conjunction and disjunction.)

c. Treat each conjunct in any conjunction as a separate sentence, and list the various sentences, as transformed above, which are to be the members of the whole conjunction whose consistency is to be tested. Each of these sentences will be either a quantification or a disjunction of quantifications.

The next steps consist of making lists of the requirements for transcribing the various sentences involved on to one possibility-diagram which involves all the predicate-letters in the argument. The numbers referred to will be the numbers of the descriptions in this diagram.

d. For sentences which are universal quantifications or disjunctions of universal quantifications: List the set of descriptions which would be required to be crossed in transcribing the quantification; or the alternative sets of descriptions, the members of one of which would have to be crossed, in the case of a disjunction of quantifications. E.g.:

"Cross *1, 3, 4, 7*" (for a single quantification)

or:

"Cross *1, 5, 6 or* cross *1, 6, 7, 8*" (for a disjunction)

e. For sentences which are existential quantifications or disjunctions of existential quantifications: List the various descriptions which would satisfy the one quantification, or which in the case of a disjunction would satisfy any one of the quantifications, as alternatives to be circled. E.g.:

"Circle *2 or 3 or 5 or 8*" (This may be put down as either the result of, e.g., transcribing one quantification which would be satisfied by *#2, 3, 5,* or *8*, or as a result of transcribing a disjunction of quantifications of which one would be satisfied by *#2* or *#3*, while the other would be satisfied by *#5* or *#8*.)

f. For sentences which are disjunctions of one or more universal quantifications with one or more existential quantifications: List the separate requirements for each disjunct as alternative requirements. E.g.:

"Cross *1, 5, 6 or* cross *1, 6, 7, 8 or* circle *2 or 3 or 5 or 8*" (The cross-requirements are listed in virtue of two universal quantifications in the disjunction, and the circle-requirements in virtue of one or more existential quantifications in the disjunction.)

g. Determine whether the premisses are consistent with the negation of the conclusion by determining the answer to the following questions: (1) Do the requirements, all together, for transcribing the various sentences necessitate crossing *all* descriptions in the diagram? (2) Is there some set of descriptions which fulfils all the circle-requirements without any of its members being included in a set of descriptions which must be crossed to fulfil the cross-requirements? In other words, can all requirements be fulfilled without crossing and circling the same description?

If the answer to (1) is "Yes", then the argument is valid (except in an empty universe, possibly) for a positive answer to (1) implies that the existence of *any* thing will falsify either one of the premisses or the negation of the conclusion. If the answer to (1) is "No", then (2) remains still to be answered.

If the answer to (2) is "Yes", then the argument is invalid (given, of course, that the answer to (1) is "No"), for this means that the premisses are consistent with the negation of the conclusion. Therefore, if the answer to (2) is "No", the argument is valid.

Only inspection may be required to determine the answers to these two questions. For more complicated cases, however, a definite method seems advisable (although the correct determination of validity through the correct answering of the two questions specified, does not turn on the use of any one particular method.) The following method seems the best.

Procedure for determining consistency, given requirements for transcribing the various sentences. Each of the requirements (i.e. the requirement for each of the various sentences involved) may be put into one of three categories: (1) *pure-cross*, requiring that one set of descriptions be crossed, or that one of alternative sets of descriptions be crossed; (2) *mixed* cross and circle, requiring that one set of descriptions be crossed (perhaps one out of alternative sets), or else that one description be circled (perhaps one out of a number of alternative descriptions); (3) *pure-circle*, requiring that one description be circled (perhaps one out of alternative descriptions.) E.g., the following are numbered according to the category in which they belong:

(1) "Cross *1, 2, 3 or* cross *2, 3, 6*

also

"Cross *4, 5, 8*"

(2) "Cross *3, 6, 7 or* circle *2, or 6 or 7*"

(3) "Circle *4 or 5 or 7 or 8.*"

First consider the pure-cross requirements. Determine which descriptions (if any) *must* necessarily be crossed according to these requirements. These will consist of all the members of any *single* set of descriptions the crossing of which is required by a certain sentence, as well as any descriptions shared by *all* the alternative sets of descriptions, one of which is required to be crossed in virtue of a certain sentence. E.g., in the case of the above two pure-cross requirements, the descriptions which *must* be crossed would be *4, 5, 8* (as a result of their being the members of a single set the crossing of which is required), *2* and *3* (as a result of their belonging to every one of the alternative sets of descriptions, one of which must be crossed.) If the list of descriptions which must be crossed includes *all* the descriptions which would make up the diagram concerned, then the answer to question (1), above, ("Do the requirements necessitate crossing all descriptions?") is "Yes", and the argument is valid; otherwise, the answer is so far "No". And if this list does *not* include all the descriptions in the diagram, then the descriptions which it does include must still be excluded from consideration as descriptions to be circled, in succeeding steps.

The next step is to list all distinct minimal sets of descriptions which, if crossed, would satisfy all the pure-cross requirements (if there are any pure-cross requirements). E.g., suppose the pure-cross requirements, *in toto*, are

(1) "Cross *1, 3, 4* *or* cross *2, 6*"

(2) "Cross *2, 5, 7* *or* cross *5, 6*"

Then the list of sets of descriptions fulfilling these would be as follows:

1, 2, 3, 4, 5, 7 (combining the first alternatives of (1) and (2).)

1, 3, 4, 5, 6 (combining the first alternative of (1) with the second alternative of (2).)

2, 5, 6, 7 (combining the second alternative of (1) with the first alternative of (2).)

2, 5, 6 (combining the second alternative of (1) with the second alternative of (2).)

Next, list all the possible sets of descriptions the circling of which will satisfy the pure-circle requirements (if there are any.) But eliminate from this list all sets of descriptions which (a) contain descriptions which *must* be crossed, according to some pure-cross requirement, or (b) contain *some* description(s) from *each* alternative cross-requirement required by some one sentence. For example, suppose the pure-circle requirements are:

(1) Circle *2* *or* *4* *or* *5*
(2) Circle *3* *or* *4*

and suppose the pure-cross requirements are those given as examples just above. Then the full original list of descriptions fulfilling the pure-circle requirements would be:

2,3	*4*	*2,3,4*
2,4	*4,5*	*3,4,5*
3,4	*2,3,5*	*2,3,4,5*
	2,4,5	

But from this list must be eliminated: (a) all sets containing description #5—since it is a description which *must* be crossed, as belonging to both of the alternative sets of descriptions in a pure-cross requirement (in fact, #5 may simply be excluded while making up the list), and (b) sets {2,3}, {2,4}, and {2,3,4}—since these each contain some description included in all of the alternative cross-requirements required by some sentence. The final list, then, will contain merely two sets: {3,4} and {4}.

Next, consider in turn each of the sets on this final list of eligible sets of descriptions. Whatever set is under consideration will be called "the set being checked." First determine whether any of the sets of descriptions on the list of those totally fulfilling the pure-cross requirements is *free of* any of the descriptions in the set being checked. If there *is* some set fulfilling the pure-cross requirements which is free of any description in the set being checked, then check the latter against each mixed requirement (if there are any) in turn. To do this, determine first whether the set of descriptions will fulfil the circle-requirements among the mixed requirement; if *so*, the check is successful, i.e. a way has been found in which the mixed requirement may be fulfilled, consistent with fulfilling the requirements already accounted for. But if such is *not* the case, then determine whether any cross-requirement which is an alternative to the circle-requirements (in the mixed requirement) is free of any description in the set being checked. If there is such a cross-requirement, then the check is still successful. On the other hand, if the set of descriptions being checked does not fulfil the circle requirement, and the alternative cross-requirement(s) include a description included in the set being checked, then the check is unsuccessful.

Taking each such set of descriptions in turn, check it against the pure-cross requirements and against each mixed requirement, until an unsuccessful check is encountered, or until the entire list of mixed requirements is gone through. If *one* such set of descriptions can be found which checks successfully with *all* mixed requirements (as well as with the pure-cross requirements), then the answer to the original question (2) is "Yes", the requirements are all consistent, and the argument is invalid. Otherwise—i.e., if no such set of descriptions can be found which checks successfully with all mixed requirements—then the answer is "No", and the argument is valid.

If there are no mixed requirements, then the procedure is modified simply by omitting the steps concerning mixed requirements; the argument will be invalid if there is merely a set of descriptions, fulfilling all pure-circle requirements, which shares no descriptions with some set which would fulfil all pure-circle requirements. If there are no pure-cross requirements, then the argument is assuredly invalid—for there is certain to be some set of descriptions which will fulfil all mixed and pure-circle requirements, and this in such a case is sufficient to allow the requirements to be consistent with one another and thus to allow the argument to be valid. Finally, if there are no pure-circle requirements, then, in place of the list of sets of descriptions which would fulfil the pure-circle requirements, one begins with the list of all sets of descriptions which would fulfil the circle-requirements among some or all of the mixed requirements; from this list are eliminated all which are made ineligible by the pure-cross requirements, and then each of the resulting sets is checked against each of the mixed requirements, as described above.

Sample test of validity. The following argument will be used as the sample argument:

(1) $(x)(Ax \supset Bx)$
(2) $(x)(Bx \cdot - Cx)\ v\ (\exists x)(Ax)$
(3) $(\exists x)(Bx) \supset (x)(Cx)$
(4) $(\exists x)(Bx\ v\ Cx)$

$\overline{}$

$(x)(Ax)\ v\ (x)(Bx)$

Predicate-letters involved: A, B, C

First, negate the conclusion. Eliminating negations of quantifications, the result is:

$$(\exists x) - (Ax) \cdot (\exists x) - (Bx)$$

Then transform any premiss—*viz.*, #3—which is not a disjunction or conjunction. The sentence into which #3 is transformed will be:

$$(x) - (Bx)\ v\ (x)(Cx)$$

Treating the two conjuncts which compose the negated conclusion as separate sentences, we have the following list of sentences to test for consistency:

1. $(x)(Ax \supset Bx)$
2. $(x)(Bx \cdot - Cx)\ v\ (\exists x)(Ax)$
3. $(x) - (Bx)\ v\ (x)(Cx)$
4. $(\exists x)(Bx\ v\ Cx)$
5. $(\exists x) - (Ax)$
6. $(\exists x) - (Bx)$

Since there are three predicate-letters involved (A, B, and C), the diagram involved will have eight descriptions, and will be as follows:

	A	*B*	*C*
1	2	3	4
AB	*AC*	*BC*	*ABC*
5	6	7	8

Following are the requirements for transcribing the above sentences on to this diagram. The particular requirements are numbered to correspond to their respective sentences.

1. Cross *2, 6*
2. Cross *1, 2, 4, 6, 7, 8 or* circle *2 or 5 or 6 or 8*
3. Cross *3, 5, 7, 8 or* cross *1, 2, 3, 5*
4. Circle *3 or 4 or 5 or 6 or 7 or 8*
5. Circle *1 or 3 or 4 or 7*
6. Circle *1 or 2 or 4 or 6*

(Nos. *1* and *3* are pure-cross requirements; no. *2* is a mixed requirement; nos. *4*, *5*, and *6* are pure-circle requirements.)

The following descriptions must be crossed, in virtue of belonging to the one set of descriptions which requirement *#1* requires to be crossed, or belonging to both alternatives in *#3*: descriptions *2, 3, 5, 6*.

It is *not* the case that the sentences are inconsistent by reason of necessity of having all descriptions crossed.

The minimal combinations which will fulfil the pure-cross requirements are as follows: *2, 3, 5, 6, 7, 8* and *1, 2, 3, 5, 6*. (The former is a combination of requirement *#1* with the first alternative of *#3*; the latter is a combination of requirement *#1* with the second alternative of *#3*.)

The combinations which will fulfil all pure-circle requirements are as follows (eliminating from consideration all descriptions which are among those necessarily crossed):

4,	*4,7*	*1,7,8*
1,4	*4,8*	*4,7,8*
1,7	*1,4,7*	*1,4,7,8*
1,8	*1,4,8*	

Of these, the following must be eliminated because each contains one description in either of the alternative sets to be crossed according to requirement *#3*: {*1,7*}, {*1,8*}, {*1,4,7*}, {*1,4,8*}, {*1,7,8*}, {*1,4,7,8*}. The following combinations remain to be considered:

4	*4,7*	*4,7,8*
1,4	*4,8*	

These sets of descriptions are taken up, in turn. It will happen, as a matter of fact, that all of these sets are such that all their component descriptions are excluded from one or the other of the combinations which will fulfil the pure-cross requirements. But the sets {*4*}, {*1,4*}, and {*4,7*} all fail because they do not fulfil the circle-requirement in the mixed requirement, while the cross-requirement which is the alternative to those circle-requirements does contain a description (*viz.*, *#4*, and *#1* in the one case) which is contained in the set in question. It is not until we get to set {*4,8*} that we find a combination of descriptions which is successful. This is successful because it fulfils the circle-requirement of the mixed requirement, in virtue of description *#8*. (Set {*4,7,8*} would be successful also, for the same reason.)

Since this one set of descriptions has successfully checked with the mixed requirement, as well as with the pure-cross requirements, the sentences to be tested are consistent with each other, which means that the original argument is invalid. More explicitly, the test has shown that all the sentences to be tested for consistency can be correctly transcribed, without inconsistency, if description *1*, *2*, *3*, *5* and *6* are crossed, while descriptions *4* and *8* (or *4*, *7*, and *8*) are circled.

Justification for this procedure. The justification for the validity of this overall procedure should be apparent. The argument to be tested is invalid if the negation of the conclusion is consistent with the premisses, and valid otherwise. A set of sentences is consistent if it is possible that all the sentences together be true. Consistency is thus indicated in a possibility-diagram by the symbolization of the conditions which make all the sentences in question true, while avoiding the symbolization of two impossibilities: crossing and circling the same description, and (given that the universe is non-empty) crossing every description. If there is a way in which this symbolization can be accomplished, the sentences in question are consistent and the argument is invalid; if there is no way in which this can be done, the sentences in question are inconsistent and the argument is valid.

The particular method described is designed to show whether the sentences in question can be symbolized without symbolizing the impossible. It consists therefore of a set of procedures which exhaust the possibilities for accomplishing that task, so that either a specific way of accomplishing it will be recognized, or the impossibility of accomplishing it will be recognized. Specifically, there is a determination of all possible ways in which the pure-circle requirements may be fulfilled; a determination of all possible ways in which the pure-cross requirements may be fulfilled; a comparison of the one against the other, in order to find whether there is some way of fulfilling both types of requirement without contradiction; and finally a comparison with the mixed requirements in order to find whether any way of fulfilling the pure-cross and pure-circle requirements together will also fulfil the mixed, still without contradiction (i.e., without symbolizing the impossible.) If any way is found for doing these things, the sentences are consistent and the argument is invalid; if all possible ways are found to be ineffective, then the argument is invalid. And in case the sentences in question do not fall into all three categories, a modified procedure is followed, to the same end.

University of Massachusetts
Amherst, Massachusetts
and
Somerville, Massachusetts