

ON THE INDEPENDENCE OF CERTAIN AXIOMS IN THE DEFINITION
 OF AN m -ARRANGEMENT

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It was shown in [1] that of the axioms defining an m -arrangement, i.e. **3.1-3.9**, **3.4**, **3.5** and **3.9** are independent. The purpose of this note is to show that after certain trivial modifications, **3.1**, **3.2** and **3.3** are also independent. **3.7** and **3.8** are known to be independent as a pair [1]. Nothing is yet known about the independence of **3.6**; indeed, this particular problem seems to be extremely complex.

The following modifications of **3.1-3.9** are called for. They in no way affect any of the proof in [1].

3.5: If x, y and z are distinct points of a 1-flat and if \overline{xy} , \overline{xz} and \overline{yz} all exist, then $\overline{xy} \cup \overline{yz} = \overline{xy}$, \overline{yz} , or \overline{xz} .

3.6: If $S = \{x_0, \dots, x_k\}$ is linearly independent and $k \geq 2$, then . . .

3.7: If $C(S)$ is a k -simplex, $k \geq 2$, . . .

3.8: If $C(S)$ is a k -simplex, $k \geq 2$, . . .

If **3.1-3.5** hold, then **3.6-3.8** are theorems for $k = 1$.

The independence of 3.1: Let $X = [0, 1]$, the closed interval between 0 and 1 with its usual topology. Set $F^0 = \{\{x\} \mid x \in (0, 1)\} \cup \{\{0, 1\}\}$, $G = \{F^{-1}, F^0\}$.

The independence of 3.2: Since the definition of a topological geometry involves two distinct assumptions, two counterexamples are required:

a) Each flat is closed: Let $X = \{a, b, c\}$ with the topology $\{\{a, b, c\}, \{a, b\}, \{b, c\}, \{b\}, \emptyset\}$. Set $F^0 = \{\{x\} \mid x \in X\}$, $G = \{F^{-1}, F^0\}$.

b) The intersection of any family of convex sets is convex: Let $X = \{(x, y) \mid x^2 + y^2 = 1\} \subset \mathbb{R}^2$ with the usual topology. Set $F^0 = \{\{x\} \mid x \in X\}$, $G = \{F^{-1}, F^0\}$.

The independence of 3.3: Let X be the set of real numbers with the discrete topology. Set $F^0 = \{\{x\} \mid x \in X\}$, $G = \{F^{-1}, F^0\}$.

REFERENCE

- [1] M. C. Gemignani; "Topological Geometries and a New Characterization of \mathbf{R}^m ", *Notre Dame Journal of Formal Logic*, V. VII (1966), pp. 57-100.

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