INDEPENDENCE OF FARIS-REJECTION-AXIOMS

IVO THOMAS

[1] questions the independence of the rejection-axioms in [2]. This system for non-void classes, based on the primitive expressions: 1xy (x and y are co-extensive), 2xy (x is properly included in y), 3xy (x and y include a common subclass and each a distinct subclass), 5xy (x and y have no common subclass), was shown equivalent to the syllogistic of [3] in [4] where some alternative assertion-axioms were given. The non-independence of the original set of assertion-axioms is proved in [5]. The resulting, independent set, with original numbering, is:

1. $1aa$ 3. $C1abC3cb3ac$ 4. $C1abC2bc2ac$ 5. $C1abC5cb5ac$

6. $C2abC2bc2ac$ 7. $C2abC5bc5ac$ 8. $CN1abCN2abCN3abCN2ba5ab$

9. $C1abKN2abKN3abN5ab$ 10. $C3abKN2abN5ab$

The rejection-axioms, which will here be proved independent, are:

51. $C2abN2bc$ 52. $C2abN5bc$ 53. $C2abC3bcN2ac$

54. $C2abC3bcN3ac$ 55. $C2abC3bcN5ac$ 56. $C2abC2cbN5ac$

57. $C3abC2bcN2ac$ 58. $C3abC3bcN3ac$ 59. $C3abC3bcN5ac$

60. $C3abC5bcN5ac$ 61. $C5abC5bcN5ac$

Besides the basic rules of rejection usual for such systems, viz. from ⊣Y and ⊣CXY to infer ⊣X, and, from ⊣Y, to infer ⊣X when Y is a substitution in X, there is a special rule (RG), discussion of which is reserved till later.

The method adopted is to transfer ⊣$-n$ from the rejection- to the assertion-axioms and find an interpretation which (always) verifies the newly augmented assertion axioms and (sometimes) falsifies the remaining rejects. In every case we shall use a subdomain of the general domain for which the system is intended, thus ensuring continued verification of the original assertion-axioms and applicability of the rules. In Tables I and II below, each capital letter represents a class exclusive of all the others, juxtaposition expressing the logical sum. For each ⊣$-n$ transferred to the assertion axioms we use one or other of the tables less line n, and the domain of interpretation is precisely the other classes that thus come to be tabled. Table I is used for ⊣51, ⊣53 – ⊣59; Table II for ⊣52, ⊣60 and ⊣61. In each table

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line n gives values for a,b,c which falsify $\dashv -n$. We shall say that X is k to Y when X and Y are values from the domain, k is a functor '2', '3' or '5' and kXY is true.

TABLE I

	a	b	c
51	A	AB	ABC
52	D	DE	F
53	G	GH	GI
54	JK	JKL	JLM
55	N	NO	OP
56	Q	QR	R
57	ST	SU	STU
58	VW	WX	VX
59	VW	WX	XY
60	JKL	JLM	A
61	A	D	G

$\vdash$51. Remove line 51. To falsify 51, the value for b will have to come from the boxed values, but none of these are 2 to any value.

$\vdash$53 Remove line 53. Again the value of b must come from the boxed values, but the only ones 3 to some value are JKL, NO. The antecedents can only be satisfied by a/JK, b/JKL, c/JLM or a/N, b/NO, c/OP but in neither case is $2ac$ satisfied.

$\vdash$54 Remove line 54. The antecedents can only be satisfied by a/G, b/GH (or GI), c/GI (or GH), or a/N, b/NO, c/OP, but in no case is $3ac$ satisfied.

$\vdash$55. Remove line 55. The antecedents can only be satisfied by a/G, b/GH (or GI), c/GI (or GH), or a/JK, b/JKL, c/JLM, but in no case is $5ac$ satisfied.

$\vdash$56. Remove line 56. To falsify, we need a value for b to which two different values are 2. ABC,STU are the only possibilities, but neither A, AB nor ST, SU are 5 to each other.

$\vdash$57. Remove line 57. To falsify, we need a value for c to which two different values are 2. The only possibilities are ABC, and QR. But neither A, AB nor Q, R are 3 to each other.

$\vdash$58. Remove line 58. There are no values 3 in pairs.

$\vdash$59. Remove line 59. The only values satisfying $3ab$, $3bc$ ($a \neq b \neq c$), are those in lines 54 and 58 but no two such are 5 to each other.

TABLE II

	a	*b*	*c*
51	ADE	ABCDE	ABCDEG
52	ABC	ABCG	DEF
53	ADE	ABCDEG	ABCDEH
54	ABCFGH	ABCFGHI	ADEGHJ
55	ACDF	ABCDF	BEN
56	AEFK	ABCDEFKN	BCDN
57	ACDEF	ACDFN	ABCDEFN
58	ACDEF	ACDEG	AFLMN
59	ACEFL	ADE	BDKMN
60	AFN	ADE	BCGKLJ
61	AFN	BDKL	CEM

⊢52. Remove line 52. To falsify, the value for *b* must come from the boxes, but none are 5 to any value.

⊢60. Remove line 60. To falsify, we need a value for *c* which is 5 to two different values which are 3 to each other. Again the boxed values are 5 to no value. Of the rest:
(i) ADE is 5 to no value;
(ii) ABC is 5 only to DEF;
(iii) ABCG is 5 only to DEF;
(iv) DEF is 5 only to ABC, ABCG but these are not 3 to each other;
(v) ACDF is 5 only to BEN, and conversely;
(vi) AEFK is 5 only to BCDN, and conversely;
(vii) ACEFL is 5 only to BDKMN, and conversely;
(viii) AFN, BDKL and CEM are 5 in pairs, but thus no two are 3 to each other, and none is 5 to any value outside the trio.
This exhausts the domain.

⊢61. Remove line 61. To falsify, we need three values 5 in pairs. As in the last proof, the boxed values are useless and (ii), (iii), (v)–(vii) still hold. Of the remaining values:
ADE is 5 only to BCGKLJ;
DEF is 5 to ABC, ABCG, BCKLJ but no two of these are 5 to each other;
AFN is 5 only to BCGKLJ;
BCGKLJ is 5 to AFN, ADE, DEF and these alone, but no two of these are 5 to each other. This exhausts the domain.

(RG). This result shows that the rejection-rule (RG), which will not be re-stated here, has a hitherto unremarked point of interest in that it is in a certain sense weaker than its syllogistic analogue in [6]. Since the two systems are inter-translatable, and a Faris-expression is asserted or rejected if and only if its syllogistic version is asserted or rejected, it is evident that a Faris-translation of Słupecki's rule and the sole syllogistic rejection-axiom would constitute a sufficient rejection-basis for this system. But the Faris-version of the axiom is inferentially equivalent, by the assertion-rules alone, to ⊣56, so that on this alternative basis the other rejection-axioms become superfluous.

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University of Notre Dame
Notre Dame, Indiana