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A STUDY IN BURLEIGH: TRACTATUS DE REGULIS GENERALIBUS CONSEQUENTIARUM

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There is perhaps no other prominent logician in the late Middle Ages who would realize the nature, the scope, and the importance of propositional logic better than Walter Burleigh (1275-1345). Not only the content, but the very arrangement of his tracts shows that his conception of logic is entirely different from that of the commentators on Aristotle's *Organon.* He placed the tract on consequences at the beginning of his logical treatise, and his tract *contains* the syllogistic rules as a very minor part. This is significant, for many of Burleigh's contemporaries, such as Ralph Strodus, William of Ockham, John Buridan, William of Shyreswood, and Albert of Saxony, may have written lengthier tracts on consequences, but placed them after the tract on categorical syllogistic or even appended them toward the end of their *Summae*. Burleigh seems to have realized that we may and indeed must consider the relations among unanalyzed propositions prior to considering the relations among analyzed propositions constituting a syllogism.

By *consequentia* a conditional proposition is meant, the antecedent and the consequent of which may themselves be complex. Like most logicians of his time, Burleigh distinguished several types of them: those that hold in virtue of an extrinsic means or logical rules and those that hold in virtue of an intrinsic means; formal and material; and absolute and factual.[1] These divisions are not mutually exclusive.

This paper is concerned only with the last mentioned division of consequences. An attempt is made to present Burleigh's views in the language of contemporary logic. Wherever modalities are *not* in question, I propose to utilize the notation of the system of material implication. This procedure may have its objections, since the examples illustrating several of the rules given by Burleigh seem to indicate that he had in mind formal connections between propositions such as found between a premiss-set and the conclusion of a categorical syllogism; yet, unless words such as 'cannot', 'must', 'may', etc., cannot be shown to have a modal function, non-modal symbolism will be employed, with the following proviso: that, depending

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on the context, *Cpq* may be interpreted either as *NKpNq* or as $\sim \Diamond(p \, . \sim q)$; *Apq* either as *NKNpNq* or as *KNKNpNqNKpq*; and *Kpq* either as *NANpNq* or as *poq*.

Two editions of *De puritate artis logicae* are available now:[2] a shorter one, *Tractatus brevior* (from now on referred to as TB) and a longer one, *Tractatus longior* (from now on referred to as TL), published by the Franciscan Institute under editorship of Father Boehner. Both of these tracts seem to be incomplete versions of a single *summa* on logic which the realist Burleigh may have attempted to write in order to purify the logic of the "nominalist" Ockham.[3]

TB begins abruptly with a paragraph distinguishing *simplex* and *ut nunc* consequences: "That consequence is simple (or absolute) which holds for all times, as: 'Man is running, therefore animal is running'; while factual consequence holds for a certain time and not always, as: 'Every man is running, therefore Socrates is running'; for this consequence does not hold always, but only so long as Socrates is man."[4] Then he announces that there are ten major rules which he procedes to state and to discuss. TL, on the other hand, lists only five principal rules, although the corollaries to these are even more numerous than in the TB. Also, Burleigh opposes here a simple consequence not only to the factual but also to a compound one: "Simple consequence, as distinguished from the compound, is that which consists of two categorical propositions, [while] a compound consequence [consists] of two hypothetical propositions or of a hypothetical and a categorical one. Hence, in some conditional propositions both the antecedent and the consequent are hypothetical [e.g., *CCpqCNqNp*; *CApqAqp*]; in others, the antecedent is hypothetical and the consequent categorical [e.g., *CKpqp*], and in still others just conversely, namely, the antecedent being a categorical and the consequent a hypothetical proposition [*CpApq*]."[5]

In the TL Burleigh also subdivides simple consequence itself:

> A [simple] consequence is natural when the antecedent includes the consequent; and such consequence holds in virtue of intrinsic means. A [simple] consequence is accidental if it holds in virtue of an extrinsic means, and this happens when the antecedent does not include the consequent, but is valid because of some extrinsic rule, as: 'If man is an ass, you are sitting'; this consequence is valid and it holds because of this rule: From the impossible anything may follow.[6]

Let us now list a number of principal rules and their corollaries—in the form of theses or laws—rather than in the form of rules, together with the Latin text and translation, and the instances exemplifying the rules, if given by Burleigh. Since the order of rules in the two tracts differs, the numbering of them for the sake of cross-reference will be imposed arbitrarily.

1.00 $p \dashv q \, . \dashv . \sim \Diamond (p \, . \sim q)$

In omni consequentia bona simplici antecedens non potest esse verum sine consequente (In a valid simple consequence the antecedent cannot be true without the consequent).[7]

The implication defined in the statement of this first principal rule (it is given as the first in both TB and TL) is absolute or simple and 'cannot' is therefore interpreted as a modal functor. The *ut nunc* or factual consequence defined above, however, would seem to correspond to a merely material implication:

1.10 *CCpqNKpNq*

Consequentia ut nunc tenet pro tempore determinato et non semper (The factual consequence holds only for a certain time and not always).[8]

While Burleigh explicitly admitted that from an impossible proposition any other may follow:

1.20 $\sim \Diamond p \dashv q$

Ex impossibili sequitur quodlibet (From the impossible anything follows),[9]

he nowhere states that from a factually false proposition any other follows [i.e., *CNpCpq*] or that a factually true proposition is implied by any other proposition [i.e., *CqCpq*]. His characterization of accidental consequence, however, is too cursory to form a definite opinion as to what sorts of logical rules may constitute the "extrinsic means" in virtue of which a consequence may hold. With a single exception, all the instances he gives are instances of consequence where there is a conceptual connection between the antecedent and the consequent. E.g., 'Every man is running, therefore Socrates is running'; introducing the implied premiss 'Socrates is a man' yields a necessary consequence, for '$[(x)(Mx \supset Rx) \,.\, Ms] \supset Rs$' is an analitically true expression.

In the TL, further discussion of the nature of *consequentia* is given: ". . . if in a certain case the antecedent could be (*possit*) true without the consequent, the consequence is not valid":[10]

1.30 $\Diamond (p \,.\sim q) \,.\dashv .\sim (p \dashv q)$

And, "in a factual consequence the antecedent cannot (*non potest*) as of now be true without the consequent."[11] He adds that "this rule is based on the fact that from truth never follows falsehood":[12]

1.31 *CKpNqNCpq*

It is then sufficient to prove *any* consequence invalid simply by showing that *KpNq* is true. To show that a consequence is valid, however, it *may* be that *NKpNq* proves a factual consequence; but it is not sufficient to show that an absolute consequence is valid: for while $p \dashv q$ does entail *NKpNq*, the latter does not entail the former.

From *1.00*, Burleigh claims, follow two other rules:

1.40 $\Diamond p \,.\sim \Diamond q : \dashv .\sim (p \dashv q)$

Ex contingenti non sequitur impossibile in consequentia simplici (In a simple consequence the impossible does not follow from the contingent).[13]

1.41 $\sim \Diamond \sim p \,.\, \Diamond \sim q : \prec . \sim (p \prec q)$

Ex necessariis non sequitur contingens (From the necessary does not follow contingent).[14]

The justification he gives is that "the contingent may be true without the impossible, and the necessary without the contingent."[15] No examples illustrate this explanation. 1.40 says that if a given statement is not logically false, then any statement which it implies is not logically false either. 1.41 states that if a given statement is logically true, then any statement which it logically implies is also logically true. We could add to the former that if the consequent were impossible, the antecedent would be impossible [$p \prec q \,.\prec.\, \sim \Diamond q \prec \sim \Diamond p$; cf. Lewis *18.5*]; and to the latter that if the consequent is not necessary, the antecedent is not necessary [$p \prec q \,.\prec.\, \Diamond \sim q \prec \Diamond \sim p$; cf. also Lewis *18.52*].

2.00 *CCpqCCqrCpr*

Quidquid sequitur ad consequens sequitur ad antecedens (Whatever follows from the consequent follows from the antecedent).[16]

2.10 *CCpqCCrpCrq*

Quidquid antecedit ad antecedens, et ad consequens (Whatever implies the antecedent implies the consequent).[17]

We are warned not to confuse the above two rules with two others which are invalid: *CCpqCCprCqr* (*Quidquid sequitur ad antecedens, et ad consequens*); and *CCpqCCrqCrp* (*Quidquid antecedit ad consequens, et ad antecedens*). In virtue of *2.00* the consequence known as *de primo ad ultimum* holds, viz.:

2.20 *CKKCpqCqrCrsCps,*

provided that in the chain of conditionals the consequent in the preceding conditional is exactly the same as the antecedent of the next conditional. Burleigh's example: "If man exists, animal exists; if animal exists, body exists; if body exists, substance exists; thus, *de primo ad ultimum*, if man exists, substance exists."[18] But if the specifications of such a consequence are not met, we may get involved into an argument such as this: the larger something is, from the greater distance it is seen; and from the greater distance it is seen, the less it is seen; therefore, *a primo ad ultimum*, the larger something is, the less it is seen. Burleigh points out that in this chain of conditionals the last consequent is not the same as the following antecedent; for the Latin tongue utilizes adverbs *tanto* in the former and *quanto* in the latter, and this is sufficient to spoil the inference. For the same reason it is not legitimate to argue: "the uglier you are, the more you adorn yourself, and the more you adorn yourself, the more beautiful you are, therefore . . ."[19]

Applications of 2.00 cited are: (1) "If a man is running, an animal is running; therefore, if Socrates is running, an animal is running;" (2) "If all men are running, Socrates is running, therefore, if all animals are running, Socrates is running."[20] It seems that Burleigh was misled here by his logic of classes. From "All *a* is *b*" we can certainly argue to "This *a* is *b*"; but from the fact that *p* implies *q* we cannot argue that something, say *r*, which is implied by *p*, implies *q*.

One of the several corollaries of *2.00* is the following:

2.30 *CCpqCCKpqrCpr*

> *Quidquid sequitur ex antecedente et consequente, sequitur ex antecedente per se* (Whatever follows from both the antecedent and the consequent follows from the antecedent by itself).[21]

Burleigh justifies the above rule thus: "Every proposition infers itself along with its consequent [*CCpqCpKpq*]. For example, it follows: 'Socrates is running, therefore Socrates is running and a man is running'; since therefore the antecedent infers both itself and the consequent and since whatever follows from the consequent follows from the antecedent [*CCpqCCqrCpr*, by *2.00*], it follows that whatever follows from both the antecedent and the consequent, follows from the antecedent alone."[22]

On the basis of the first statement we may add to the collection of theses expressing consequential rules:

2.31 *CCpqCpKpq*

> *Quaelibet propositio infert seipsam cum suo consequente* (Every proposition infers itself along with its consequent).

Another corollary of *2.00* expresses one of the laws of factorization:

2.40 *CCpqCCKqrsCKprs*

> *Quidquid sequitur ad consequens cum aliquo addito, sequitur ad antecedens cum eodem addito* (Whatever follows from the consequent with a proposition added follows from the antecedent with the same proposition added).[23]

Explanation given by Burleigh: "For it follows: 'Socrates is running and you are sitting; therefore a man is running and you are sitting.' Since, therefore, whatever follows from the consequent follows from the antecedent, it is necessary that whatever follows from the consequent with some proposition added, follows also from the antecedent with the same proposition added."[24]

That the above two laws are logically true can be shown schematically by modern methods of conditional proof:

CCpqCCKpqrCpr

(1)	*Cpq*	Assumptions, C.P.
(2)	*CKpqr*	
(3)	*p*	
(4)	*q*	1,3 M.P
(5)	*Kpq*	3,4 Conj.

(6)	*r*	2,5 M.P.
(7)	*Cpr*	3-6 C.P.
(8)	*CCKpqrCpr*	2-7 C.P.
(9)	*CCpqCCKpqrCpr*	1-8 C.P.

CCpqCCKqrsCKprs

(1)	*Cpq*	Assumptions, C.P.
(2)	*CKqrs*	
(3)	*Kpr*	
(4)	*p*	3, Simpl.
(5)	*q*	1,4 M.P.
(6)	*r*	3, Simpl.
(7)	*Kqr*	5,6 Conj.
(8)	*s*	2,7 M.P.
(9)	*CKprs*	3-8 C.P.
(10)	*CCKqrsCKprs*	2-9 C.P.
(11)	*CCpqCCKqrsCKprs*	1-10 C.P.

A simpler (in the sense of utilizing fewer logical elements) method of demonstrating *2.40* (and other rules) is utilized by A. N. Prior.[25] Following Łukasiewicz he assumes only the rule of substitution and the rule of detachment and shows that if we assert *2.00* and *2.31* we must also assert *2.30*:

1. *CCpqCCqrCpr.*
2. *CCpqCpKpq.*
   1 p/Kpq = 3.
3. *CCpKpqCCKpqrCpr.*
   1 p/Cpq, q/CpKpq, r/CCKpqrCpr = C2 – C3 – 4.
4. *CCpqCCKpqrCpr.*

While this proof may be said to have been suggested by the paragraph quoted in connection with *2.30*, we must not think that Burleigh invented a methodic way of utilizing the substitution rule to deduce every subsequent thesis on the basis of the previous ones. Nor is it claimed, of course, that the conditional proofs given above were constructed (even in a verbal form) by Burleigh himself; yet one might wish to check the validity of the theses and perhaps wonder at the accuracy, in face of the absence of a symbolic language, of the medieval logician.

Burleigh proceeds to examine four sophisms arising from the misapplication of *2.40*. I will give only the examination of one, the second one, –and not for the sake of scrutinizing the fallacy, but in order to point out another rule *implicitly* utilized by Burleigh. Consider the following passage: "For this disjunctive proposition: 'Socrates is running or else he is not running,' follows from this one: 'Socrates is not running.' And yet something follows from this disjunctive proposition which does not follow from 'Socrates is running' . . ."[26] While he questions the sophism based on the disjunctive premiss, he does not question the way this disjunctive premiss was derived. We may add, then, to his collection of rules another one that corresponds to the thesis

2.50 *CpApq*,

which is exemplified by a statement of the form *CNpAqNp* quoted above.

The third principal rule listed in the TB (and as a corollary to another rule in the TL) is the following:

3.00 *CCpqCNqNp*

> *In omni consequentia bona, quae non est syllogistica, ex opposito consequentis contradictorie sequitur oppositum antecedentis* (In every valid non-syllogistic consequence the contradictory opposite of the antecedent follows from the contradictory opposite of the consequent).[27]

Burleigh establishes it by the *reductio ad absurdum* method:

> If from the contradictory opposite of the consequent, the opposite of the antecedent did not follow [*NCNqNp*], then the antecedent would be consistent with (*staret cum*) the contradictory of the consequent [*KpNq*]; for if one opposite does not follow, the remaining one stands. For whatever is consistent with the antecedent is consistent with the consequent. Therefore, if the contradictory of the consequent were consistent with the antecedent, it would follow that the contradictory of the antecedent would be consistent with the consequent, and thus the contradictories would stand together–which is impossible.[28]

This justification of *3.00* certainly resembles the *reductio ad absurdum* and the indirect proofs of such a thesis in our text-books (note especially steps (2) and (4) and compare them with the first two sentences of the text):

(1) *Cpq*	*CNqNp*	
(2) *NCNqNp*	Assumption	
(3) *KNqNNp*	2,	Denial of Implication
(4) *KNqp*	3,	Double Negation
(5) *Nq*	4,	Simplification
(6) *p*	4,	Simplification
(7) *q*	1,6,	Modus Ponens
(8) *AqCNqNp*	7,	Addition
(9) *CNqNp*	8,5,	Disjunctive Syllogism

In the TL, a lengthier discussion of *3.00*, or a special case of it, and its converse, is found:

> This rule has to be understood in the enthymematic consequence, as when it follows: 'Man is running, therefore animal is running', and so from the contradictory of the consequent follows the contradictory of the antecedent; for it follows: 'No animal is running, therefore no man is running.' Nor does it suffice for the validity of the consequence that from the opposite of the consequent the contrary of the antecedent follows; because if this were so, it would follow: 'Every man is running, therefore every animal is

running', because from the opposite of the consequent the contrary of the antecedent follows. And whatever opposite of the antecedent, either contrary or any other, follows from the contradictory of the consequent, it suffices for the validity of the consequence, because if any other opposite of the antecedent follows, it is necessary that the contradictory of the antecedent follows, for every opposite to anything, of whatever sort be its opposite, implies the contradictory of the same."[29]

We have here a consideration of consequences involving analyzed propositions. The following theses are proposed as valid:

3.10 *CCAabAcbCNAcbNAab*
3.11 *CCAabAcbCEcbNAab*
3.12 *CCAabAcbCOcbOab*
3.13 *CCAabAcbCEcbOab*
3.14 *CCAabAcbCOcbOab*,

while

CCAabAcbCEcbEab
CCAabAcbCOcbEab

are rejected as invalid. If we should have a case where *CCAabAcbCEcbEab* does hold (on non-formal grounds), we could deduce from the contrary of the antecedent (*Eab*) the contradictory of it (*Oab*) by the *dictum de nullo*.

In the TB we find a clear statement of the converse of *3.00*, viz.

3.30 *CCNqNpCpq*

Si ex opposito consequentis contradictorie sequitur oppositum antecedentis, tunc prima consequentia fuit bona (If from the opposite of the consequent the contradictory of the antecedent follows then the original consequence is valid).[30]

3.00 and *3.30* in conjunction yield

3.40 *ECpqNqNp*,

although it must be stressed that the last thesis holds only if we do not interpret the consequence as a strict implication.

Concerning syllogistic consequence *CKpqr* (whose validity depends on the internal structure of propositions involved), Burleigh states rules corresponding to the following theses:

3.50 *CCKpqrCKqNrNp*
3.51 *CCKpqrCKpNrNq*
3.52 *CCKpqrCNrANpNq*

Ex opposito conclusionis cum altera praemissarum sequitur oppositum alterius praemissae (From the [contradictory] opposite of the conclusion and either of the premisses follows the [contradictory] opposite of the other premiss).[31]

The converse of this rule is also given:

3.60 *CCKNrApqANpNqCKpqr*
3.61 *CCKNrpNqCKpqr*
3.62 *CCKNrqNpCKpqr;*

from *3.50* - *3.62* we get an equivalence

3.63 *ECKNrApqANpNqCKpqr*

Si ex opposito conclusionis cum una praemissarum vel altera sequitur oppositum alterius, tunc primus syllogismus fuit bonus (If from the contradictory of the conclusion together with one of the premisses the contradictory of the remaining premiss follows, the original syllogism is valid).[32]

The passage in which a defense of *3.00* is given is rich in logical elements which Burleigh at least implicitly admitted: (a) the indirect argumentation; (b) the rule that "if one opposite does not follow, the remaining one stands"; (c) the rule that "whatever is consistent with the antecedent is consistent with the consequent"; (d) the rule that the contradictories cannot stand together. We formulate, accordingly, these additional theses:

3.70 *CKApNpNpNp*
3.71 *CKApNppp*
3.80 *CCpqCKprKqr*
3.81 *CKCpqCprCqr*
3.90 $\sim\Diamond(p \cdot \sim p)$

As a matter of fact, we find *3.80* in the TL explicitly stated as the fourth principal rule: "Quidquid stat cum antecedente stat cum consequente." We also find there a clarification of Burleigh's notion of 'consistency'; 'being consistent with something' means to him 'be capable of being true with it'. And the justification for this rule is: ". . . if the antecedent is true, the consequent is true. Hence, whatever can be true while the antecedent is true can be true while the consequent is true."[33]

The next principal rule (the fourth in the TB) concerns the denial: *Formale affirmativum debet negari in reliquo* (That which is formally affirmative in one of the contradictories must be negated in the other).[34] Every proposition has a certain determinate form: it may be *de inesse* or modal; simple or compound; causal, reduplicative, etc.; conditional, copulative, disjunctive, or conjunctive; universal, particular, or singular; and so on, according to the various bases of division. And to every affirmation there is a single denial. The form of a *de inesse* proposition is determined by the copulating verb, hence that verb cannot remain of the same quality in the two contradictory propositions concerning the same subject. In modal propositions, the mode is principal, the dictum is secondary; thus, contradictories are formed by adding negation to the mode rather than the dictum; the contradictory of $\sim\Diamond\sim p$ is $\Diamond\sim p$ rather than $\sim\Diamond p$. Similarly, in copulative propositions there is one contradictory to *Kpq* and not two, i.e., *NKpq* rather than *Np* and *Nq*. The same holds for disjunctives, conditionals, reduplicatives, and others: in every case the negation should be applied to that which makes a given proposition the kind of proposition it is.

Burleigh gives the following equivalences (aequipollentia):

4.00 *ENKpqANpNq*

Contradictorium copulativae valet unam disjunctivan habentem partes contradicentes partibus copulativae (Contradictory of a conjunctive proposition is equivalent to a disjunction of the same components sublated).[35]

For instance: The contradictory of 'Socrates is running and Plato is running' is 'Either Socrates is not running or Plato is not running (or neither).' A add 'or neither' because Burleigh has definitely in mind a non-exclusive disjunction, otherwise he could not give the following:

4.10 *ENApqKNpNq*

Contradictorium disjunctivae aequipollet copulativae factae ex contradictoriis partium disiunctivae (The contradictory of a disjunctive is equivalent to a conjunctive proposition composed of the contradictories of each of the components of the original).[36]

The example given is: 'Socrates is running or Plato is running' is equivalent to 'Neither Socrates nor Plato is running.' We have here, then, precursors of the laws of duality which later came to be known as De Morgan Laws,[37] except that *4.00* and *4.10* apply to proportions rather than to classes.

4.20 *ENCpqKpNq*

Contradictoria conditionalis valet unam propositionem quae significat oppositum sui consequentis stare cum suo antecedente (Contradictory of a conditional proposition is equivalent to a proposition signifying the consistency of the contradictory of its consequent with its antecedent).[38]

"The contradictory of 'If Socrates is running, man is running' is 'It is not the case that if Socrates is running, man is running'; to this proposition the following is equivalent: 'The following are compatible: Socrates is running and no man is running.'"[39]

4.30 *Contradictoria propositionis reduplicativae habet duas causas veritatis: potest enim contradictoria propositionis reduplicativae esse vera, vel quia consequens non sequitur ad antecedens, aut quia antecedens non est causa consequentis* (Contradictory of a reduplicative proposition has two causes of truth: for the contradictory of a reduplicative proposition could be true either because the consequent does not follow from the antecedent or because the antecedent is not the cause of the consequent).[40]

EN(pREq) ANKpqNCpq

Burleigh explains: "The contradictory of 'Inasmuch as you are an ass, you are an animal' is 'Not: inasmuch as you are an ass, you are an animal'; and this proposition has two causes of truth: either because it does not follow that if you are an ass you are an animal, or because proposition 'You are an ass' is not the cause of the truth of 'You are an animal.'"[41] Thus writing "*RE*" between '*p*' and '*q*' to indicate that the truth of the first is the cause of the truth of the second proposition, we can show schematically that the denial of reduplicative proposition amounts to the affirmation of a disjunction of *NKpq* and *NCpq*.

From *pREq* we may infer *Kpq* as well as *Cpq*; but the converse relations do not hold. It must also be noted that both of these two reduplicative propositions are false: 'Inasmuch as you are a man, you are an ass,' and 'Inasmuch you are a man, you are not an ass.' *pREq* and *pRENq* are not contradictories because they both affirm reduplication, the *formale* which alone must be negated if we are to arrive at any contradictory opposition.

In the TB we find a related and perhaps more general rule:

4.40 *A propositione habente pluras causas veritatis ad unam illarium non tenet consequentia* (From a proposition which has several causes of truth the inference to one of those is not valid).[42]

Both *CN(pREq)NKpq* and *CN(pREq)NCpq* would thus be invalid. But Burleigh seems to consider all negative propositions to have two causes of truth and consequently two causes of falsehood: either because the contradictory (or contrary) affirmative proposition is true or because the subject does not exist. From 'Socrates is not ill it does not follow that Socrates is well.' "For the proposition 'Socrates is not ill' has two causes of truth, namely these: Socrates does not exist and consequently is not ill, and: Socrates is well."[43] If we infer either of the two we commit the fallacy of the consequent.

There is no indication that Burleigh would consider all universal propositions to be disguised conditionals without existential import (unless special assumptions concerning the existence of the subject be added to them). Yet holding the view he did concerning negative propositions would make his square of opposition look very different from the one which is usually presented as the "traditional" square.

In connection with 4.50 we find a discussion of the view that from a *purely* negative proposition never follows an affirmative one. He agrees that the consequence: 'Socrates is not ill, hence Socrates is well' is not valid; while "Socrates is not ill, and Socrates exists, therefore Socrates is well" is valid. Nevertheless, there is a sense in which an affirmative proposition follows from a negative one: "'Some proposition is true' is an affirmative proposition, and yet this very one follows from any negative proposition inasmuch as it is negative; for it follows: 'Socrates is not running, therefore some proposition is true.' For it follows: 'Socrates is running, therefore that Socrates is running is true,' because every proposition asserts itself to be true. Again, it follows: 'That Socrates is not running is true, therefore some proposition is true.' And the same is true of any negative proposition . . ."[44]

There is still another sense in which an affirmative proposition follows from a negative one: "Any negative proposition infers a disjunction of which the negative proposition is a component [i.e., *CNpANpq*]; for it follows: 'Socrates is not running, therefore Socrates is running or else Socrates is not running.' And this one is affirmative, and thus every negative proposition infers an affirmative."[45]

Some other interesting *consequentiae* are listed in the TL which are not found in the TB:

5.00 *CCpqCCrNqCrNp*

Quidquid repugnat consequenti repugnat antecedenti (Whatever contradicts the consequent contradicts the antecedent).[46]

From *5.00* and *3.80*, three other rules are derivable:

6.00 *CKCpqCrsCNKqsNKpr*

Si consequentia repugnent, antecedentia repugnant (If the consequents are incompatible, the antecedents are incompatible).[47]

Burleigh's reasoning behind *6.00* runs as follows: "If the consequents are incompatible [*NKqs*], then each consequent implies the negation of the other [*CqNs*; *CsNq*], and thus the negation of the antecedent [*CqNr*; *CsNp*]; as a result, antecedents are mutually incompatible [*KCpNrCqNr*]. And thus if the consequents are incompatible, the antecedents must be incompatible."[48]

6.10 *CKCpqCrsCKprKqs*

Si antecedentia stent simul, oportet quod eorum consequentia stent simul (If the antecedents are compatible, the consequents are compatible).[49]

6.20 *CCpqNKNqp*

In omni consequentia bona oppositum consequentis repugnat antecedenti (In every valid consequence the opposite of the consequent is repugnant to the antecedent).[50]

Burleigh proposes this last rule as a means of testing the validity of consequences:" . . . to find out whether a consequence is valid or not, one must check whether the contradictory of the consequent is repugnant to the antecedent or not; if it is, then the consequence is valid, if not, the consequence is not valid."[51] Thus we may add the converse

6.21 *CNKNqpCpq*

and the equivalence

6.22 *ECpqNKNqp*.

When speaking of this method of testing, Burleigh takes into account the fact that the consequent and the antecedent may be contradictorily or merely contrarily opposed, and he makes it a point to distinguish between inferring from an opposite of a consequent the contradictory of the corresponding antecedent *in a valid consequence* and making the same step to check *whether the consequence* is valid: "... it is not sufficient for the truth of a consequence that the opposite of the consequence contrarily opposes the antecedent, for then this would follow: 'Every man is running, therefore every animal is running', because the contrary of the consequent opposes the antecedent... Nevertheless I say, that any opposite of the consequent, of whatever sort of opposition it be, opposes the antecedent; the same, however, is not sufficient to show the validity of a consequence, namely, that any opposite of the consequent opposes the antecedent, but it is sufficient and necessary that the opposite of the consequent contradictorily opposes the antecedent."[52]

In the TL Burleigh "deduces" two corollaries from *3.00*:

7.00 *CCpqCCNprCNqr*

Quidquid sequitur ad oppositum antecedentis sequitur ad oppositum consequentis (Whatever follows from the contradictory of the antecedent follows from the contradictory of the consequent).[53]

The proof offered is: "If the consequence is valid, then from the opposite of the consequent the opposite of the antecedent follows [*CCpqCNqNp*, by *3.00*]; thus the opposite of the antecedent becomes consequent, and the opposite of the consequent becomes antecedent [*CNqNp*]; but whatever follows from the consequent follows from the antecedent [*CCpqCCqrCpr*, by *2.00*]; therefore, whatever follows from the opposite of the antecedent follows from the opposite of the consequent." Burleigh also points out that "if something followed from the opposite of the antecedent which would not follow from the opposite of the consequent, the original consequence would not be valid,"[54] i.e.,

7.10 *CKCNprNCNqrNCpq*

The second derived rule from *3.00* is

7.20 *CCpqCCrNqCrNp*

"The reason for this rule," he says, "is that the opposite of the consequent is the antecedent to the opposite of the antecedent [*CCpqCNqNp*, by *3.00*]; now whatever implies the antecedent implies the consequent [*CCpqCCrpCrq* by *2.10*]; therefore, whatever implies the opposite of the consequent implies the opposite of the antecedent." He adds that "if something implied the opposite of the consequent which did not imply the opposite of the antecedent, the original consequence would not be valid,"[55] which we may add to our list as

7.21 *CKCrNqNCrNpNCpq*

In the TB we find an interesting rule which makes it clear that Burleigh was aware of the distinction between the *use* and mention of linguistic symbols. This is the rule that concerns the distinction between the acts of predication, or affirmation and denial in general, and the naming or mentioning such acts, between the *actus exercitus* and *actus significatus.*

8.00 *Ad omnem actum exercitum sequitur actus significatus et econverso* (From an implicit act we may infer an explicit one, and conversely).[56]

Let '*a*' and '*b*' be names of the terms *a* and *b* respectively; and '*p*' and '*q*' be names of the propositions *p* and *q*. We can make then the following sorts of inferences:

Si (*a* est *b*) ('*b*' praedicatur de '*a*')
Si (*Cpq*) ('*p*' entails '*q*')

The verb 'est' exercises praedication, while the verb 'praedicatur' signifies the act of predication. Generally, the syncategorematic terms exercise the acts, while adjectives signify the corresponding acts. The quantifier 'every' in 'Every *a* is *b*' exercises the act of distribution, distributes, the subject term; while 'distributes' in the statement "'Every' distributes the subject term in 'Every *a* is *b*'" signifies the act of distribution. Similarly, 'if' in 'if *p* then *q*' states or posits the condition, while 'entails' in "'*p*' entails '*q*'" signifies a relation between two statements.

Yet, Burleigh is aware that this rule may in some cases be misleading since it may happen that the *actus exercitus* does not infer the corresponding *actus significatus*, and conversely: "For this proposition is true: 'The highest genus is truly predicated of a species'; yet this one is false: 'Species is the highest genus'. Nevertheless, *actus exercitus* does hold for that for which *actus significatus* holds, and vice versa: 'The highest genus is predicated of a species,' because 'substance' is predicated of 'man' and the proposition 'Man is a substance' is true."[57]

This study may be concluded with Burleigh's remarks on the categorical syllogistic rules which he made *at the very end of his tract* "De regulis generalibus consequentiarum" in the TB. Since this is all that Burleigh says on the categorical syllogistic, and since he accords it a place subordinated to the logic of unanalyzed propositions, I will quote the whole passage in P. Boehner's translation:

> After having spoken about the general rules for every consequence, a few special remarks on syllogistic consequence must be added. I say, therefore, that there are two general rules for every syllogism, no matter in which figure or mode they happen to be, that is, providing that the syllogism has one universal proposition and one affirmative proposition, since nothing follows syllogistically from either a particular or a negative proposition.

Besides these rules common to every figure, there are certain special rules for each figure. In the first figure there are two rules, viz. that in the modes concluding directly the major must be universal and the minor must be affirmative.

In the second figure there are other rules. One of these is that the major must be universal and either one of the propositions must be negative.

In the third figure there are two other rules, viz. the minor must always be affirmative and the conclusion particular. If this figure is executed in any other way, the syllogism is invalid.

These remarks about the consequences may suffice.[58]

Regardless of what one may think of the nature of logic, a glance at Burleigh will convince him that traditional logic is much more picturesque and much richer than some works on "traditional" logic might suggest.

NOTES

1. On this subject see P. Boehner, *Medieval Logic,* 1952; P. Boehner, "Does Ockham Know of Material Implications?" 1951; E. Moody, *The Logic of William of Ockham,* 1935; E. Moody, *Truth and Consequence in Medieval Logic,* 1952.

2. Walter Burleigh, DE PURITATE ARTIS LOGICAE, 1951; and DE PURITATE ARTIS LOGICAE TRACTATUS LONGIOR (With a Revised Edition of the Tractatus Brevior), 1955. The references in this paper will be to either or to both editions.

   The prologue to the TB gives a promise of a treatment of the following topics: 1. General Rules [a) general rules of consequences, b) on the nature of syncategorematic terms, c) on the supposition of terms]; 2. On the Sophistic Rules; 3. On the Art of Obligation; 4. On the Art of Demonstration. However, only 1a) and 1b) are actually given.

   TL, on the other hand, begins with a discussion of properties of terms (supposition, appellation, copulation) and continues with the tracts on hypothetical [conditional] propositions and syllogisms—where 1a) of the TB is partly repeated—and on other hypothetical syllogisms. It is the contention of P. Boehner that the two works are not abbreviations or summaries but incomplete versions of a single logical *summa.* Cf. his "Introduction" to either edition.

3. See P. Boehner's "Introduction" to either edition of the DE PURITATE ARTIS LOGICAE for the significance of the fact that the "realistic" and the "nominalistic" logics may not be so different after all.

4. "Consequentiarum quaedam est simplex, quaedam est ut nunc. Consequentia simplex est ista, quae tenet pro omni tempore, ut: 'Homo currit, igitur animal currit.' Consequentia ut nunc tenet pro tempore

determinato et non semper, ut: 'Omnis homo currit, igitur Sores currit'; illa enim consequentia non tenet semper, sed solum dum Sortes est homo." TB, p. 1. According to E. Moody, the distinction between "simple" and "as of now" consequence corresponds to Diodorean and Philonian conceptions of implication respectively. See his *Truth and Consequence in Medieval Logic*, p. 75. See also Sextus Empiricus, Loeb Library, Vol. 2, pp. 113, 115, in which the information concerning much of Stoic logic is preserved. See further B. Mates' *Stoic Logic*, 1953, for an account of the heated discussions on the nature of implication in ancient times.

If Moody's contention on this point is correct, then we may not interpret Cpq as $p \prec q$ even when "simple" consequence is definitely intended, for Diodorean implication seems to be stronger than Lewis' strict implication. Compare the latter with the former: $p \prec q \,.\, = \,.\, \sim\Diamond (p \,.\, \sim q)$; and $p \rightarrow q \,.\, = \,.\, (t) \,.\, p(t) \supset q(t)$.

5. TL, 61.
6. *Ibid.*
7. TB, 1; see also TL, 61.
8. TB, 1; TL, 61.
9. TL, 61.
10. TB, 1; TL, 61.
11. TL, 61.
12. *Ibid.*
13. TB, 1f.; TL, 62.
14. TB, 2; TL, 62.
15. TB, 2; see also TL, 62 and, for objections to the rule and answers to them, pp. 79f. and 81ff.
16. TB, 2; TL, 62.
17. *Ibid.*
18. TB, 2. This law belonging to the logic of propositions was known in the Stoic-Megaric school and had been anticipated by Aristotle and Theophrastus as a law of quantificational logic where it served to extend the Principle of Syllogism (Cf. Lewis *11.7*) from the case of two antecedents to the case of n antecedents (Cf. Lewis *12.78*).
19. TB, 3.
20. TB, 3f.
21. TB, 1951 edition, p. 4, reads here: "Quidquid sequitur ex consequente et ex antecedente, sequitur ex consequente per se," which, as A. N. Prior shows, is invalid. Cf. his "On Some *Consequentiae* in Walter Burleigh," *The New Scholasticism*, 27 (1953), pp. 433-446. In the revised edition of TB (1955) the rule has been corrected: "Quidquid

sequitur ex consequente et ex antecedente, sequitur ex antecedente per se" (TL, p. 203). The statement of *2.30* as given in the text of this paper is that of TL, p. 62.

22. TB, 5; TL, 62.

23. TB, 5f; TL, 62.

24. TB, 6; TL, 62.

25. "On Some Consequentiae in Walter Burleigh," *The New Scholasticism*, 27 (1953), pp. 436f. Prior gives a similar proof for *2.40*. He also observes that "although *Principia Mathematica* has all of the propositions *CCpqCCqrCpr* (**2.06)*. *CCpqCpKpq* (strengthened to an equivalence, **4.7*) and *CCpqCKprKqr* (Peano's principle of the factor' **3.45*) [i.e. theses *2.00*, *2.31* and *3.80* of the present paper], it has neither of the more complex proposition which Burleigh proves from them" [namely, *2.30* and *2.40* of this paper], pp. 438f.

26. TB, 7; see also TL, 81.

27. TB, 8.

28. TB, 8f.

29. TL, 64f.

30. TB, 8.

31. TB, 9; see also TL, 65, 80-1, and 86 for the objections to the rule and Burleigh's replies.

32. TB, 9; TL, 65. Burleigh evidently has in mind the type of syllogistic rules which Aristotle had utilized to prove the validity of certain syllogistic forms by the *reductio ad impossible* method. Cf., J. Łukasiewicz, *Aristotle's Syllogistic*, (2nd ed.), 1957, and I. M. Bocheński, *Ancient Formal Logic*, 1957. A similar rule is found in the ancient writer Apuleius (fl. 150 AD): "If the drawing of any conclusion be challenged, and either of the two propositions be granted, the other will be denied." Cf. R. Houde, *Readings in Logic*, 1957, p. 175.

    On the level of propositional logic, however, we find several of the laws *3.50-3.62* stated by Stoics. Cf. Bocheński, *op. cit.*, on Stoic-Megaric logic. The rules analogous to the following laws are listed: *CCKpqrCKNrqNp*, *CCKpqrCKpNrNq;* as well as a derived mode *CKCKpqrNrNq*. See also B. Mates, *Stoic Logic*, 1953.

33. TL, 63, 82, and 87.

34. TB, 9.

35. TB, 10.

36. *Ibid.*

37. P. Boehner, in his Introduction to *De puritate artis logicae* says that the first formulation of these laws "has still to be credited to Ockham." p. xiii. See Ockham's *Summa Logicae*, II, c. 32, where he states: "Opposita contradictoria copulativae est una disiunctiva composita ex

contradictoriis partium copulativae"; and in c. 33: "Opposita contradictoria disiunctivae est una copulativa composita ex contradictoriis partium ipsius disiunctivae." At least one other medieval logician, viz., Albert of Saxony, stated the same equivalences. Cf. *Perutilis Logica* (Venice, 1522), III, 5. Quotations may be found in E. Moody, *op. cit.*, p. 41, footnote**.

38. TB, 11.

39. *Loc. cit.*

40. *Loc. cit.*

41. *Loc. cit.*

42. TB, 15.

43. *Ibid.*

44. *Ibid.*

45. TB, 15f.

46. TL, 63.

47. TL, 63. Burleigh adds a defence of it: "Et ratio huius est, quia quod repugnat consequenti, destruit consequens et destructo consequente destruitur antecedens, et quod destruit antecedens, repugnat antecedenti, ideo quidquid repugnat consequenti, repugnat antecedenti." *Ibid.*

48. *Loc. cit.*

49. *Loc. cit.*

50. *Loc. cit.*

51. TL, 64.

52. *Ibid.*

53. TL, 65.

54. *Ibid.*

55. TL, 65f.

56. TB, 21.

57. TB, 21f.

58. *Medieval Logic*, p. 88. P. Boehner comments on the position of the tract on consequences, and on the place of this passage in the treatise on logic: "For the first time in medieval logic—as far as we know—a logician places the tract on consequences, which in turn contains syllogistics as a minor part, at the beginning of his system of logic." *Ibid.*, p. 89. Here, "syllogistics are swallowed up, as it were, into a tract which is considered more basic. This tract is the theory of consequences." p. 88.

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