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WHAT PEIRCE MEANS BY LEADING PRINCIPLES

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C. S. Peirce contributed an article on what he called 'Leading Principle' to Baldwin's *Dictionary of Philosophy and Psychology* of 1901 (reprinted in 2. 588-589)*. He had earlier, in 1880, developed the notion in an article on the algebra of logic for the *American Journal of Mathematics* (reprinted in 3. 154-251). The *Collected Papers* contain several other places in which he treats the same subject. In this note I propose to analyze what he means by a 'leading principle' and to indicate the mediaeval antecedents for the major division that he makes of it.

Peirce introduces the notion of a leading principle to make clear the nature of inference. Representing the general type of inference from premiss to conclusion by the form, '$P \therefore C$', he claims that this passage "takes place according to a habit or rule" (3. 162-163). It is this habit or rule that he calls a 'leading principle'. It is logically good, he goes on to say, "provided it would never (or in case of a probable inference, seldom) lead from a true premiss to a false conclusion". From this it would appear that by a 'leading principle' Peirce means no more than what would now usually be referred to as a 'rule of inference'.

Yet that he does mean something more than this would seem to be indicated by his going on to speak of using the leading principle as a premiss for a new inference (3. 164). Strictly speaking, a rule of inference cannot be a premiss for an inference. Although both have a function as logical statements, they belong, as it were, to different orders. A premiss is a statement *from which* conclusions are drawn and is expressed within a logical system in the object-language of that system. A rule of inference, on the other hand, is a rule-statement *in accord with which* conclusions are drawn and has to be expressed in the meta-language of the system; it is incapable of expression in the object-language of the system because it is a directive stating how the operation of inferring is to be performed within the system; it is a matter of choice.

*Here and henceforth I shall quote Peirce by volume number and paragraph of the *Collected Papers*, full bibliographical data for which is given in the references at the end of this paper.

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However, although a rule of inference may be of our choosing, the choice is not entirely arbitrary. To be logically good, as Peirce noted, it must not lead from a true premiss to a false conclusion. What is needed for this condition to be met is readily stated, as Lewis has shown (p. 242). Using his notation (and fore-going the refinement of single-quotes or corners around the statement variables), we write

(1) '*pIq*' for '*p implies q, in any kind of implication*'

An implication of any kind, material, formal, strict, intensional, or any other, will provide a satisfactory basis for inference provided only that

(2) '(*p and pIq*) *I q*' is a tautology

If this is a tautology, i.e. always true no matter what truth-value the components have (and hence it makes no difference whether or not *pIq* is a tautology), it is impossible at one and the same time for the main antecedent to be true and the main consequent false. Since this is precisely the definition of the Lewis functor of strict implication, we may re-write (2) in the form

(2*) (*p and pIq*) $\prec$ *q*

Once we have this we have met the condition required for a good rule of inference, and *pIq* may be taken as the basis for such a rule, which may be formulated in the usual fashion, e.g.

(3) *If expressions of the form α and α I β both occur as asserted within the system, then an expression of the form β may also be asserted.*

Returning to Peirce, we now may ask whether by a 'leading principle' he means (1), (2), or (3), or now one and now another, or something else entirely. As we have already noted, he sometimes talks as though he were thinking of a leading principle as a rule, i.e. of (3). At other times he seems to be thinking primarily of (1) or (2). In his most formal statement on leading principles that appears in the published work, he writes: "We not only require the form P .·. C to express an argument, but also the form $P_i \prec C_i$ to express the truth of its leading principle. Here P_i denotes any one of the class of premisses, and C_i the corresponding conclusion. The symbol $\prec$ is the copula and signifies primarily that every state of things in which a proposition of the class P_i is true is a state of thing in which the corresponding propositions of the class C_i are true" (3. 165). This "copula" obviously expresses the modal condition: it is impossible to have P and not C. Thus on this reading a leading principle approximates to our (2) since it lays down a modal condition.

However, Peirce then proceeds to write: "This principle contains all that is necessary besides the premiss P to justify the conclusion. . . . We may therefore construct a new argument which shall have for its premisses the two propositions P and $Pi \prec C_i$ taken together, and for its conclusion,

C. This argument, no doubt, has, like every other, its leading principle, because the inference is governed by some habit; but yet the substance of the leading principle must already be contained implicitly in the premisses, because the proposition $P_i \prec C_i$ contains by hypothesis all that is requisite to justify the inference of C from P. Such a leading principle, which contains no fact not implied or observable in the premisses, is termed a *logical* principle, and the argument it governs is termed a *complete*, in contradistinction to an *incomplete* argument" (3. 166). This distinction is also used to distinguish *logical* from *extralogical* validity, the complete argument being based on a logical principle which "is said to be an *empty* or merely formal proposition, because it can add nothing to the premisses of the argument it governs, although it is relevant" (3. 168).

Here we obviously have something different from (2). As Lewis has pointed out (p. 242, 259), (2) characterizes all rules of inference, whereas Peirce is here allowing for different kinds of leading principles and distinguishing among them. That his leading principle now approximates to our (1) becomes clearer on closer consideration of his division of it and its mediaeval antecedents.

Elsewhere Peirce claims that the principle is "most conveniently expressed in the nomenclature of the mediaeval logicians" and thereupon calls the premiss *antecedent*, the conclusion *consequent*, and "the leading principle that every. . . such antecedent is followed by such a consequent. . . the *consequence*" (2. 669). Further, in the Baldwin *Dictionary* article, he notes that the two classes of leading principles are called *formal* and *material*, although he finds this less appropriate than *logical* and *factual* respectively (2. 589); these obviously correspond to the distinction between *logical* and *extralogical* made above.

The division of consequences into formal and material is the primary distinction made in the edition of Scotus that Peirce had in his library. A *formal consequence* is there defined as "one that holds for all terms when there is a similar disposition and form of the terms". A *material consequence* is "one which does not hold for all terms retaining a similar disposition and form but with a variation of the terms" (*Quaestiones super lib. Priorum Analyticorum Aristotelis*, I.x, p. 276v-277r).

It is this distinction, I would suggest, that Peirce takes over in the Baldwin article and renders in his own fashion in the words: "Any leading principle whose truth is implied in the premisses of every inference which it governs is called a *logical* (or, less appropriately, a *formal*) principle; while a leading principle whose truth is not implied in the premisses is called a *factual* (or *material*) leading principle (2. 589). In fact, the language of the mediaeval logician is clearer than that of Peirce. Suppose Peirce to be talking of an implication that may serve as the basis for an inference-rule, i.e. our (1) above. Then in saying that the truth of a factual leading principle is "not implied in the premisses", he is talking about logical truth and noting that there are terms in which the given implication will not hold even though the same disposition and form is kept. A logical or formal leading principle, on the other hand, is said to be "an empty or merely formal proposition", i.e. it is a tautology, or a logical law and hence for Peirce to say that its "truth is implied in the premisses of every inference which it governs" is but to note in the language of the mediaeval logician that it is an implication of such form as to "hold for all terms that keep the same form". A factual leading principle is moreover called *incomplete*, because, if it is valid, it is capable by the addition of a premiss of becoming a complete, i.e. a logical or tautologous proposition. This is only to say that when (1) is not a tautology it can become one as in (2), in which a premiss has been added.

It thus appears from the division that Peirce makes that it is best to take his leading principle as (1), i.e. as an implication capable of providing the basis for an inference-rule. If it is a tautology or logical truth, it constitutes his logical leading principle. If it is not, it is his factual leading principle, and this when valid can be completed by the addition of another premiss so as to form a tautology. Such factual or material leading principles constitute what the mediaeval logicians knew as *Topical maxims*, which I have analysed in previous numbers of this journal.

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[3] Duns Scotus: *In Universam Aristotelis Logicam Exactissimae Quaestiones*, edit. Constantius Sarnano, Venice, 1591. The *Questions on the Prior Analytics* are now known to be spurious and are usually quoted as by the Pseudo-Scot. Peirce's copy of this edition is in the Peirce Collection in the library of Johns Hopkins University.

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[5] O. Bird: "Topic and Consequence in Ockham's logic", same journal, 2 (1961), 65-78.

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