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SOLUTIONS OF FIVE MODAL PROBLEMS OF SOBOCIŃSKI

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In Sobociński's [1] five problems concerning independence are left open. These are here solved, acquaintance with [1] being presupposed.

(1) Does S3° contain *Z5*? No.
(2) Does S1$^+$ contain *G1* or *C11*? No.
(3) Does S3* contain *F5*? No.
(4) Is *L1* independent when S5 is axiomatised by {S3°, *W1*}? Yes.
(5) Is *M1* independent when S5 is axiomatised by {S4°, *W1*}? Yes.

In answering all five questions we use the matrix

K	*1*	*2*	*3*	*4*	*.N*
1	*1*	*2*	*3*	*4*	*4*
2	*2*	*2*	*4*	*4*	*3*
3	*3*	*4*	*3*	*4*	*2*
4	*4*	*4*	*4*	*4*	*1*

Ad (1). Designate the value *1* and take $M(1234) = (3334)$. The four rules and the axioms of S3° are satisfied but *Z5* $p/1$ gives $NKNM1NN1 = NKN31 = NK21 = N2 = 3$.

Ad (2). Designate the value *1* and take $M(1234) = (1314)$. The four rules and the axioms of S1$^+$ are satisfied but *G1* $p/2$ gives $NMK2NM2 = NMK2N3 = NMK22 = N3 = 2$, while *C11* (*V1*) $p/2$ gives $NMKM2NNMNM2 = NMK3MN3 = NMK33 = NM3 = N1 = 4$.

Ad (3). Designate the values *1* and *2*, and take $M(1234) = (1224)$. *Z1*-*Z4* always obtain the value *1*, *Z5* the values *1* or *2*, and the two rules are satisfied. But *F5* $p/1$, $q/3$, $r/4$ gives $NMKKNMK1N3NMK3N4NNMK1N4 = NMKKNM2NM3M1 = NMKKN2N21 = NMK31 = NM3 = N2 = 3$.

Ad (4) and (5). The bases for S5, {S3°, *W1*} and {S4°, *W1*}, can be expressed as {S1$^+$, *L1*} and {S1$^+$, *M1*}. The matrix used *ad* (2) satisfies S1$^+$ but *L1* $p/2$, $q/4$ takes the value *2*, as does *M1* $p/2$. (*L1* was misprinted in [1] and should end with *KMpNMq* rather than *KMpMq*.)

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The matrix used *ad* (2) shows also that [2] was in error in stating (7.542) that S2° contains $\mathfrak{C}\mathfrak{C}p\mathfrak{C}qr\mathfrak{C}Kpqr$. This is not even in S1^{+} now, as shown by the matrix used *ad* (1), in S3°.

The following facts about S3* may be of interest. All of theorems 1-30 in Simons's [3] hold for S3* except 11, 25, 26, 30. 11 and 30 are disproved by our matrix *ad* (3). 26 (3) is provable and with 25 would yield 26. But 26 is *F5*, disproved; so *25* is not provable. That the remainder, and 26 (3), can be proved is clear from the proofs in [3]. Our matrix also rejects $\mathfrak{C}KLpLqLKpq$.

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