A NOTE TO MY PAPER: ON CHARACTERIZATIONS OF THE FIRST-ORDER FUNCTIONAL CALCULUS

JULIUSZ REICHBACH

In [1] I have presented two characterizations of theses of the first-order functional calculus; the first characterization may be modified in the following way:[1]

D.0. $Q(k) . \equiv . Q$ is a non-empty set of tables of the rank k.

D.1. $Q/T, i_1, \ldots, i_m . \equiv . (\exists T_1)(\exists j_1) \ldots (\exists j_m) \{(T_1 \in Q) \wedge ([T_1|j_1, \ldots, j_m] = [T|i_1, \ldots, i_m])\}$.

$Q/T, i_1, \ldots, i_m$ asserts that $[T|i_1, \ldots, i_m]$ is a submodel of some $T_1 \in Q$ in the meaning of homomorphism.

D.2. $T, Q/T_1, i_1, \ldots, i_m; i . \equiv . ([T|i_1, \ldots, i_m] = [T_1|i_1, \ldots, i_m]) \wedge Q/T_1, i_1, \ldots, i_m, i$.

D.3. $Q\{r,k\} . \equiv . (r \leq k) \wedge Q(k) \wedge (i_1) \ldots (i_{m+1}) (T) \{(m < r) \wedge (i_1, \ldots, i_{m+1}$ are different numbers $\leq k) \wedge Q/T, i_1, \ldots, i_m \wedge Q/T, i_{m+1} \rightarrow (\exists T_1) (T, Q/T_1, i_1, . i_m; i_{m+1} \wedge (j_1) \ldots (j_{m-1}) \{(j_1, \ldots, j_s$ is a subsequence of $i_1, \ldots, i_m) \wedge Q/T, j_1, \ldots, j_s, i_{m+1} \rightarrow ([T_1|j_1, \ldots, j_s, i_{m+1}] = [T|j_1, \ldots, j_s, i_{m+1}])\})\}$.

The meaning of *D.2.* and *D.3.* is clear, see *D.1.*

For an arbitrary T of the rank k, for an arbitrary Q such that $Q(k)$ and for an arbitrary formula E whose indices of free variables occurring in it are $\leq k$, we introduce the inductive definition of the functional V:

(*1d*) $V\{T, Q, f_j^m(x_{r_1}, \ldots, x_{r_m})\} = 1 . \equiv . F_j^m(r_1, \ldots, r_m)$,

(*2d*) $V\{T, Q, F'\} = 1 . \equiv . \sim V\{T, Q, F\} = 1 . \equiv . V\{T, Q, F\} = 0$,

(*3d*) $V\{T, Q, F + G\} = 1 . \equiv . V\{T, Q, F\} = 1 \vee V\{T, Q, G\} = 1$,

(*4d*) $V\{T, Q, \Pi a F\} = 1 . \equiv . (i)(T_1) \{(i \leq k) \wedge T, Q/T_1, i_1, \ldots, i_{w(F)}; i \rightarrow V\{T_1, Q, F(x_i/a)\} = 1\}$.

D.4. $E \in P(Q) . \equiv . (T) \{(H) \{(H \in A\{E\}) \rightarrow Q/T, i_1, \ldots, i_{w(H)}\} \rightarrow V\{T, Q, E\} = 1\}$.

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D.5. $E \epsilon P|r, k| \,.\equiv.\, (Q)\{Q\{r, k\} \rightarrow (E \epsilon P(Q))\}$.
D.6. $E \epsilon P \,.\equiv.\, E \epsilon P\{n(E), \Sigma(E)\}$.

The meaning of the above definitions is analogous to the given in [1]. Analogously to the proof given in [1] we may also prove that *P* is the class of all theses: the whole proof is given in [2].

The theorem remains true, if in *D*.3. we assume $k = r$, $m + 1 = r$, but then in the definitions (*1d*) - (*4d*), *D*.4.-6. the table *T* has not the rank of elements of *Q*, but has the rank $n(E)$ and elements of *Q* have the rank $\Sigma(E)$, see [3]. Ones give an interesting connection between the considered calculus and $\aleph_0$ propositional calculus, see [4].

NOTE

1. We use the notation given in [1].

BIBLIOGRAPHY

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Tel Aviv, Israel