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A NOTE ON AN AXIOM-SYSTEM OF ATOMISTIC MEREOLOGY

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In [2] and [3]* the system of atomistic mereology with "**el**" as its single primitive functor is based on two axioms. Namely,

$A \quad [AB]:\cdot:A\,\varepsilon\,\mathbf{el}(B) \,.\equiv:\,:B\,\varepsilon\,B:\,:[Ta]:\,:[C]\,.\therefore C\,\varepsilon\,T\,.\equiv:[B]:B\,\varepsilon\,a\,.\supset.$
$B\,\varepsilon\,\mathbf{el}(C):[B]:B\,\varepsilon\,\mathbf{el}(C)\,.\supset.[\exists EF]\,.E\,\varepsilon\,a\,.F\,\varepsilon\,\mathbf{el}(E)\,.F\,\varepsilon\,\mathbf{el}(B)\,\therefore$
$B\,\varepsilon\,\mathbf{el}(B)\,.B\,\varepsilon\,a\,.\supset.A\,\varepsilon\,\mathbf{el}(T)$

which is Lejewski's single axiom of general mereology, *cf.* [2], section **2**, and the additional atomistic axiom:

$V \quad [A]:\,:A\,\varepsilon\,A\,.\supset\therefore[\exists B]\,\therefore B\,\varepsilon\,\mathbf{el}(A):[C]:C\,\varepsilon\,\mathbf{el}(B)\,.\supset.C = B$

Since in the field of general mereology the following formula which is shorter than axiom A:

$B \quad [AB]:\cdot:A\,\varepsilon\,\mathbf{el}(B)\,.\equiv:\,:B\,\varepsilon\,B:\,:[Ta]:\,:[C]\,.\therefore C\,\varepsilon\,T\,.\equiv:[B]:B\,\varepsilon\,a\,.\supset.B\,\varepsilon\,\mathbf{el}$
$(C):[B]:B\,\varepsilon\,\mathbf{el}(C)\,.\supset.[\exists EF]\,.E\,\varepsilon\,a\,.F\,\varepsilon\,\mathbf{el}(E)\,.F\,\varepsilon\,\mathbf{el}(B)\,\therefore B\,\varepsilon\,a\,.\supset.$
$A\,\varepsilon\,\mathbf{el}(T)$

holds, as an inspection of the proofs of *P10* and *P11* from [3], section **4.2**, can show easily, an occurrence of a subformula "$B\,\varepsilon\,\mathbf{el}(B)$" in A is rather irritating. But, up to now any endeavor to substitute A by B, as a single axiom of mereology, failed. In this note it will be proved that in the axiom-system of atomistic mereology which is presented above axiom A can be substituted by B.

Proof: Let us assume B and V. Then:

$A1 \quad [AB]:A\,\varepsilon\,\mathbf{el}(B)\,.\supset.B\,\varepsilon\,B$ $\qquad [B]$
$Z1 \quad [ABa]\,\therefore B\,\varepsilon\,a:[B]:B\,\varepsilon\,a\,.\supset.B\,\varepsilon\,\mathbf{el}(A):\supset.A\,\varepsilon\,A$ $\qquad [A1]$
$D1 \quad [Aa]\,\therefore A\,\varepsilon\,A:[B]:B\,\varepsilon\,a\,.\supset.B\,\varepsilon\,\mathbf{el}(A):[B]:B\,\varepsilon\,\mathbf{el}(A)\,.\supset.[\exists EF]\,.E\,\varepsilon\,a\,.$
$F\,\varepsilon\,\mathbf{el}(E)\,.F\,\varepsilon\,\mathbf{el}(B):\equiv.A\,\varepsilon\,\mathbf{Kl}(a)$

*An acquaintance with [2] and [3] is presupposed. An enumeration of the theorems which are appearing in this note, except for *B*, *Z1*, *Z2* and *Z3*, is the same as in those papers.

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Z2 $[ABa]:A\varepsilon\mathbf{el}(B).B\varepsilon a.\supset.A\varepsilon\mathbf{el}(\mathbf{Kl}(a))$

PR $[ABa]::\mathrm{Hp}(2).\supset\therefore$

3\. $[C]\therefore C\varepsilon\mathbf{Kl}(a).\equiv:[B]:B\varepsilon a.\supset.B\varepsilon\mathbf{el}(C):[B]:B\varepsilon\mathbf{el}(C).\supset.[\exists EF].E\varepsilon a.F\varepsilon\mathbf{el}(E).F\varepsilon\mathbf{el}(B)\therefore$ [*T1*; *D1*; *Z1*; 2]

$A\varepsilon\mathbf{el}(\mathbf{Kl}(a))$ [*B*; 1; 3; 2]

A8 $[Aa]:A\varepsilon a.\supset.A\varepsilon\mathbf{el}(\mathbf{Kl}(a))$

PR $[Aa]\therefore\mathrm{Hp}(1).\supset:$

$[\exists B].$

2\. $B\varepsilon\mathbf{el}(A).$ [*T1*; *V*; 1]

3\. $B\varepsilon\mathbf{el}(\mathbf{Kl}(a)).$ [*Z2*; 2; 1]

4\. $\mathbf{Kl}(a)\varepsilon\mathbf{Kl}(a):$ [*A1*; 3]

5\. $[B]:B\varepsilon a.\supset.B\varepsilon\mathbf{el}(\mathbf{Kl}(a)):$ [*D1*; 4]

$A\varepsilon\mathbf{el}(\mathbf{Kl}(a))$ [5; 1]

A3 $[Aa]:A\varepsilon a.\supset.[\exists B].B\varepsilon\mathbf{Kl}(a)$ [*A8*; *A1*]

Z3 $[ATa]::[C]\therefore C\varepsilon T.\equiv:[B]:B\varepsilon a.\supset.B\varepsilon\mathbf{el}(C):[B]:B\varepsilon\mathbf{el}(C).\supset.[\exists EF].E\varepsilon a.F\varepsilon\mathbf{el}(E).F\varepsilon\mathbf{el}(B)\therefore A\varepsilon a.\supset.A\varepsilon\mathbf{el}(T)$

PR $[ATa]::\mathrm{Hp}(2).\supset\therefore$

3\. $[C]\therefore C\varepsilon\mathbf{Kl}(a).\equiv:[B]:B\varepsilon a.\supset.B\varepsilon\mathbf{el}(C):[B]:B\varepsilon\mathbf{el}(C).\supset.[\exists EF].E\varepsilon a.F\varepsilon\mathbf{el}(E).F\varepsilon\mathbf{el}(C)\therefore$ [*T1*; *D1*; *Z1*; 2]

4\. $[C]:C\varepsilon\mathbf{Kl}(a).\equiv.C\varepsilon T:$ [1; 3]

5\. $A\varepsilon\mathbf{el}(\mathbf{Kl}(a)).$ [*A8*; 2]

$A\varepsilon\mathbf{el}(T)$ [*E2*; 4; 5]

A5 $[A]:A\varepsilon A.\supset.A\varepsilon\mathbf{el}(A)$ [*B*; *Z3*]

A4 $[ABa]:A\varepsilon\mathbf{Kl}(a).B\varepsilon\mathbf{Kl}(a).\supset.A=B$

PR $[ABa]:\mathrm{Hp}(2).\supset.$

3\. $A\varepsilon\mathbf{el}(A).$ [*T1*; *A5*; 1]

$[\exists E].$

4\. $E\varepsilon a.$ [*D1*; 1; 3]

5\. $E\varepsilon\mathbf{el}(\mathbf{Kl}(a)).$ [*A8*; 4]

6\. $\mathbf{Kl}(a)\varepsilon\mathbf{Kl}(a).$ [*A1*; 5]

7\. $A=\mathbf{Kl}(a).$ [*T2*; 1; 6]

8\. $B=\mathbf{Kl}(a).$ [*T2*; 2; 6]

$A=B$ [7; 8]

A2 $[ABC]:A\varepsilon\mathbf{el}(B).B\varepsilon\mathbf{el}(C).\supset.A\varepsilon\mathbf{el}(C)$

PR $[ABC]\therefore\mathrm{Hp}(2).\supset:$

3\. $C\varepsilon C.$ [*A1*; 2]

4\. $C\varepsilon\mathbf{el}(C):$ [*A5*; 3]

5\. $[V]:V\varepsilon\mathbf{el}(C).\supset.[\exists EF].E\varepsilon\mathbf{el}(C).F\varepsilon\mathbf{el}(E).F\varepsilon\mathbf{el}(V):$ [*T1*; *A5*; 4]

6\. $C=\mathbf{Kl}(\mathbf{el}(C)).$ [*T3*; *D1*; *A4*; 3; 5]

7\. $A\varepsilon\mathbf{el}(\mathbf{Kl}(\mathbf{el}(C))).$ [*Z2*; 1; 2]

$A\varepsilon\mathbf{el}(C)$ [*E1*; 6; 7]

Since, *cf.* [2], section **2.1**, $\{A1, A2, D1, A3, A4\} \rightleftarrows \{A\}$, the proof is complete. It should be remarked that without V we do not know how to obtain $A5$ which is indispensable in order to deduce $A2$, $A3$ and $A4$ from B. On the other hand, $A5$ follows from $A1$, $A2$, $D1$, $A3$ and $A4$, as it has been shown by Clay in [1].

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